

Chapters 35 & 36



위조지폐 방지 장치



35장 간섭

p. 1183



- 물리학에서 추구하는 중요한 목표 중의 하나
→ 빛의 본성을 이해 (입자성, 파동성)
- 광학적 간섭
→ 빛의 파동성의 대표적 현상



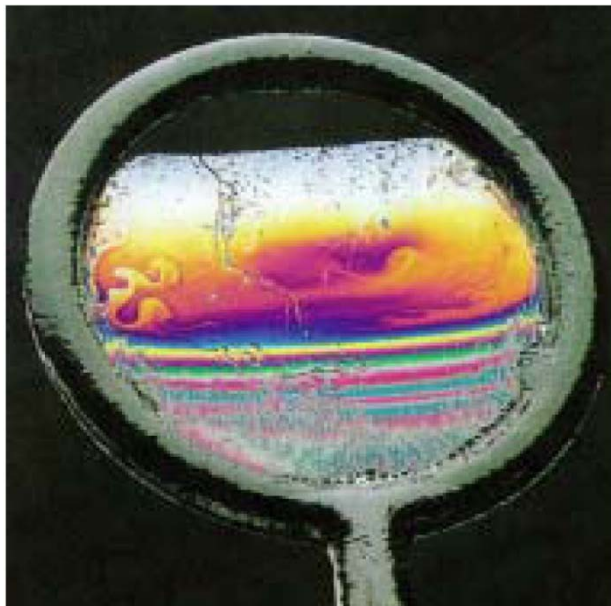
간섭; interference



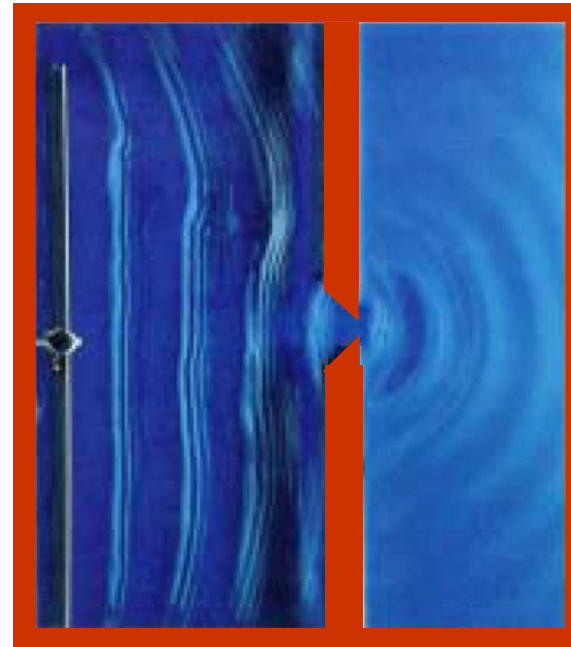
파동으로서의 빛

1. 용어의 정의

- 간섭: 둘 이상의 빛살(파동)이 겹쳐질 때 나타나는 밝기(강도)변화
- 회절: 기하광학의 법칙으로 설명할 수 없는 빛살(파동)의 전파현상



간섭 현상
(interference)



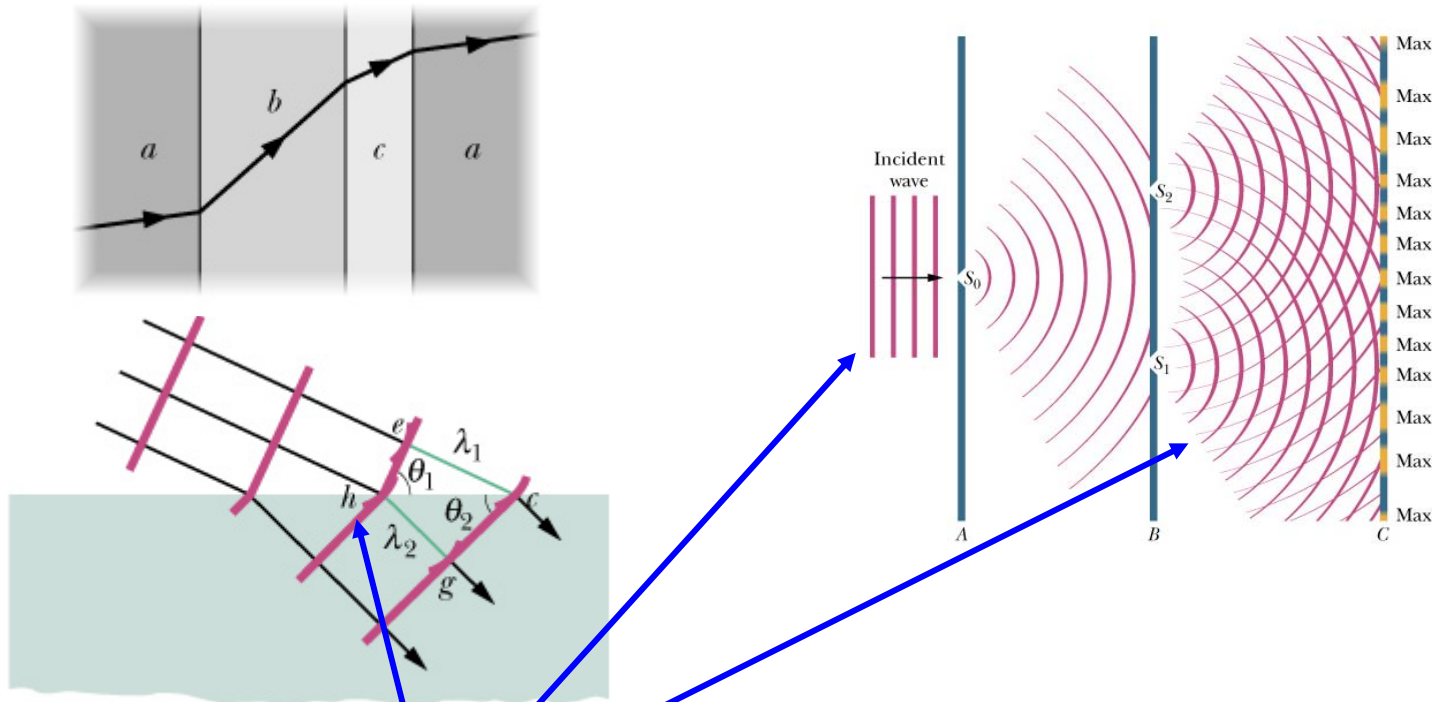
회절 현상
(diffraction)





- 기하광학

- 파동광학



※ 파동광학의 기본개념: 파면 $\Leftrightarrow$ 기하광학의 기본개념: 빛살(광선)

※ 빛살 $\perp$ 파동면 (wavefront)

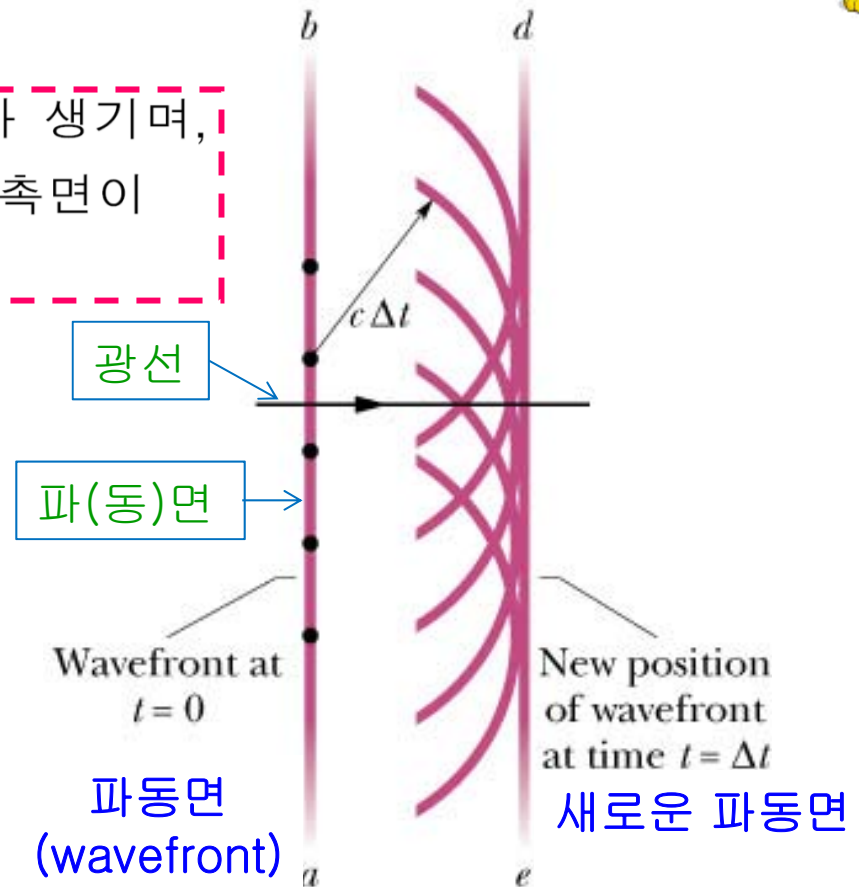




2. Huygens 원리 (1678)

"파면의 모든 점에서 제2차 구면파가 생기며,
퍼져나간 제2차 구면파에 대한 접촉면이
새로운 파면이 된다."

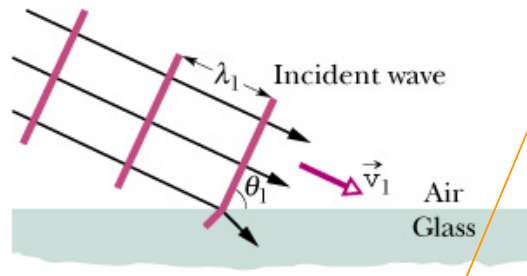
빛살 $\perp$ 파면



Huygens 원리에 따라 진공중
에서 평면파가 전파하는 모습



3. 호이겐스의 원리로 설명한 굴절법칙



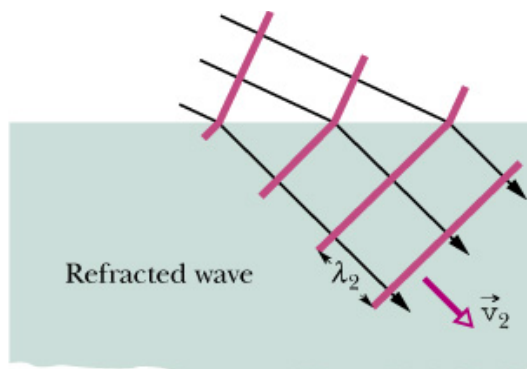
$$\tau = \frac{\lambda_1}{v_1}; \tau = \frac{\lambda_2}{v_2} \Rightarrow \frac{\lambda_n}{v_n} = \frac{\lambda}{c} \rightarrow \lambda_n = \lambda \times \frac{v_n}{c}$$

파동의 전파속도 v 와 굴절율 n

$$\text{공기: } v_1 = \frac{c}{n_1}$$

$$\text{유리: } v_2 = \frac{c}{n_2}$$

c : 진공에서의 빛의 속도



(c)

굴절법칙

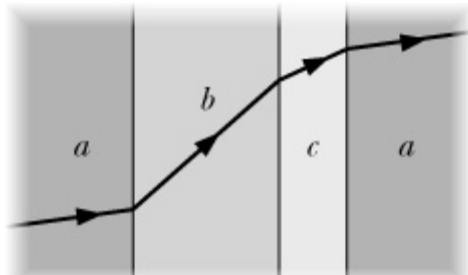
$$\Rightarrow \frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1 \Delta t}{v_2 \Delta t} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}$$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$



4. 매질의 굴절율 n 과 매질 속에서의 파장

(※ 진동수 f 는 바뀌지 않음)



a, b, c 중에 광속이
가장 빠른 영역은?

$$\tau = \frac{\lambda_1}{v_1}; \tau = \frac{\lambda_2}{v_2}$$

$$\Rightarrow \frac{\lambda_n}{v_n} = \frac{\lambda}{c} \equiv \frac{1}{f}$$

$$c = \lambda f$$

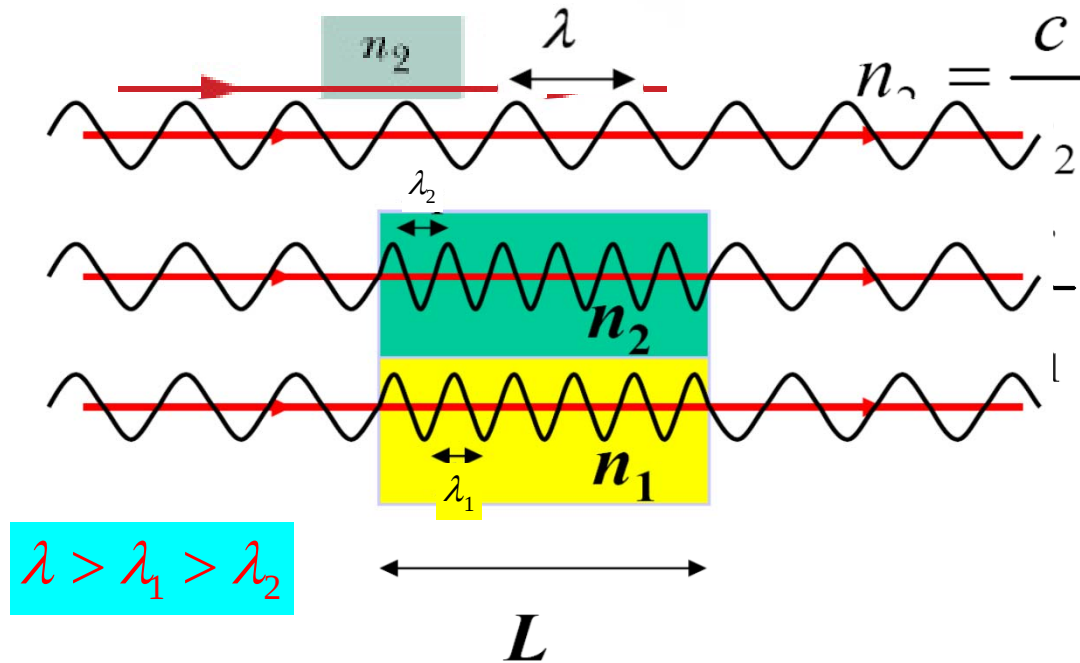
$$v_n = \frac{c}{n} = \lambda_n f$$

따라서

$$\lambda_n = \lambda \cdot \frac{v_n}{c} = \frac{\lambda}{n}$$



● 굴절률이 다른 매질을 통과한 두 파동의 위상차



$$n_2 > n_1$$

$$t = \frac{\lambda}{c} = \frac{\lambda_1}{v_1}$$

$$\lambda_1 = \lambda \frac{v_1}{c} = \frac{\lambda}{n_1}$$

❖ 주어진 거리 L 을 지나는 데 포함된 파장 수

$$N_1 = \frac{L}{\lambda_1} = \frac{Ln_1}{\lambda}$$

$$N_2 = \frac{L}{\lambda_2} = \frac{Ln_2}{\lambda}$$

$$N_2 - N_1 = \frac{Ln_2}{\lambda} - \frac{Ln_1}{\lambda} = \frac{L}{\lambda} (n_2 - n_1)$$

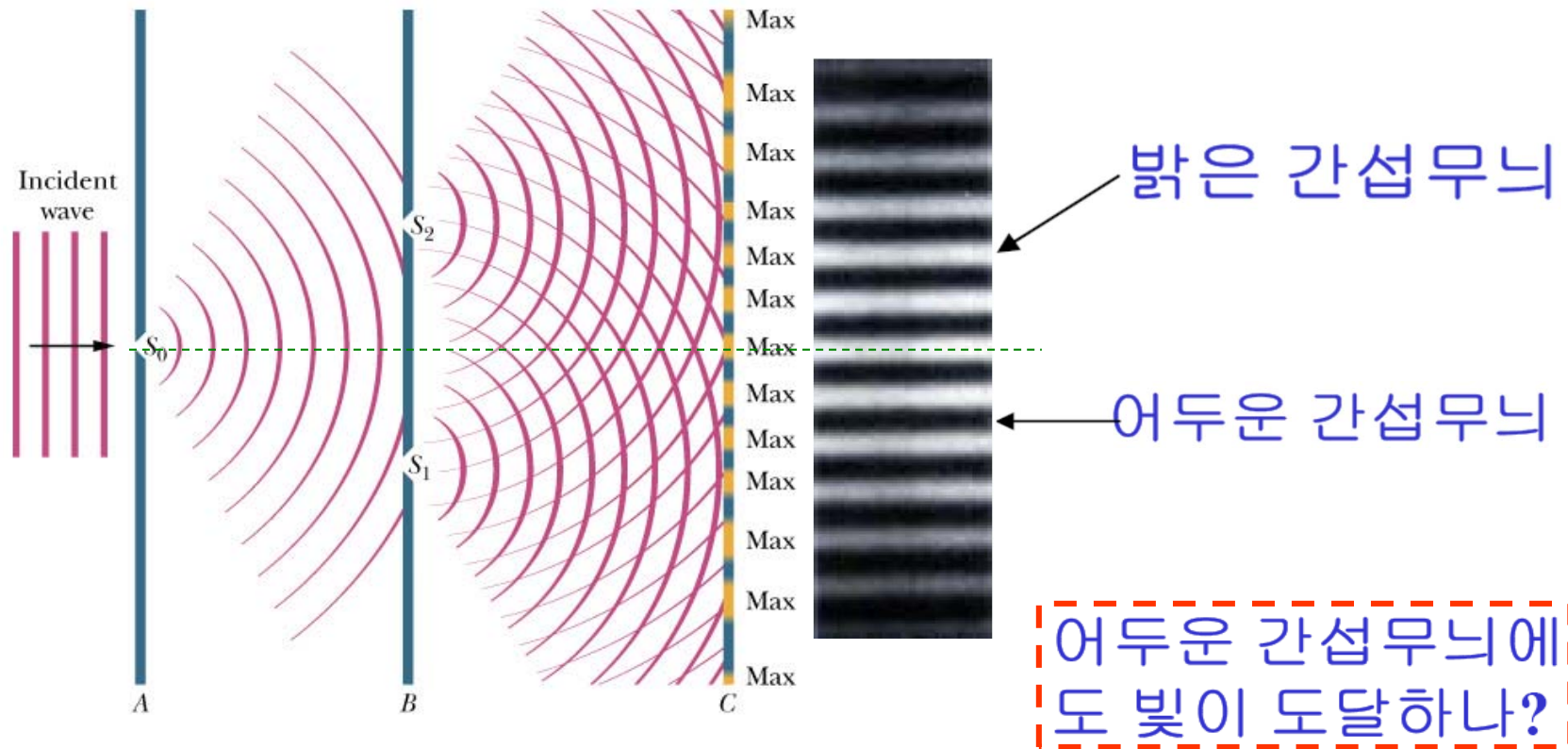


Young의 2중 슬릿 간섭실험

p. 1189



1. 실험장치 배열 및 나타나는 간섭무늬

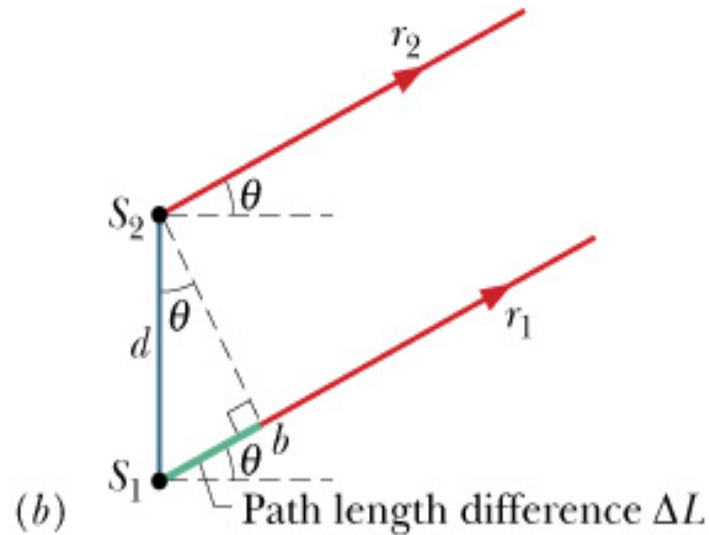
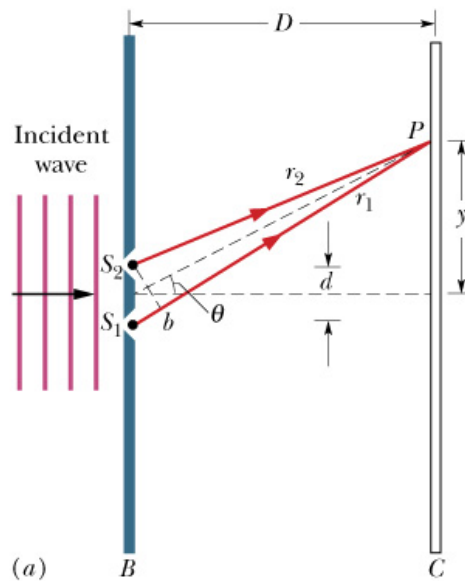


2. 간섭무늬: 생기는 원인과 밝고 어두운 곳의 위치



1) 간섭무늬가 나타나는 원인

겹쳐지는 두 파동의 경로차(ΔL)가 반파장($\lambda/2$)의 짝수배가 되면
밝아지고, 홀수배가 되면 어두워짐



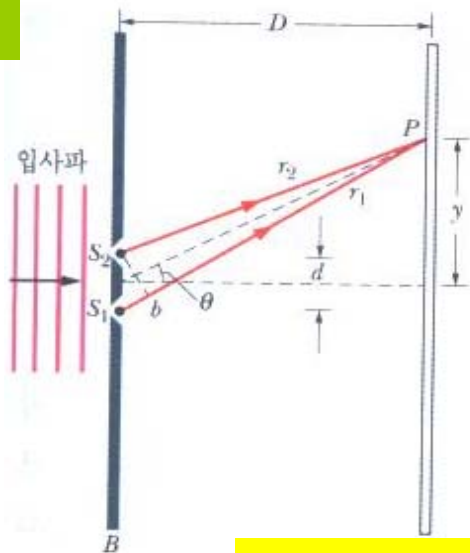
$$\Delta L = d \sin \theta$$

2) 밝고 어두운 곳의 위치

밝은 곳: $\Delta L = d \sin \theta = m\lambda$

어두운 곳: $\Delta L = d \sin \theta = \left(m + \frac{1}{2}\right)\lambda$





빛의 파장: 546nm

슬릿간격: 0.12mm

슬릿-자막 거리: 55cm

아래의 근사가 성립

$$\tan \theta \approx \sin \theta \approx \theta$$

질문: 서로 이웃한 두 극대점 사이의 거리?

풀이:

y_m : 간섭무늬 중앙으로부터 m번째 극대점까지의 수직 거리

$$\tan \theta_m \approx \theta_m = \frac{y_m}{D};$$

$$\text{한편, } \sin \theta \approx \theta = \frac{m\lambda}{d} \Rightarrow y_m = \frac{m\lambda D}{d}, \quad y_{m+1} = \frac{(m+1)\lambda D}{d}$$

$$\Delta y = y_{m+1} - y_m = \frac{\lambda D}{d} = \dots = 2.5 \text{ mm}$$





❖ 두 파동이 서로 다른 거리만큼 진행하면 위상차가 생긴다.

- Coherence 결 맞음:

$$\Delta\phi = \text{constant in time}$$

- Incoherence 결 어긋남

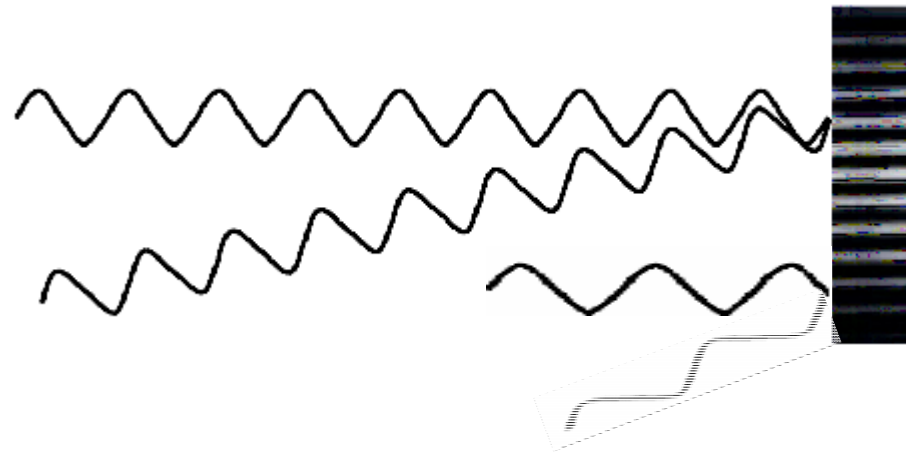




❖ **결맞음 광원 (Coherent light) : 위상이 균일함**



→ 레이저



→ 시간에 따른 위상변화 없음
간섭무늬 관측가능

❖ **비결맞음 광원 (Incoherent light) : 위상이 불균일 함**



→ 형광등, 백열등



→ 시간에 따라 위상변화
간섭무늬 관측 불가능





$$E_1 = E_0 \sin \omega t$$
$$E_2 = E_0 \sin(\omega t + \phi)$$

위상차이

$$(\text{위상차}) \equiv \frac{2\pi}{\lambda} \times (\text{경로차})$$

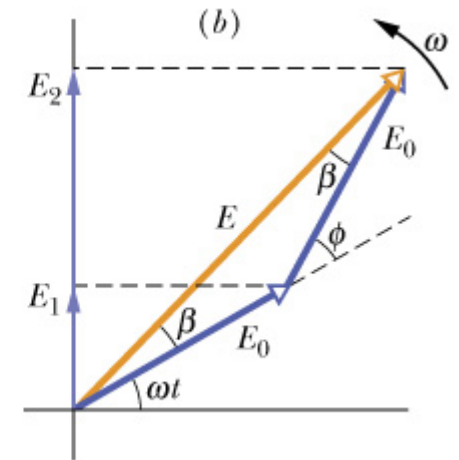
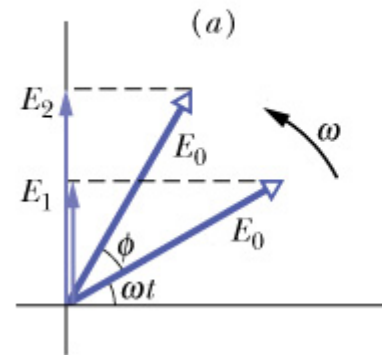
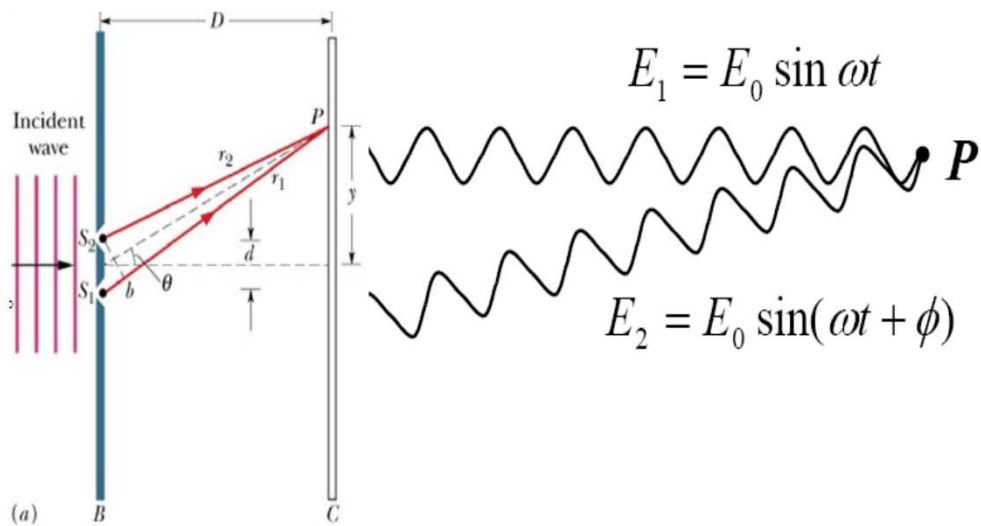


$$E_1 = E_0 \sin \omega t, \quad E_2 = E_0 \sin(\omega t + \phi)$$

" ϕ = indep. of time, but depends on θ "

$$\rightarrow \boxed{\phi = \frac{2\pi}{\lambda} \times d \sin \theta}$$





$$\begin{aligned}
 I &= S_{avg} = \frac{1}{c\mu_0} [E^2]_{avg} \\
 &= \frac{1}{c\mu_0} [(E_1 + E_2) \cdot (E_1 + E_2)]_{avg} \\
 &= \frac{1}{c\mu_0} [E_1^2 + E_2^2 + 2E_1 \cdot E_2]_{avg} \\
 &= I_1 + I_2 + I_{12} \text{ 간섭항}
 \end{aligned}$$





$$I = \frac{1}{c\mu_0} [E_1^2 + E_2^2 + 2E_1 \cdot E_2]_{avg} = I_1 + I_2 + I_{12} \quad \text{간섭항}$$

$$[E_1 \cdot E_2]_{avg} = [E_0^2 \sin \omega t \sin(\omega t + \phi)]_{avg}$$

$$\sin(\omega t + \phi) = \sin \omega t \cos \phi + \cos \omega t \sin \phi$$

$$I_0 = I_1 = I_2 = \frac{E_0^2 / 2}{c\mu_0}; \quad I_{12} = \frac{E_0^2 \cos \phi}{c\mu_0} = 2I_0 \cos \phi$$

$$\rightarrow I = 2I_0(1 + \cos \phi) = 4I_0 \cos^2\left(\frac{\phi}{2}\right) \quad \text{← 위상차이}$$



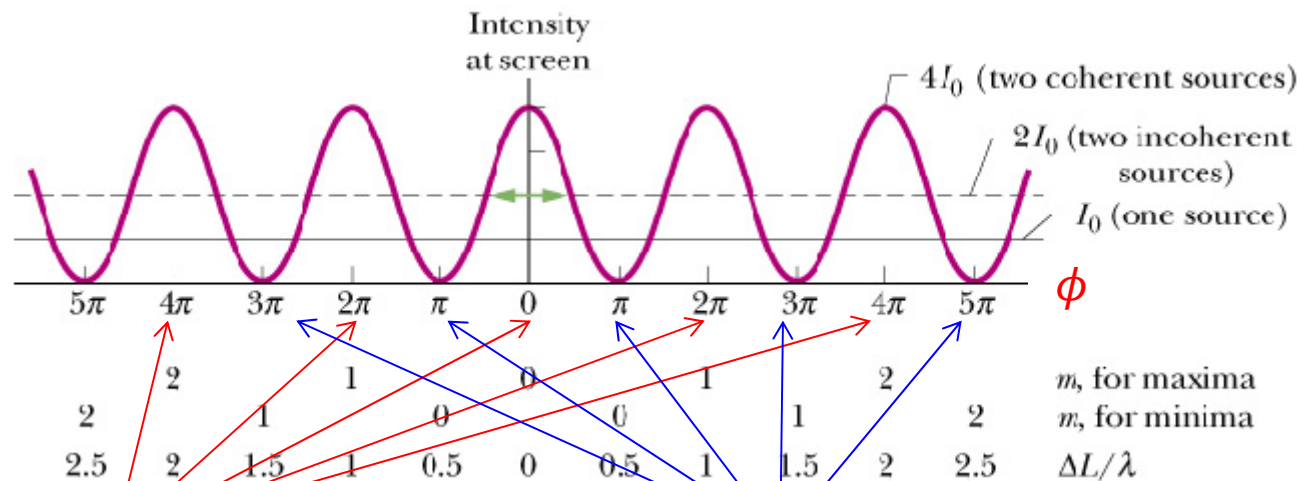


$$E_1 = E_0 \sin \omega t, \quad E_2 = E_0 \sin(\omega t + \phi)$$

$$\rightarrow E = 2E_0 \cos \beta \quad \Rightarrow I = 4I_0 \cos^2 \frac{1}{2} \phi$$

$$\phi = \frac{2\pi}{\lambda} d \sin \theta$$

위상차이



❖ 극대점

$$d \sin \theta = m\lambda, \quad m = 0, 1, 2, 3, \dots$$

$$\frac{1}{2}\phi = m\pi \quad \phi = 2m\pi$$

❖ 극소점

$$d \sin \theta = (m + \frac{1}{2})\lambda, \quad m = 0, 1, 2, 3, \dots$$

$$\frac{1}{2}\phi = (m + \frac{1}{2})\pi$$

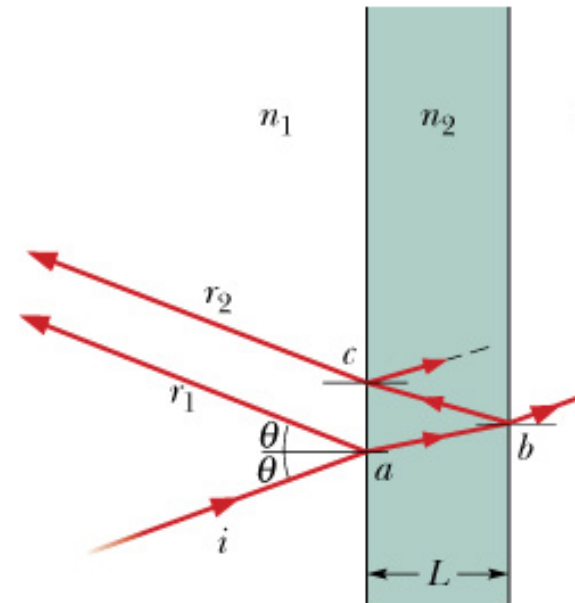
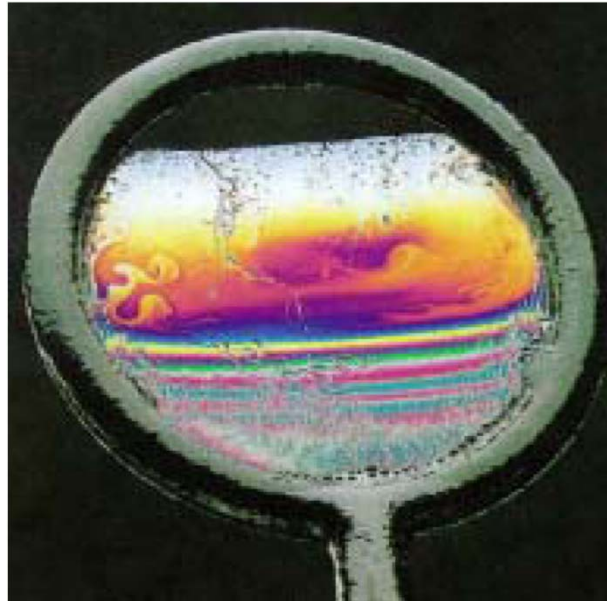


박막간섭

p. 1199



박막의 앞면과 뒷면에 의한 간섭
→ 비누막, 기름막에서의 간섭



위의 그림에서 맨 위쪽 가장 얇은 곳이 어두운 까닭?



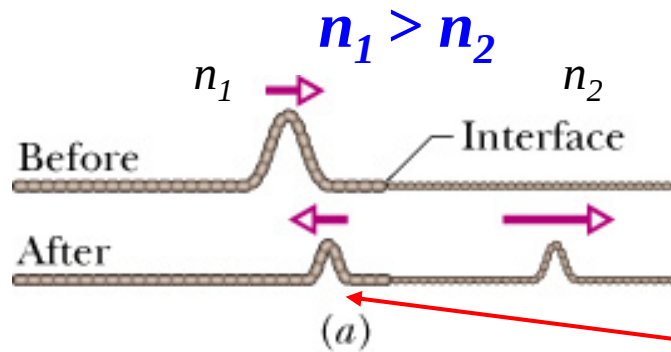
파동 r_1 과 r_2 의 위상차 = (반사 위상차) + (경로차 위상차)





※ 반사 위상차

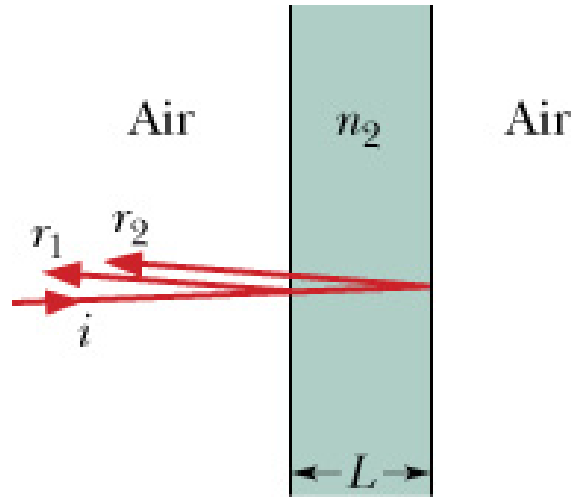
굴절율이 높은 매질에서 반사될 때 위상반전이 일어남
(위상 π 에 해당)



Reflection
Off lower index :

Reflection Phase Shift
0





보강간섭 :

$$2L = \frac{\text{홀수}}{2} \times \text{파장}$$

$$= \frac{\text{홀수}}{2} \times \lambda_{n_2} = \left(m + \frac{1}{2}\right) \frac{\lambda}{n_2}$$

소멸간섭 :

$$2L = \text{정수} \times \text{파장}$$

$$= m \times \lambda_{n_2} = m \frac{\lambda}{n_2}$$

$$n_2 > n_1$$

반사 위상차 주의!!

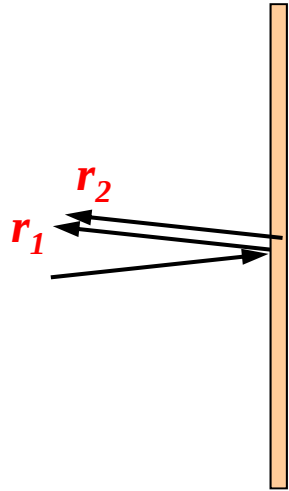
$$2L = \left(m + \frac{1}{2}\right) \frac{\lambda}{n_2}, \text{ for } m=0, 1, 2, \dots : \text{ 밝은 필름}$$

$$2L = m \frac{\lambda}{n_2}, \text{ for } m=0, 1, 2, \dots : \text{ 어두운 필름}$$

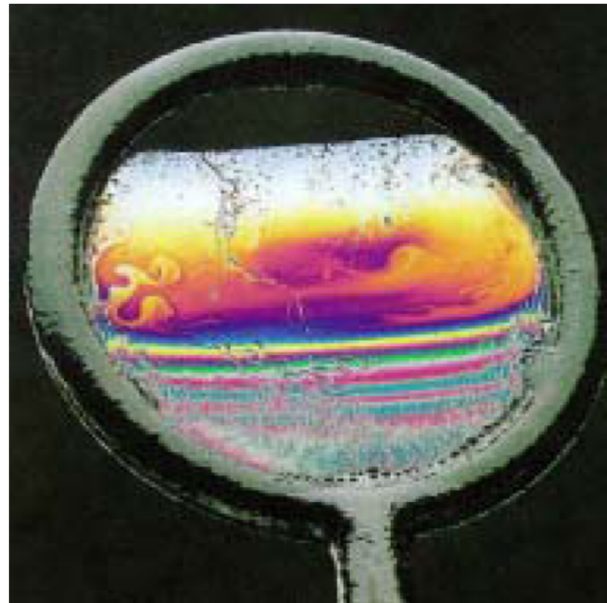




Film Thickness Much Less Than λ



- ✓ If $L \ll \lambda$, for example $L < 0.1\lambda$, then phase difference due to the path difference $2L$ can be neglected.
- ✓ Phase difference between r_1 and r_2 will always be $\frac{1}{2}\lambda$
→ destructive interference → film will appear **dark** when viewed from illuminated side.





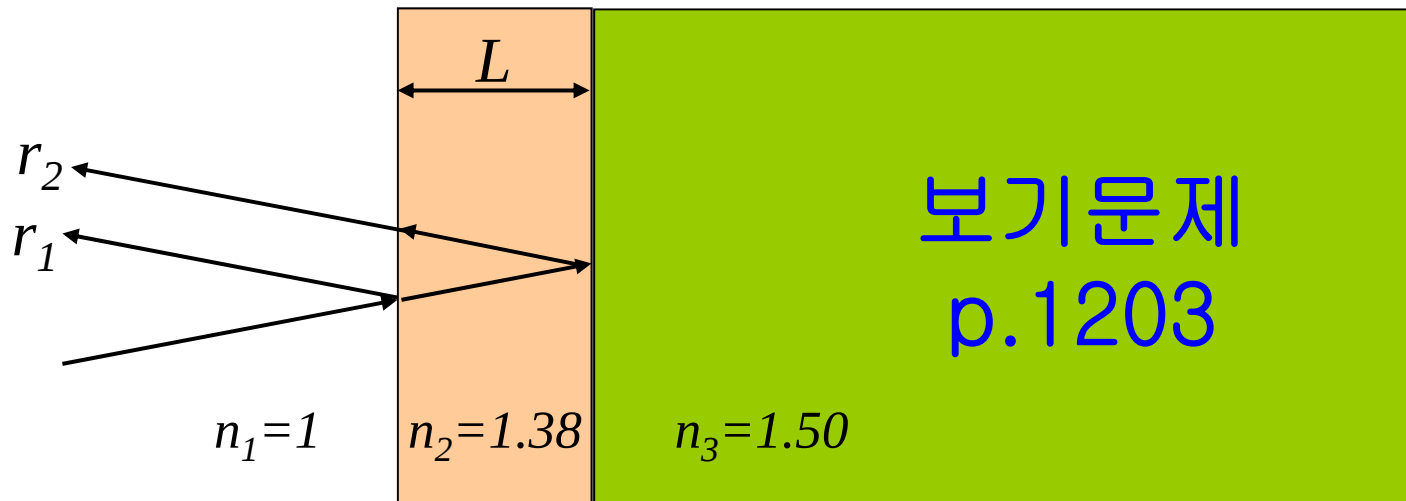
- ✓ For the special case of a higher index film flanked by air on both sides

주의!

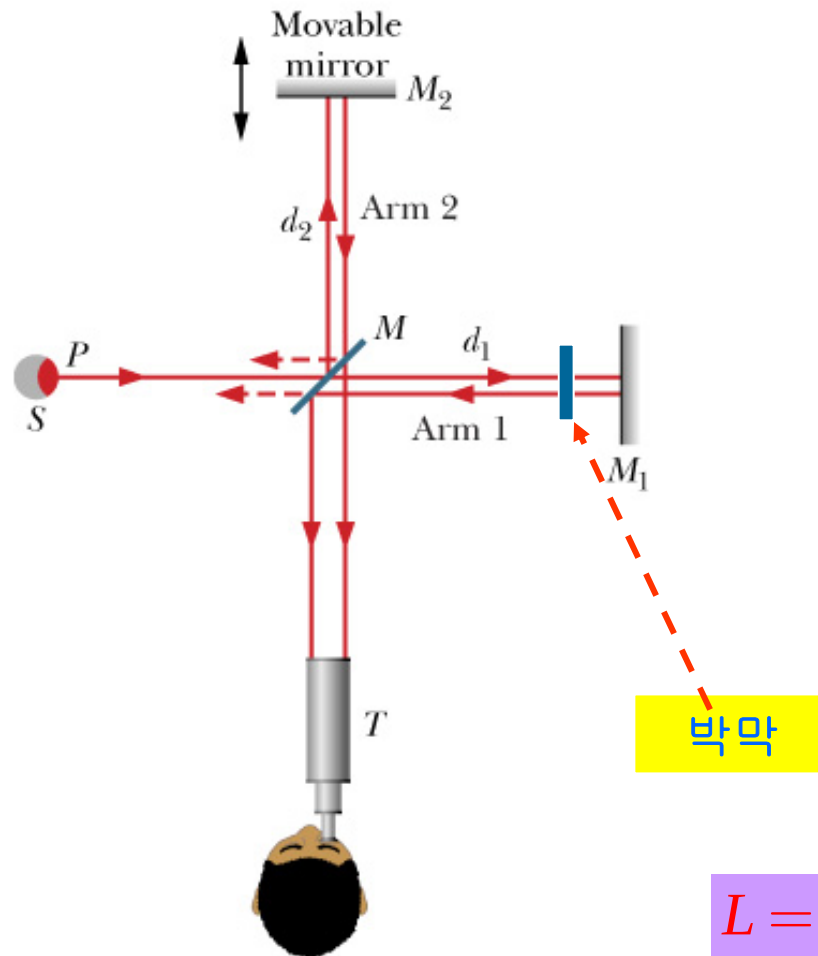
$$2L = \left(m + \frac{1}{2}\right) \frac{\lambda}{n_2}, \text{ for } m=0, 1, 2, \dots : \text{ 밝은 필름}$$

$$2L = m \frac{\lambda}{n_2}, \text{ for } m=0, 1, 2, \dots : \text{ 어두운 필름}$$

- ✓ For multilayer systems, this is not always the case and these equations are not appropriate.



Michelson's Interferometer



통과 파장 갯수

박막의 두께

$$N_{\text{박막}} - N_{\text{공기}} = \frac{2L}{\lambda} (n - 1)$$

박막의 굴절률

빛의 파장

⇒ shift in fringes

$$L = \mu\lambda \Rightarrow 1 \text{ m} = 1,533,163.5 \lambda_{\text{빨강}}$$

(Michelson:1907 Nobel Prize in Physics)

For each change in path by 1λ , the interference pattern shifts by one fringe at T . By counting the fringe change, one determines $N_{\text{박막}} - N_{\text{공기}}$ and can then solve for L in terms of λ and n .





Diffraction of ocean water waves

Ocean waves passing through slits in Tel Aviv, Israel

36장 회절, 에돌이, diffraction



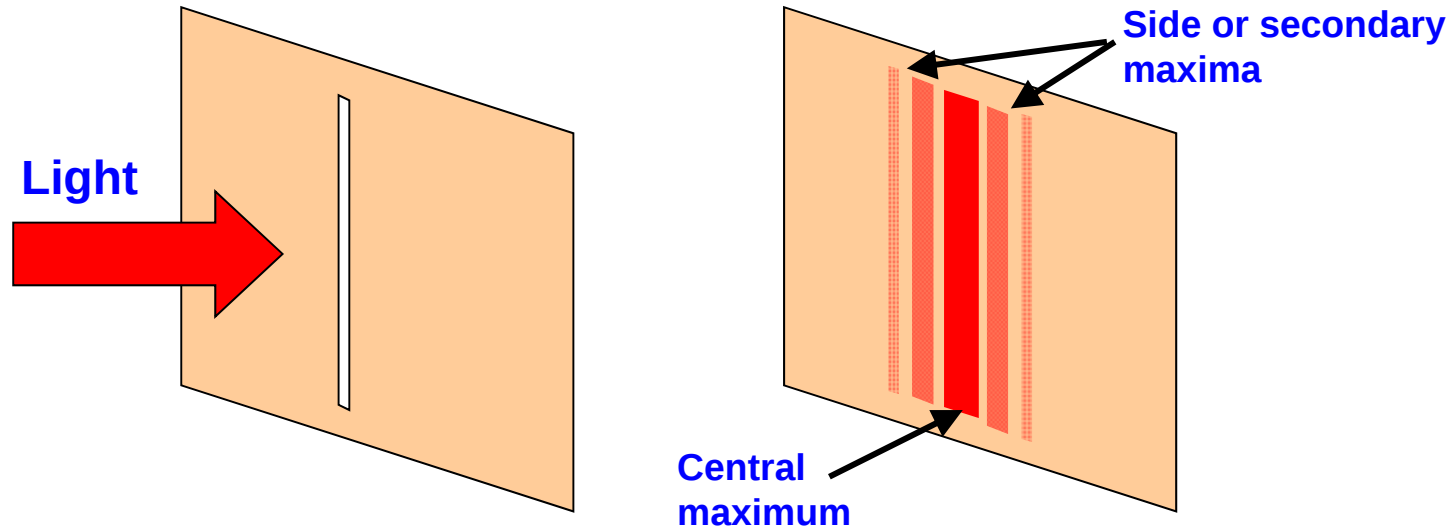
Diffraction occurs for all waves, whatever the phenomenon.



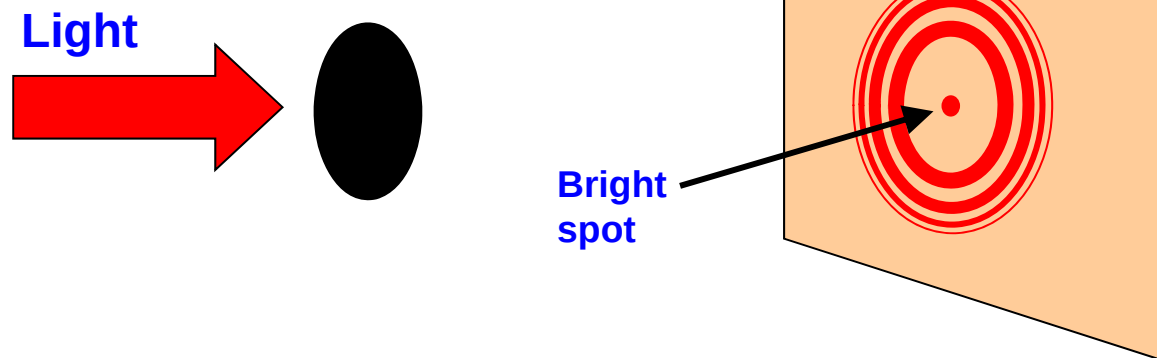
Diffraction and the Wave Theory of Light



Diffraction Pattern from a single narrow slit



Fresnel Bright Spot



These patterns cannot be explained using geometrical optics (Ch. 34)!

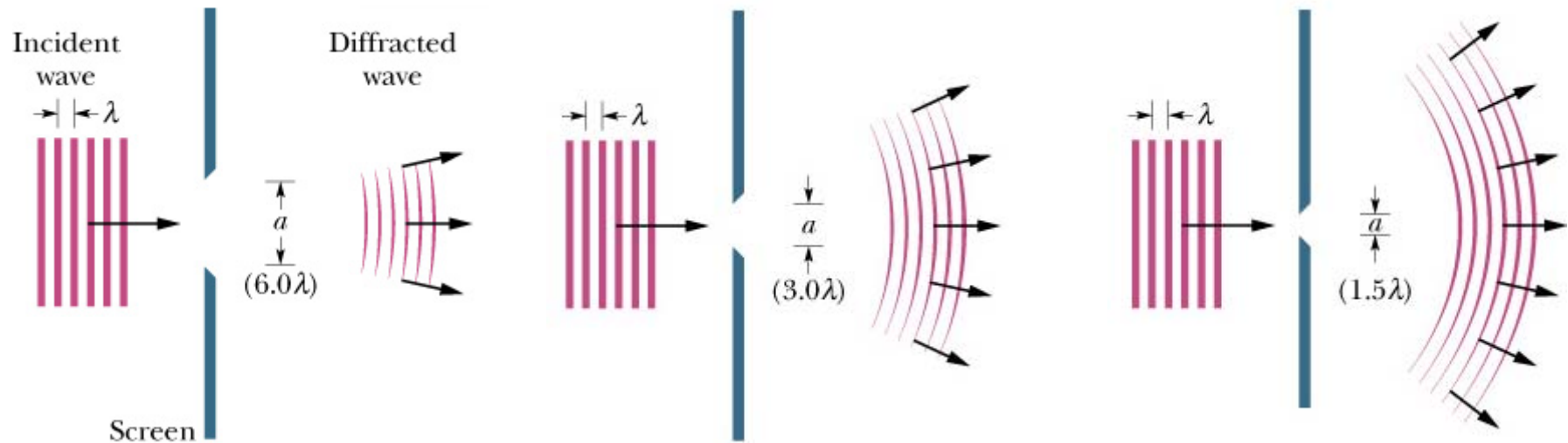




회절 (에돌이)

에워싸고
돌아서

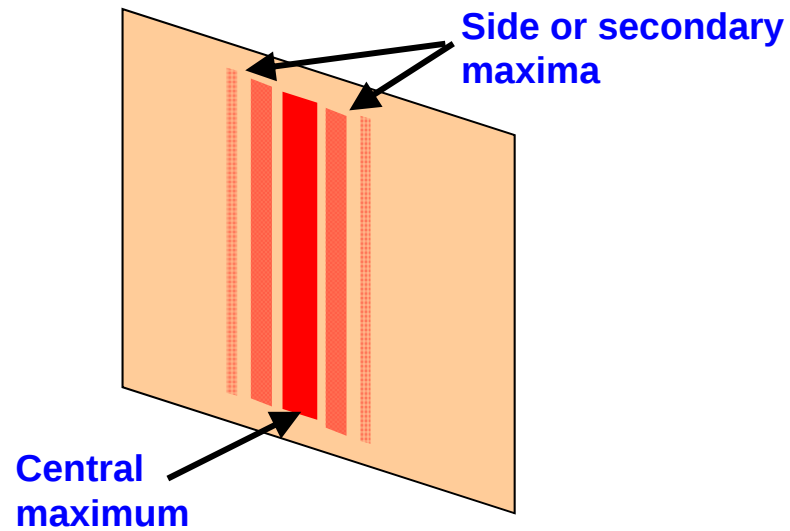
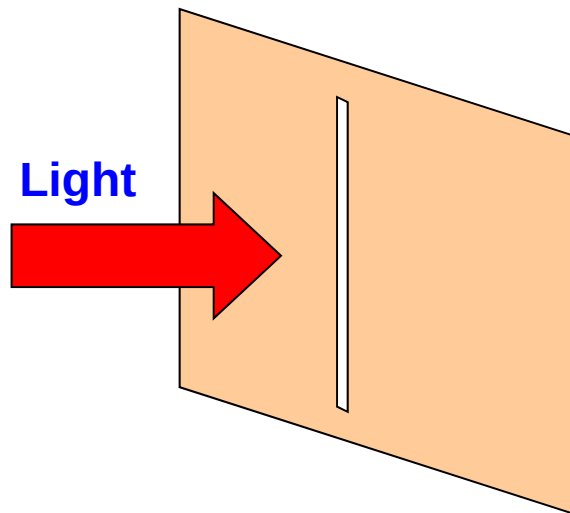
파동이 진행하다 장애물을 만나면, 그것을 돌아서 전파하는 현상

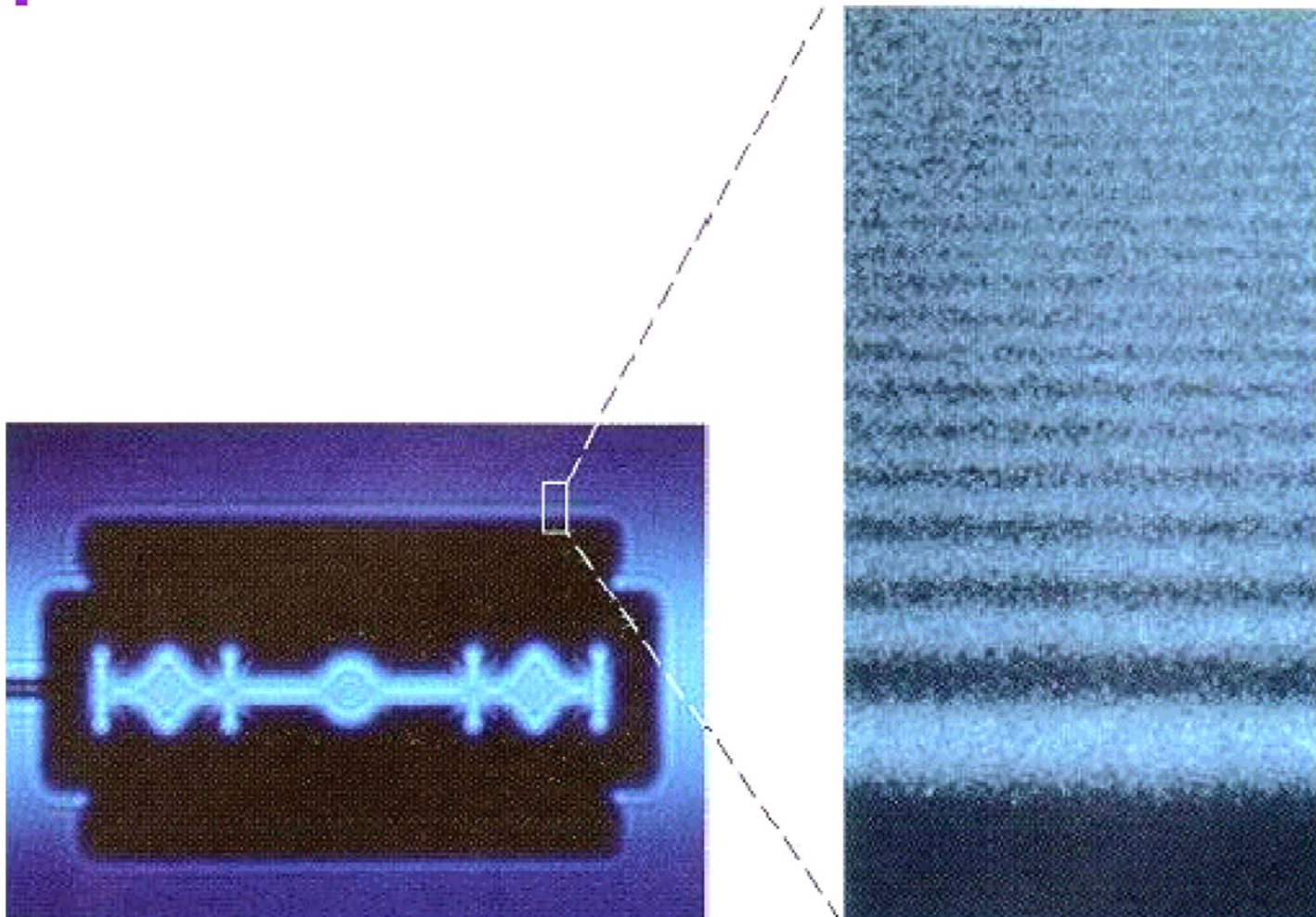


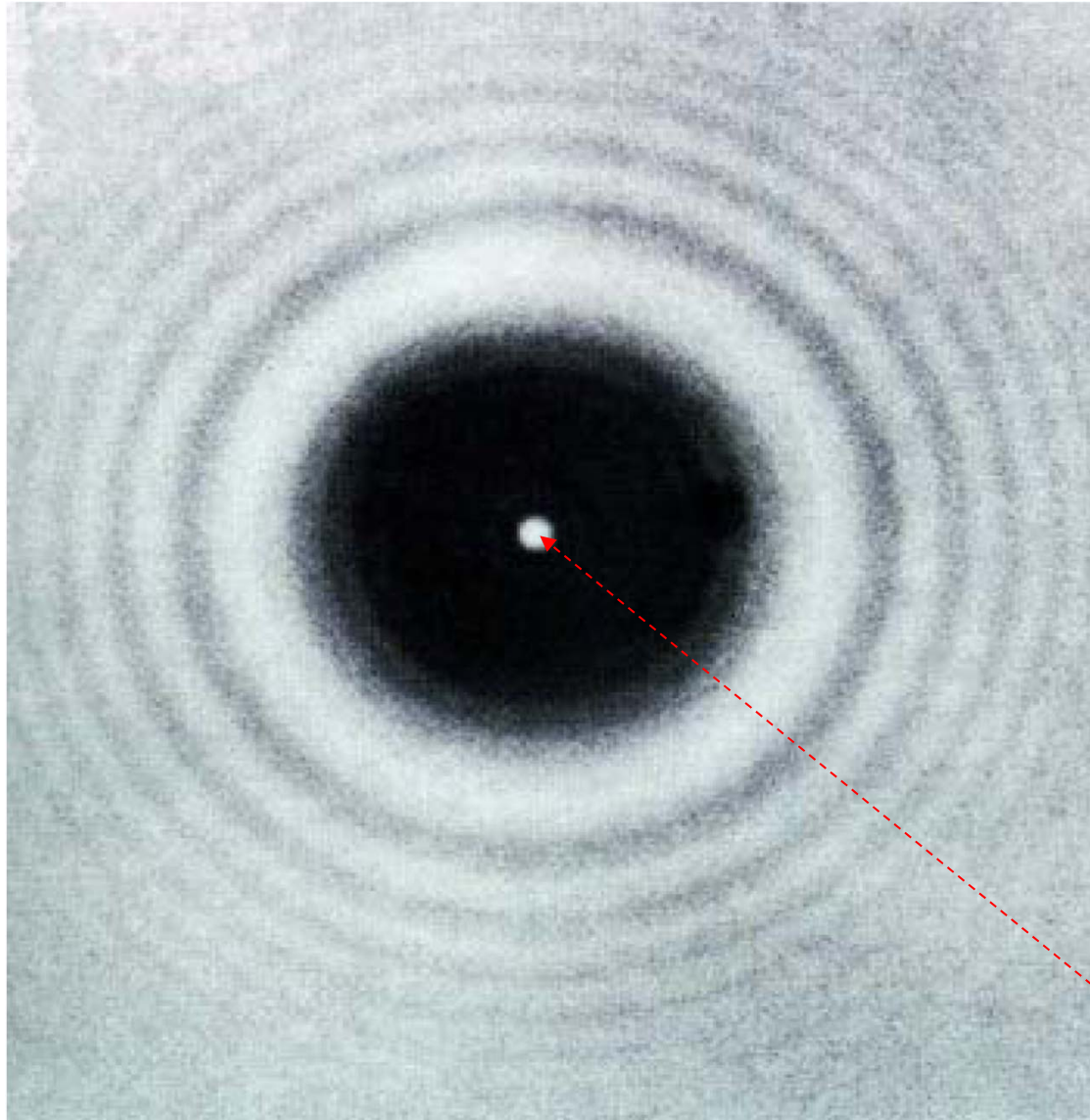
(Huygens 원리로 설명)



회절무늬: 회절과 빛의 파동이론







원반 뒤쪽에 형성되는 회절무늬

- Huygens(1678):
wave optics
- Newtonian group
(1819): 빛의 입자성

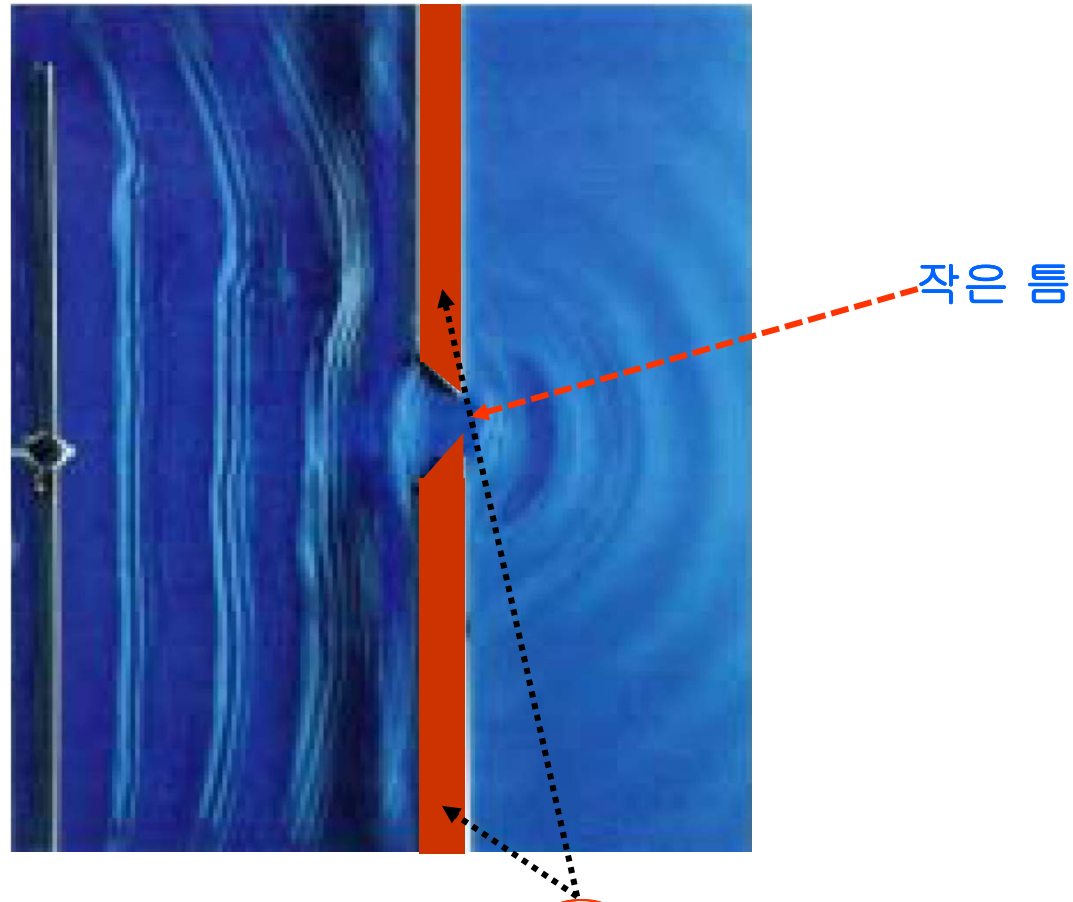
“Poisson” 광점

Fresnel Bright Spot



회절; diffraction

p. 1220



물결파의 회절현상. 장벽에 난 작은 틈을 통과한 물결파가 틈의 끝을 돌아서 장벽의 뒷쪽으로 퍼져나간다.

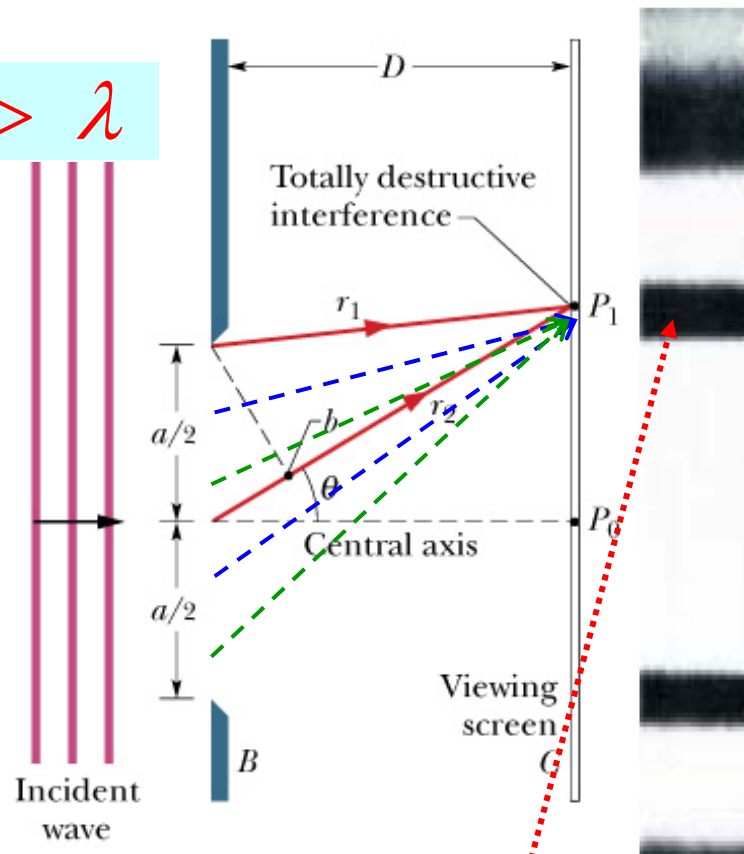


단일 슬릿 회절 무늬

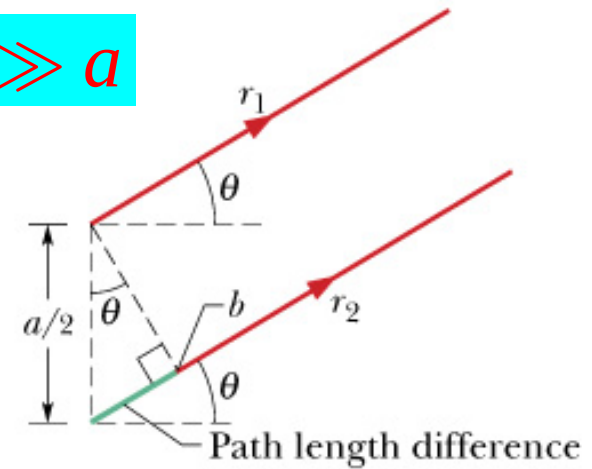
p. 1221



$$a > \lambda$$



$$D \gg a$$



What do you expect if $\lambda = a$?

1. 첫 번째 어두운 띠의 위치
슬릿 가장자리와 가운데를 지나온 두 빛살의 위상이 처음으로
완전히 어긋나는 곳 (경로차 = 반파장)

$$\frac{a}{2} \sin \theta_1 = \frac{\lambda}{2}$$



$$\sin \theta_1 = \frac{\lambda}{a}$$



2. 어두운 띠의 일반적인 위치



$$a > \lambda$$

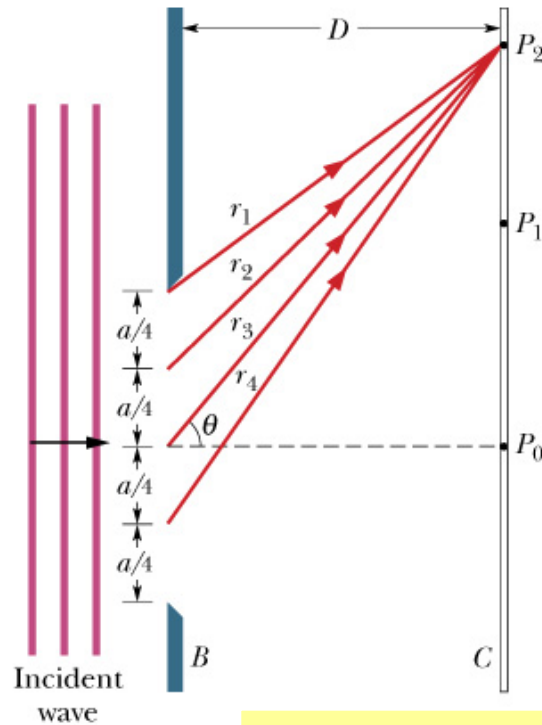
$$\frac{a}{2m} \sin \theta_m = \frac{\lambda}{2}$$

m 번째 극소무늬

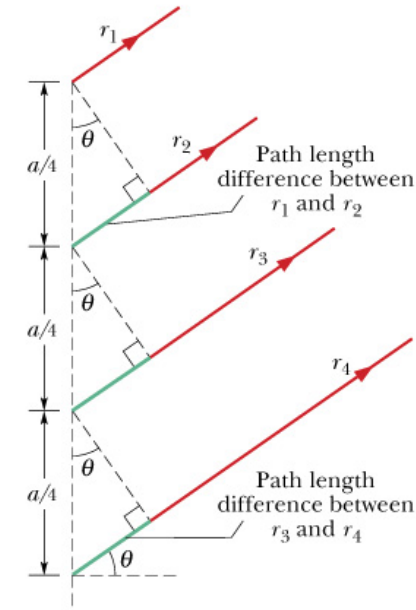
$$\sin \theta_m = \frac{\lambda}{a} m ;$$

$$m = \pm 1, \pm 2, \dots$$

$m = 2$ 인 경우:



$$D \sim a$$



$$\frac{a}{4} \sin \theta_2 = \pm \frac{\lambda}{2} \rightarrow \sin \theta_2 = \pm \frac{2\lambda}{a} : m = 2$$

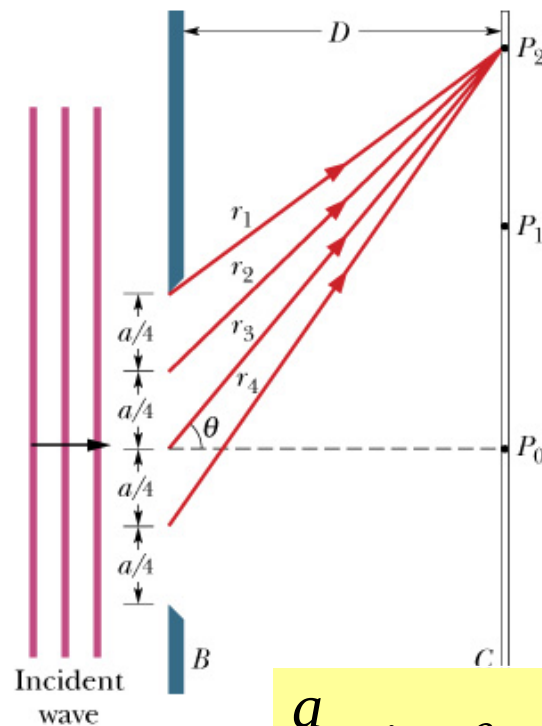


복습

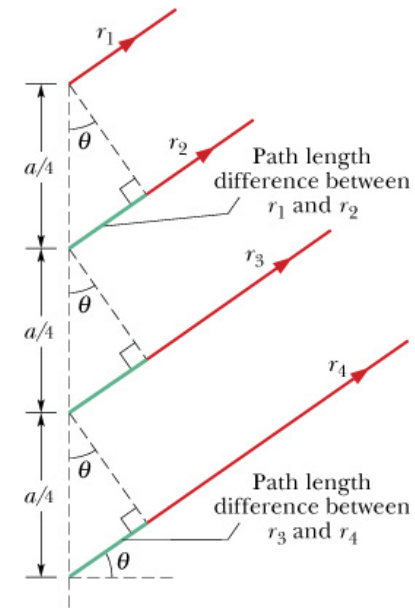
$a > \lambda$ 일 때



$m = 2$ 인 경우:



$D \sim a$



$$\frac{a}{4} \sin \theta_2 = \pm \frac{\lambda}{2} \rightarrow \sin \theta_2 = \pm \frac{2\lambda}{a} : m = 2$$

어두운 띠의 일반적인 위치:

$$\sin \theta_m = \frac{\lambda}{a} m ;$$

$$m = \pm 1, \pm 2, \dots$$

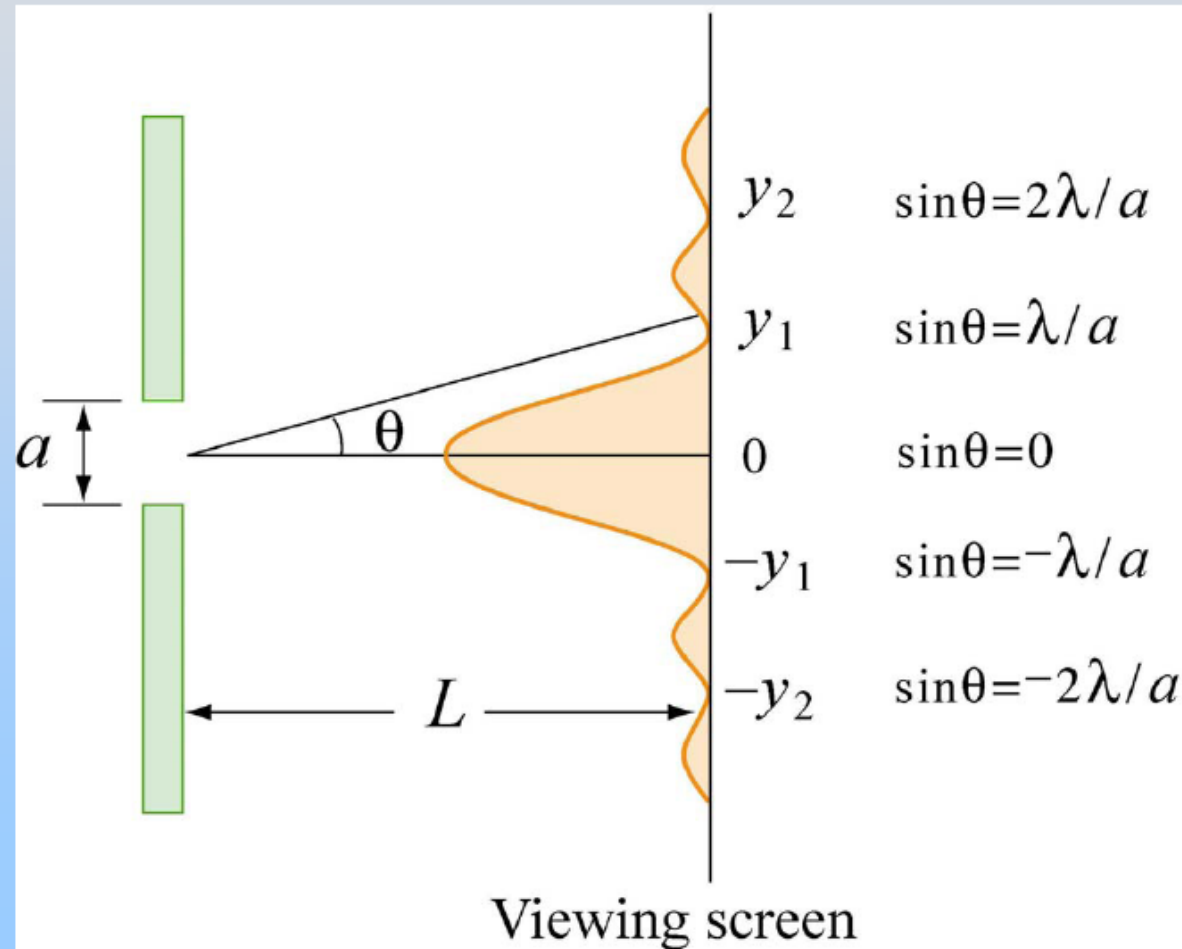
m 번째 소멸(극소)





Intensity Distribution

Destructive Interference: $a \sin \theta = m\lambda$ $m = \pm 1, \pm 2, \dots$



Intensity in Single-Slit Diffraction

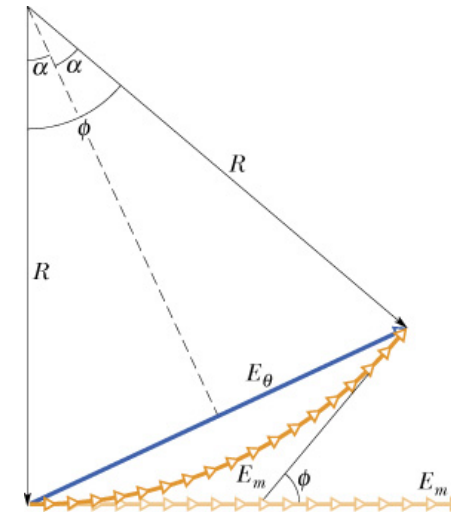
p. 1225



$$\Delta\phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x \sin \theta)$$

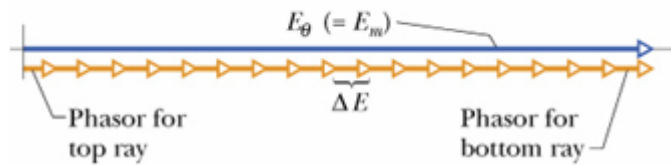
: 이웃영역에서 오는
빛의 위상차

이웃 영역에서 오는
빛의 경로차

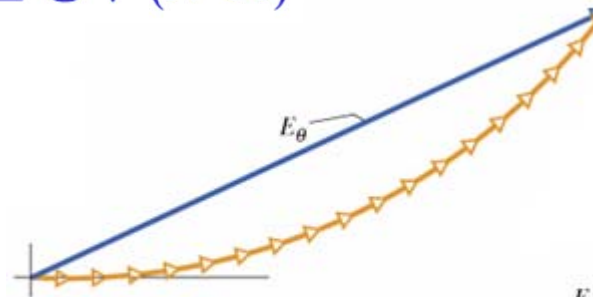


- 단일슬릿을 18개 영역으로 쪼갠 경우 (N=18)

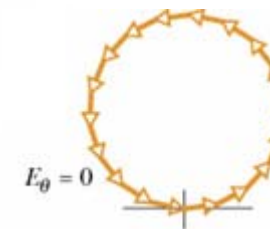
$N = 18$:



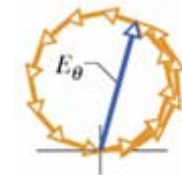
주극대 ($\theta=0$)



small θ

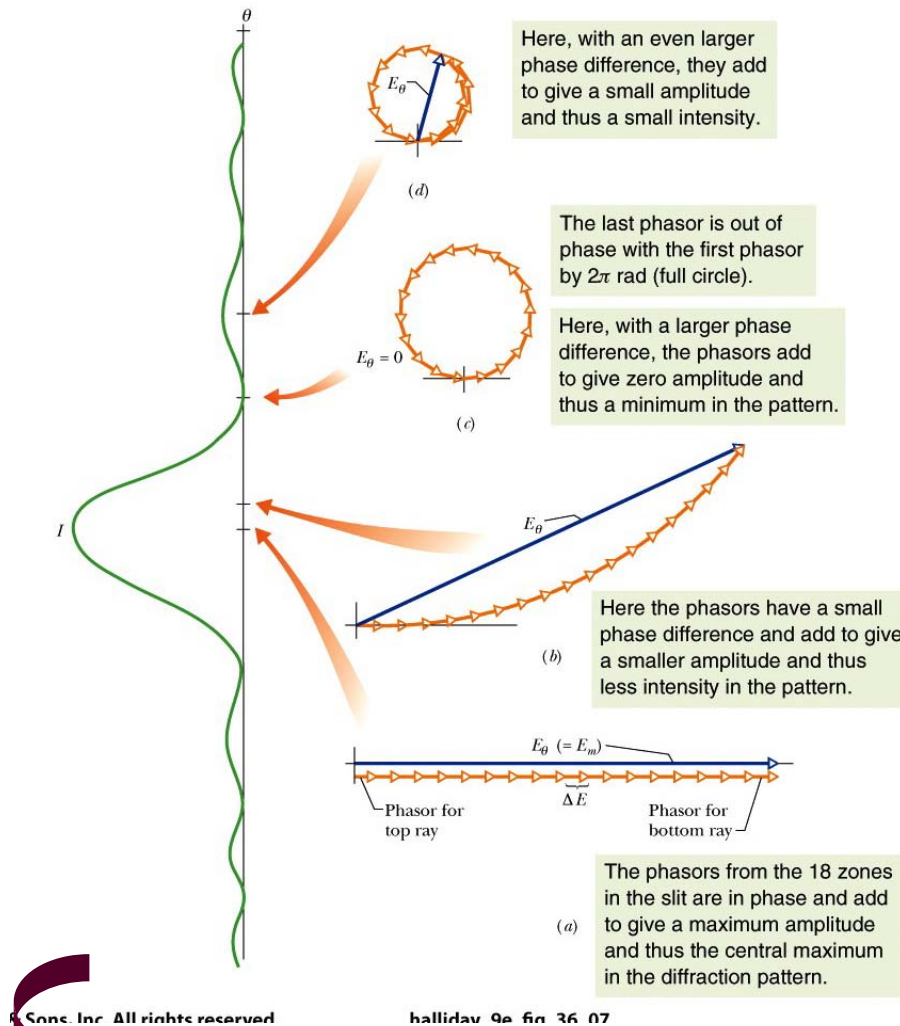


제1극소



제1부극대





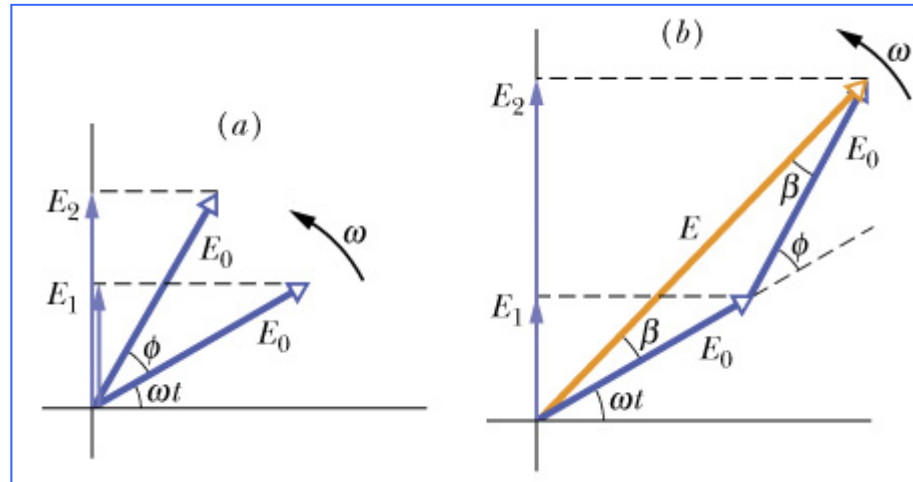
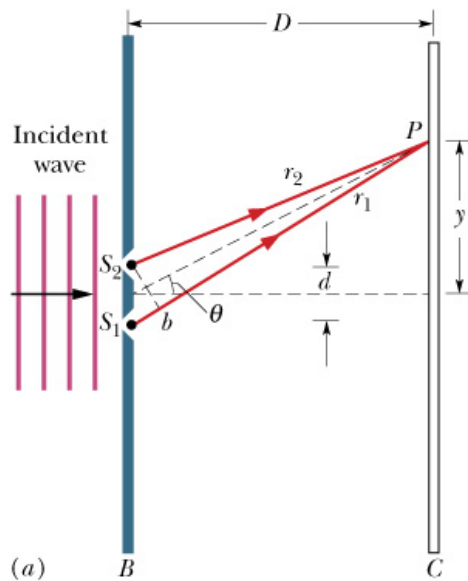
(p. 1227-1229 참고)

$$I(\theta) = I_m \left(\frac{\sin \alpha}{\alpha} \right)^2, \quad \alpha = \frac{1}{2} \phi = \frac{\pi a}{\lambda} \sin \theta$$

$\rightarrow \alpha = m\pi; \quad a \sin \theta = m\lambda \quad \text{for } m = 1, 2, 3 \cdots \Leftarrow \text{minima-dark fringes}$



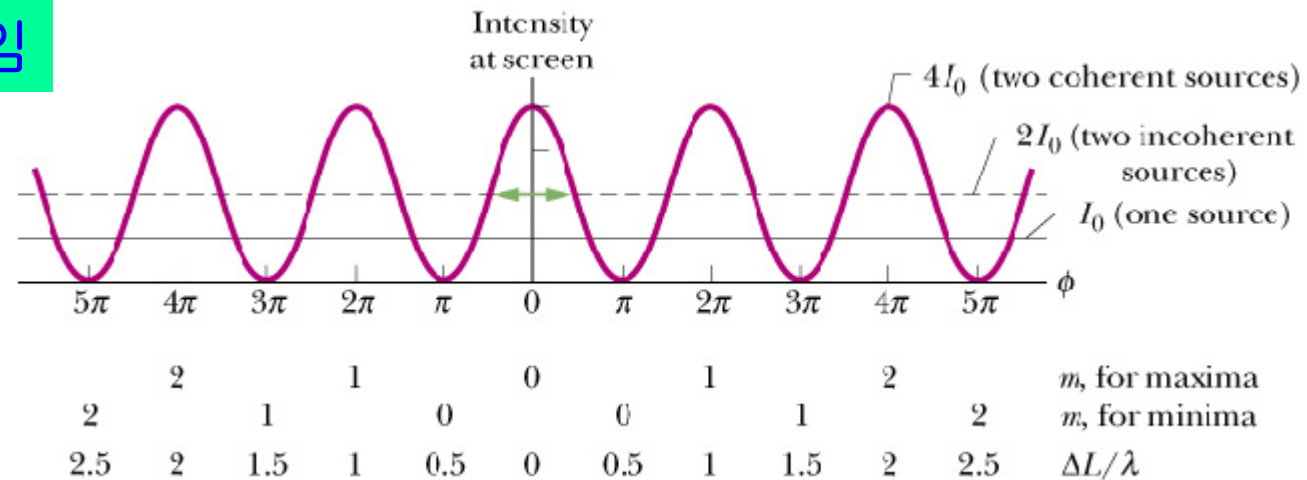
Young의 2중 슬릿 간섭무늬 (p.1190-1197)



$$d \sin \theta = m\lambda, \quad m = 0, 1, 2, \dots : \text{maxima}$$

$$d \sin \theta = (m + \frac{1}{2})\lambda, \quad m = 0, 1, 2, \dots : \text{minima}$$

• 슬릿의 폭을 무시한 경우임

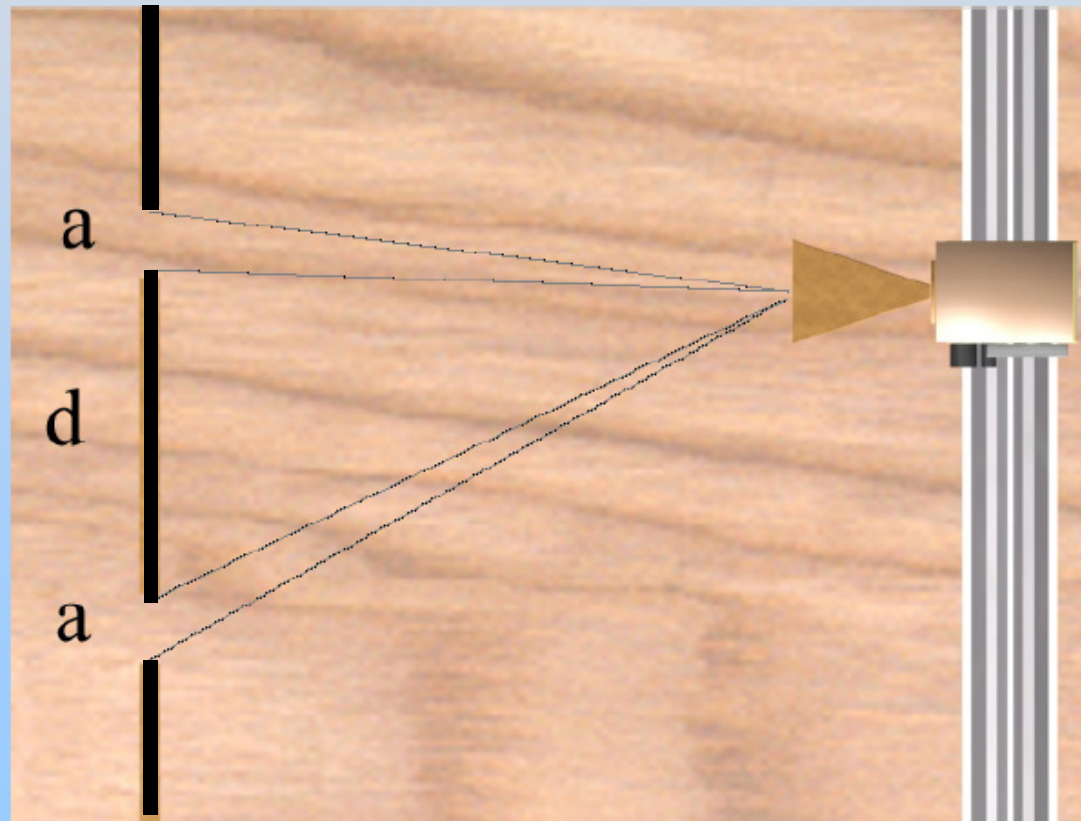




Two Slits With Finite Width a

With more than one slit having finite width a , we must consider

1. Diffraction due to the individual slit
2. Interference of waves from different slits



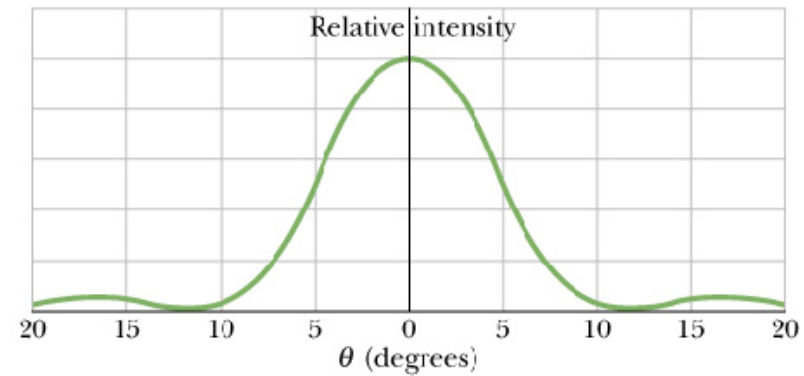
이중슬릿이 만드는 회절무늬

(p.1234-1236)



슬릿 하나의 회절무늬

X

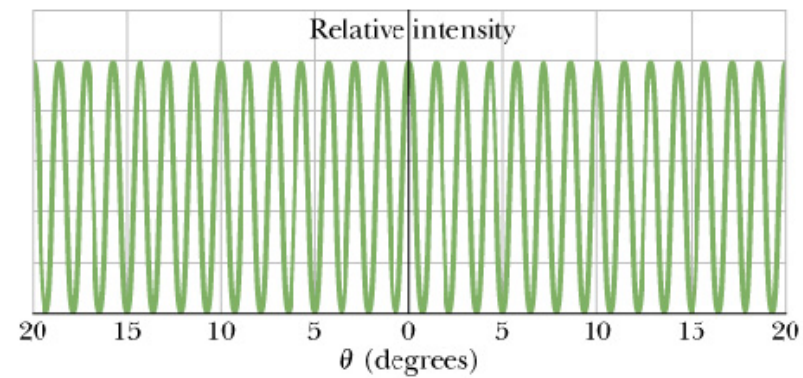


두 빛의 간섭무늬

$$a \sin \theta = \lambda$$

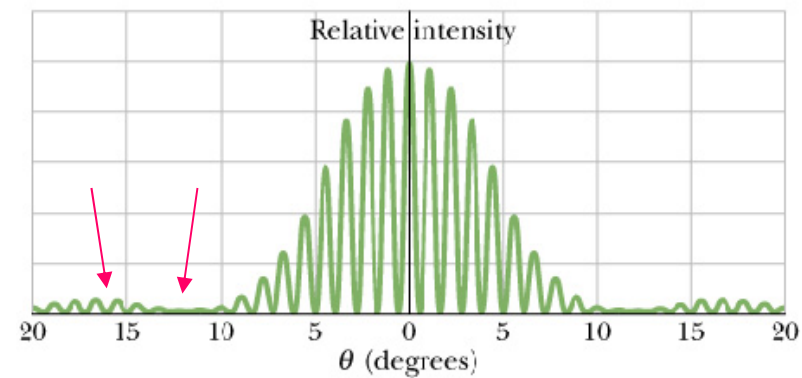
(a : 슬릿간격)

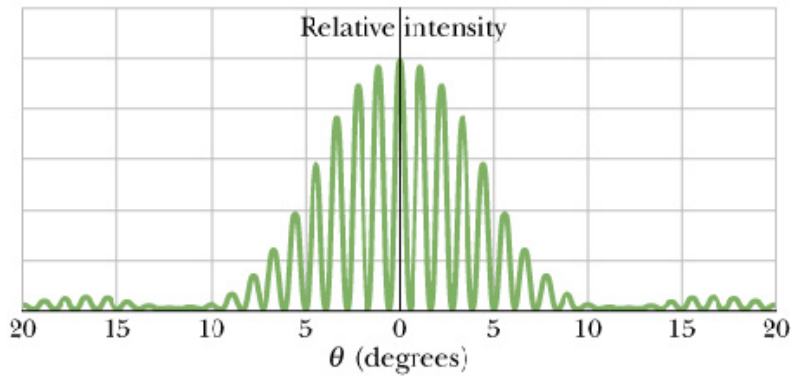
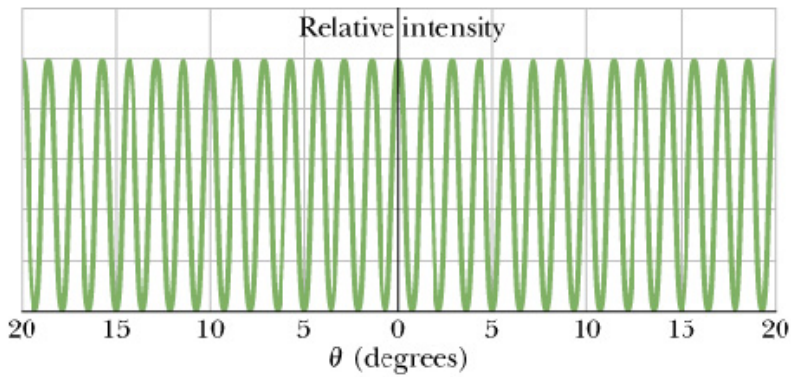
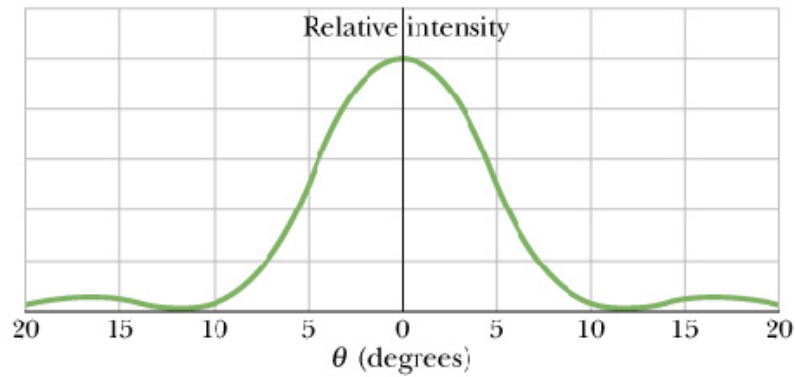
||



두 슬릿이 만드는 무늬

(회절 × 간섭)





간섭인자

$$I(\theta) = I_m \cos^2 \beta \left(\frac{\sin \alpha}{\alpha} \right)^2$$

회절인자

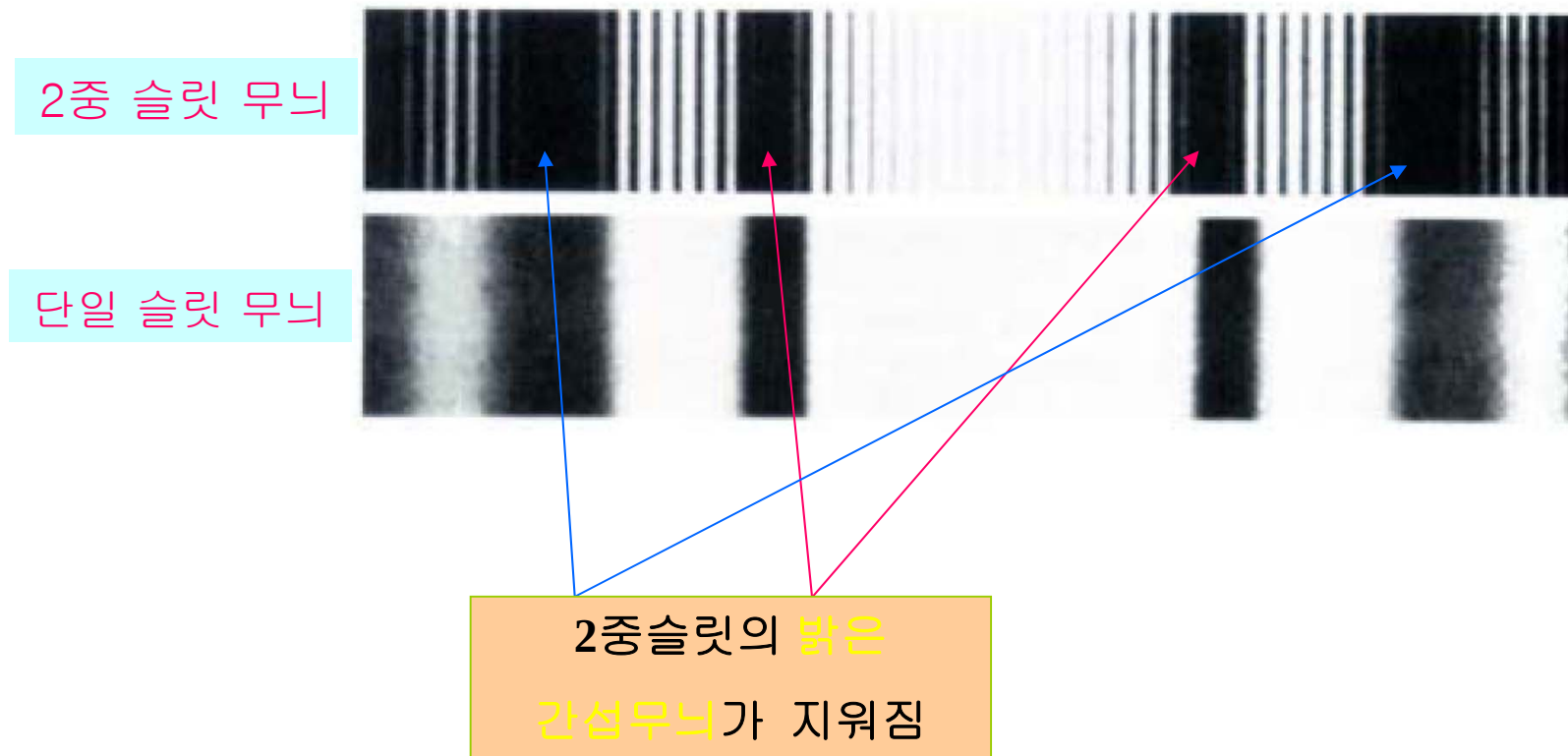
$$\beta = \frac{\pi d}{\lambda} \sin \theta$$

$$\alpha = \frac{\pi a}{\lambda} \sin \theta$$





전체 무늬의 모양 = 회절무늬(슬릿 하나) $\times$ 간섭무늬(두 슬릿)





Two Slits With Finite width a

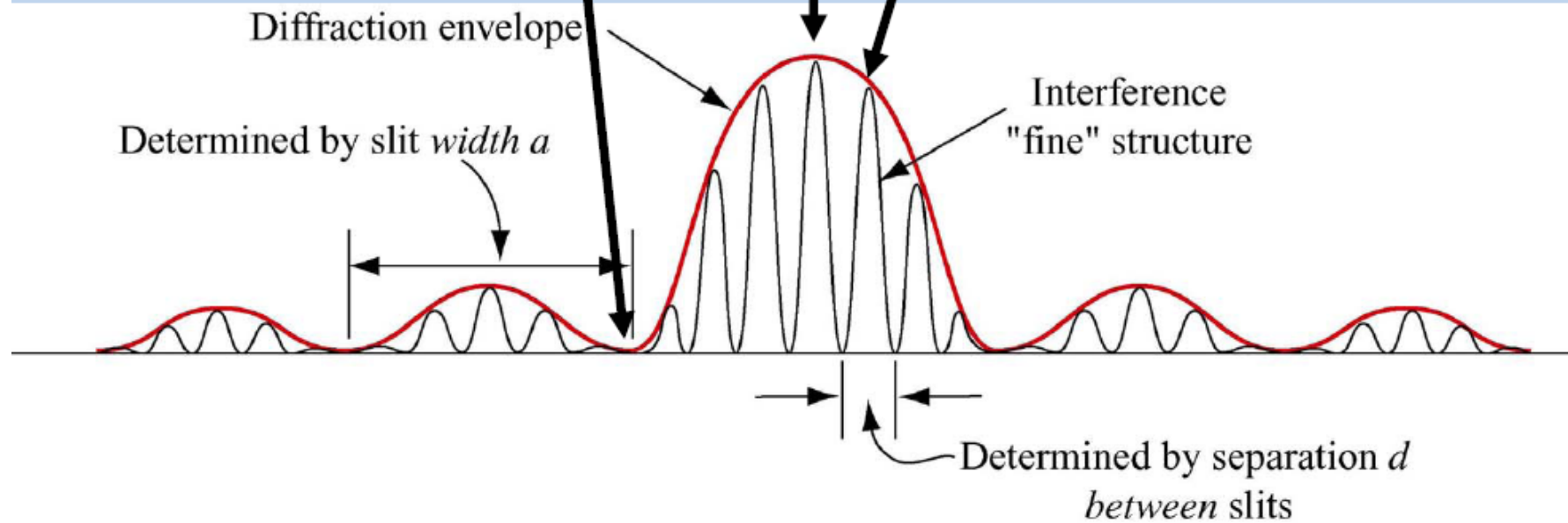
Zero Order Maximum

First Diff. Minimum

$$a \sin \theta = \lambda$$

First Order Maximum

$$d \sin \theta = \lambda$$

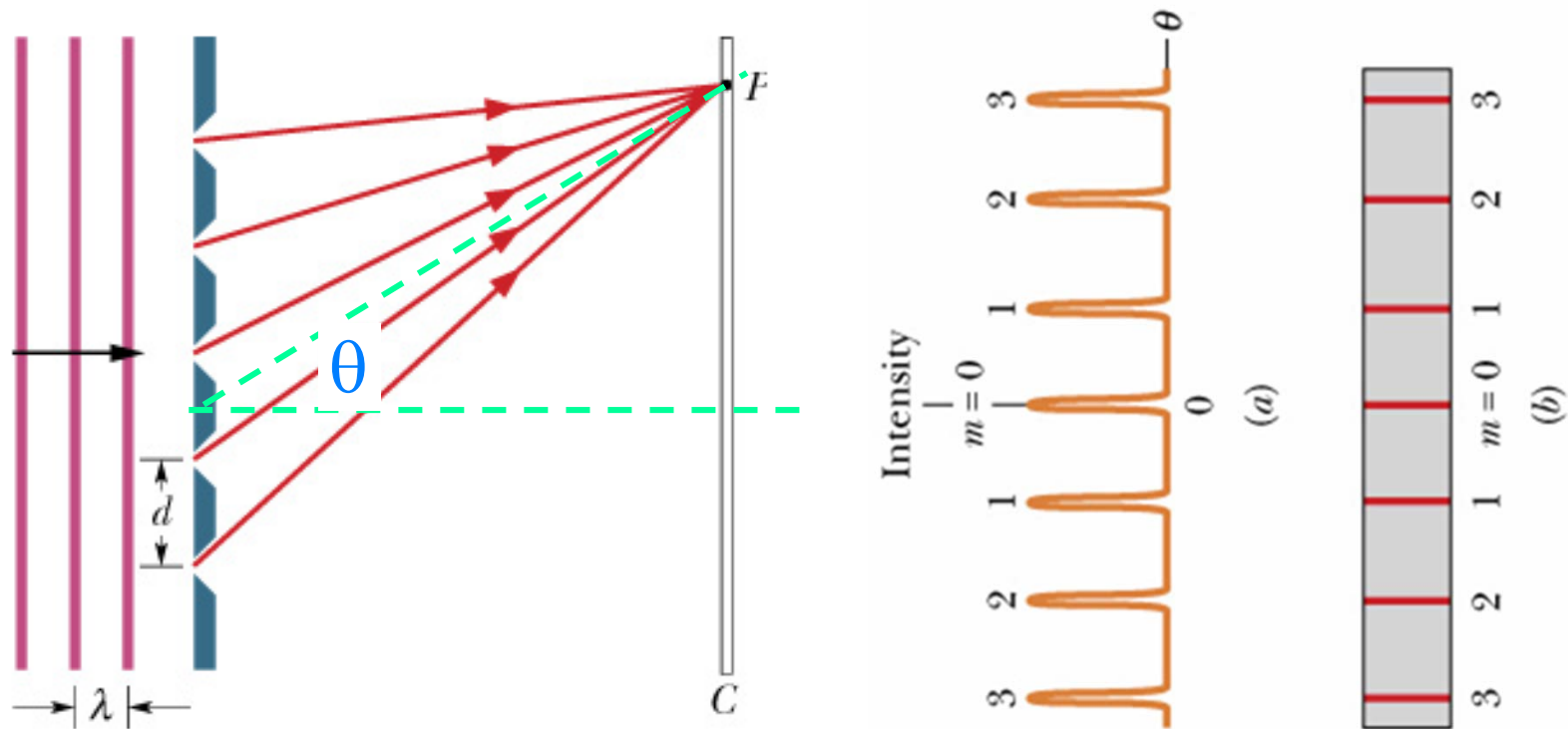


회절격자 (p.1237-1240)



회절격자, gratings : 간격이 고른 여러 개의 슬릿

1. 회절격자가 만드는 회절무늬

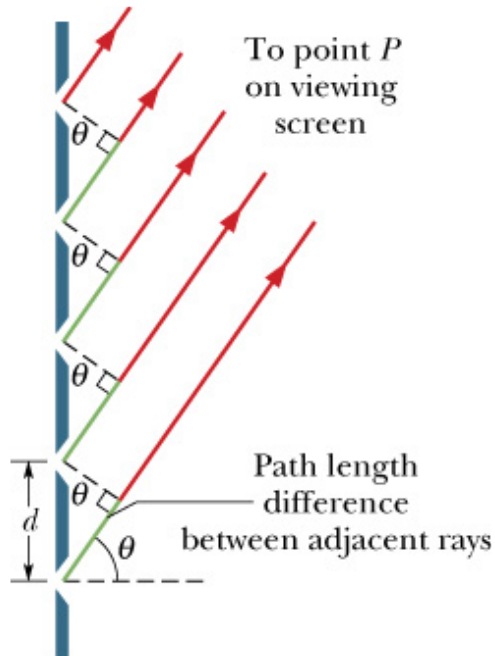


• m = order, 차수

• 격자(발)의 수 N 이 클수록 예리한 회절선



2) 회절무늬의 변화: 띠가 더 밝아지고, 폭이 더 좁아짐

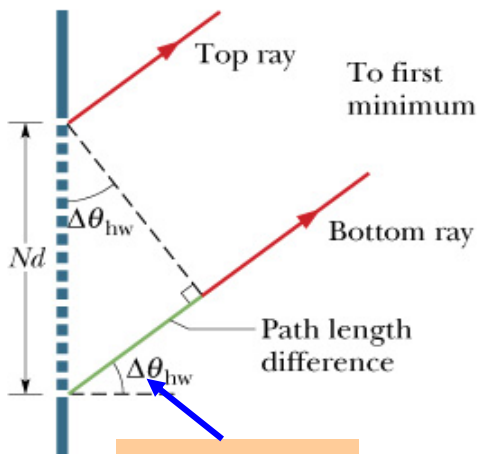
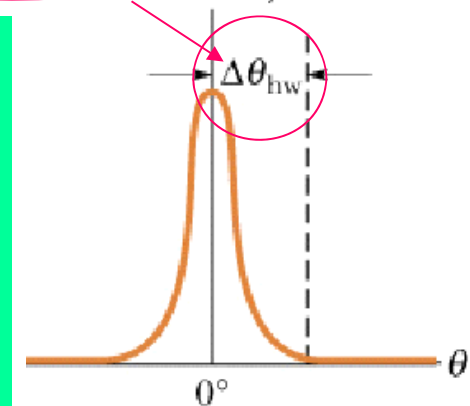


$$d \sin \theta = m\lambda \quad \text{for } m = 0, 1, 2, \dots \quad (\text{maxima-lines})$$

중앙선의 반너비 각; half width

- 격자(발)의 수 N 이 클수록 예리한 회절선
- 밝은 띠의 밝기:

슬릿의 개수의 제곱에 비례



1차 극소
반너비 각

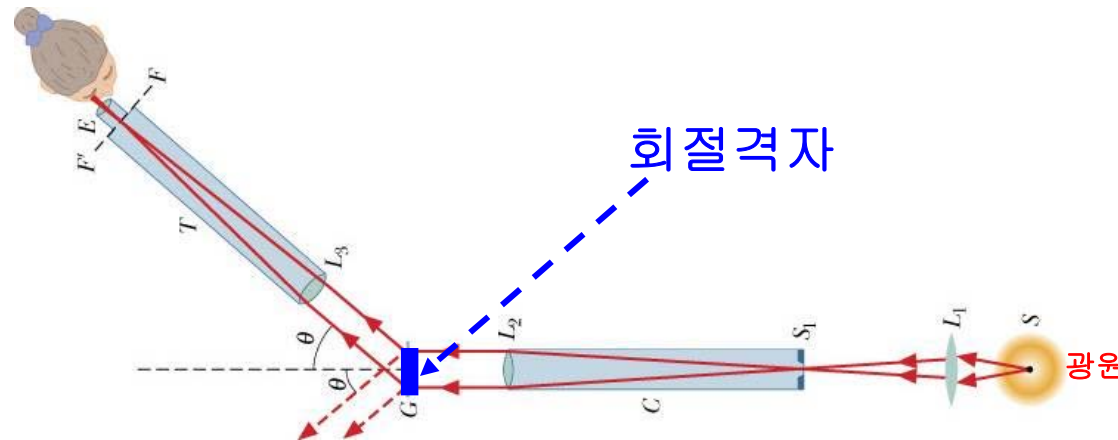
제1극소의 위치:

$$\text{단일슬릿: } a \sin \theta = 1 \cdot \lambda \rightarrow \text{회절격자: } Nd \sin \theta = 1 \cdot \lambda$$

Grating Spectroscope; 회절 분광기



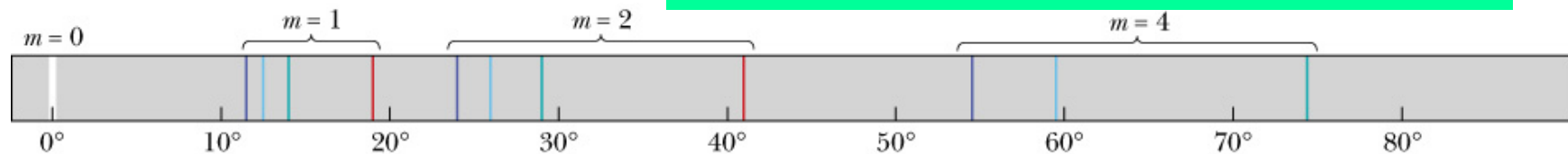
(p.1213)



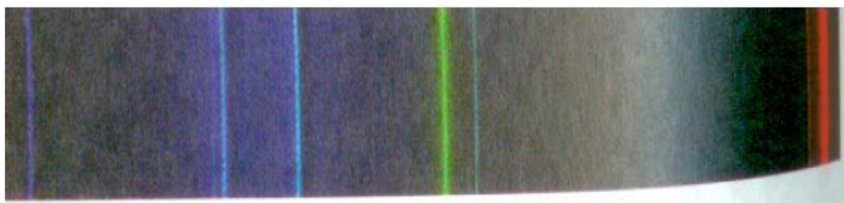
$$d \sin \theta = m\lambda \quad \text{for } m = 0, 1, 2 \dots \text{ (maxima-lines)}$$

• 수소의 방출 스펙트럼 (회절무늬)

• separates different wavelengths for a given m



• 에돌이발 분광기를 이용한 카드뮴 방출선



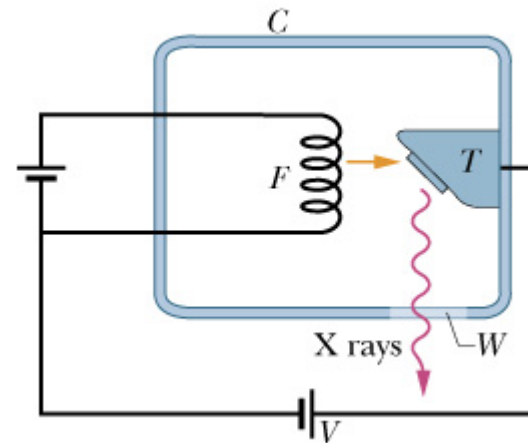
X-선 회절: 결정구조 분석에 활용 (p.1244-1245)



1) X-선의 파장: 약 $1 \text{ \AA} (= 10^{-10} \text{ m})$

※ 가시광의 중심파장: $550 \text{ nm} (= 5.5 \times 10^{-7} \text{ m})$

2) X-선 발생장치의 구조도



$$\theta = \sin^{-1} \left(\frac{m\lambda}{d} \right) \approx 0.0019^\circ \text{ for } \lambda \approx 0.1 \text{ nm}, d \approx 3000 \text{ nm}$$

X-선 파장

광학용
회절격자

300 slits/mm

Too small to be distinguished
from the central maximum !!

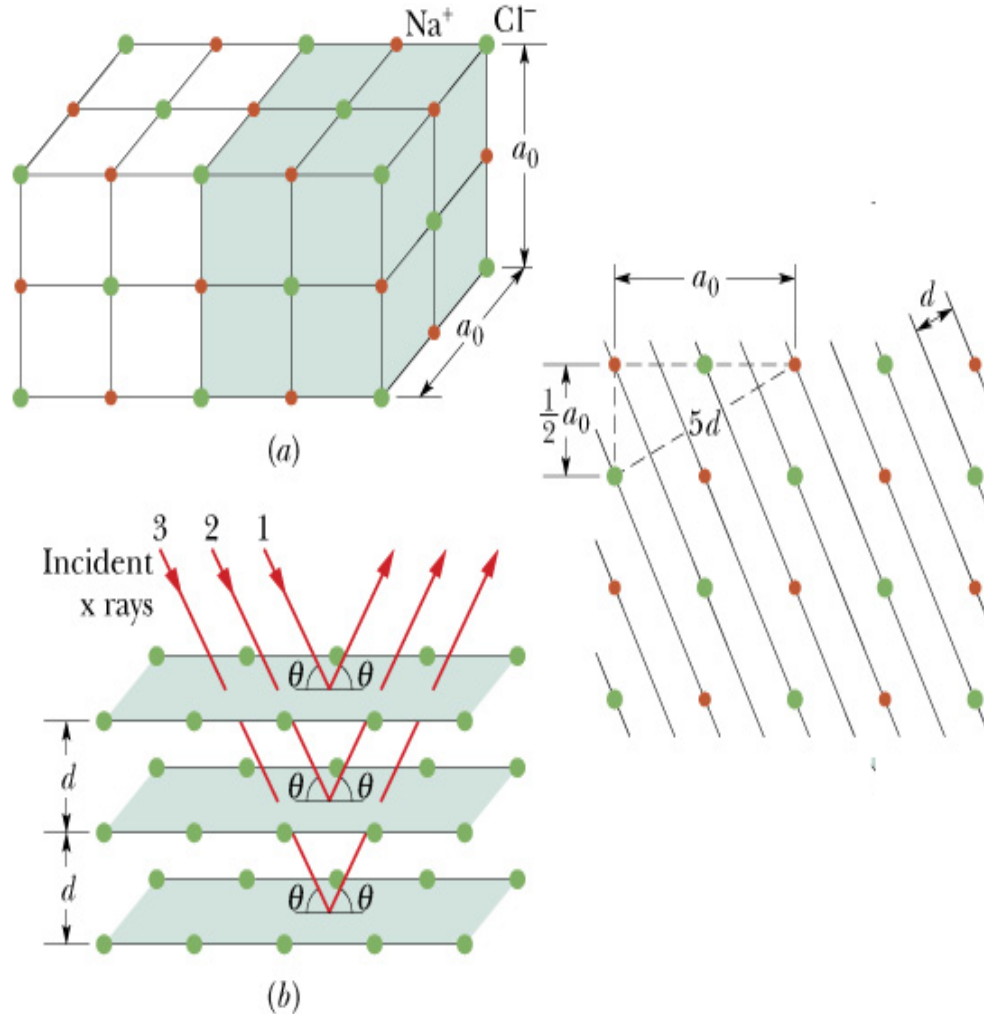


Diffraction of x-rays by crystal: spacing d of the order of 0.1 nm



3) X-선 회절을 쓴 결정구조 분석 (Max von Laue, 1912)

$$2d \sin \theta = m\lambda, \quad m=0, 1, 2, \dots$$



X-Ray Diffraction, cont'd



$$2d \sin \theta = m\lambda, \quad m=0, 1, 2, \dots$$

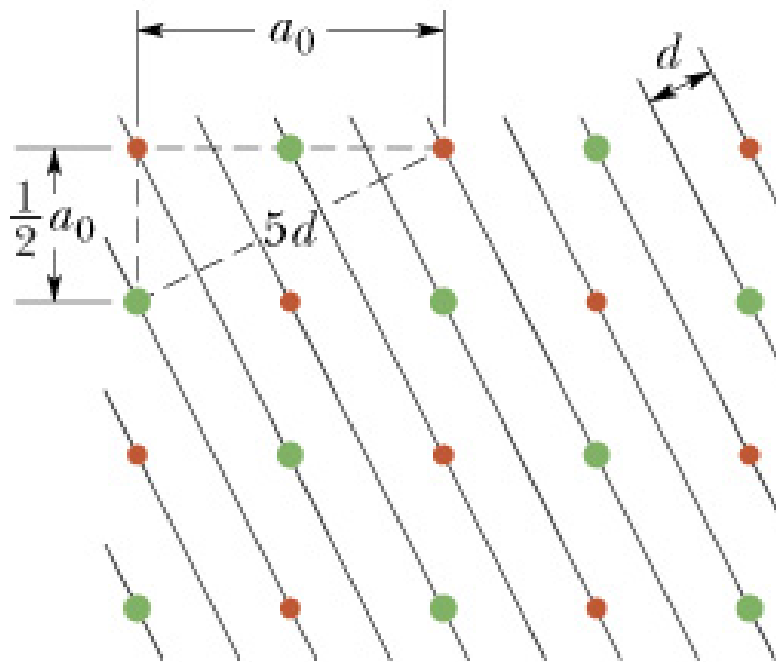


Fig. 36-29

inter-planar spacing d is related to the unit cell dimension a_0

$$5d = \sqrt{\frac{5}{4}} a_0^2 \quad \text{or} \quad d = \frac{a_0}{20} = 0.2236a_0$$

Not only can

- 1) **crystals** be used to separate different x-ray wavelengths, but
- 2) **x-rays**, in turn, can be used to **study crystals**, for example determine the type crystal ordering and a_0





대상의 범위

양자장 이론

- Pauli Adrien Maurice [Dirac](#) (1902. 8. 8-1984. 10. 20),
- Wolfgang [Pauli](#) (1900. 4. 25-1958. 12. 15),
- Julian Seymour [Schwinger](#) (1918. 2. 12 -),
- Richard Phillips [Feynman](#) (1918. 5. 11-1988. 2. 15),

상대론적 역학

- Albert [Einstein](#) (1879. 3. 14-1955. 4. 18),
- Hermann [Minkowski](#) (1864. 6. 22-1909. 1. 12),

•
•
•

양자역학

- Niels Hendrik David [Bohr](#) (1885. 10. 7-1962. 11. 18),
- Erwin [Schrödinger](#) (1887. 8. 12-1961. 1. 4),
- Werner [Heisenberg](#) (1901. 12. 5-1976. 2. 1),
-
-

고전역학

- Galileo [Galilei](#) (1564. 2. 15-1642. 1. 8),
- Isaac [Newton](#) (1642. 12. 25-1727. 3. 20),
- Joseph Louis [Lagrange](#) (1736. 1. 25-1813. 4. 10),
- Pierre Simon de [Laplace](#) (1749. 3. 23-1827. 3. 5),

→
계의 크기

