Chapter 8. Module and Process Design (Part II)
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8.6 Reverse Osmosis

- Solvent flux \( J_w = A (\Delta P - \Delta \pi) \) (8-6)

  where \( A \) = permeability constant

- Solute flux \( J_s = B (c_f - c_p) \) (8-7)

  where \( B \) = permeability coefficient

- Permeate concentration \( c_p \),

\[
\frac{c_p}{J_w} = \frac{J_s}{J_w} = \frac{B (c_f - c_p)}{J_w} = \frac{B}{J_w + B} \cdot \frac{c_f}{c_f} = \frac{B}{J_w + B}
\] (8-8 & 8-9)

  Combining Eq(8-9) with Eq(8-4) ⇒

\[
R = 1 - \frac{B}{c_f (J_w + B)} = 1 - \frac{B}{J_w + B}
\] (8-10)

  or

\[
\frac{J_w (1 - R)}{R} = B
\] (8-11)

- For high value of retention coefficient \( R > 90\% \)

  Eq(8-11) ⇒ \( J_w (1 - R) = \text{constant} \) (8-12)

『Meaning』 \( \Delta P \uparrow \Rightarrow J_w \uparrow \Rightarrow R \uparrow \)
**8.6 Reverse Osmosis**

- **Equations for cross-flow RO**
  - **<Assume> Process conditions remain constant**
    - No pressure drop
    - No change in osmotic pressure
    - Rejection coefficient $R \neq f(\text{feed concentration})$
  - **Mass balance under steady state**
    - Water: $q_f = q_p + q_r$  \hspace{1cm} (8-13)
    - Solute: $q_f \cdot c_f = q_p \cdot c_p + q_r \cdot c_r$  \hspace{1cm} (8-14)
    - Eq(8-1) and (8-13) $\rightarrow$ Eq(8-14) $\Rightarrow$

$$c_r = \frac{(c_f - S \cdot c_p)}{1 - S}$$  \hspace{1cm} (8-15)

where $S = \text{recovery} = \frac{q_p}{q_f}$  \hspace{1cm} (8-1)

**<Figure 8-30>** Schematical representation of RO process.
8.6 Reverse Osmosis

✓ For segment of <Figure 8-30>

\[ c' = \frac{(c_f - S' c_{p'})}{1 - S'} \]  

(8-16)

where \( c' \) = retentate concentration of segment

\( c_{p'} = \) average permeate concentration of segment

\[ c_{p'} = \frac{1}{S'} \int (1 - R) c' \, dS \]  

(8-17)

✓ Substitution of Eq(8-16) into (8-17) ⇒

\[ c' = \frac{1}{1 - S'} \left[ c_f - \int (1 - R) c' \, dS' \right] \]  

(8-18)

✓ Differentiation with respect to \( S' \) ⇒

\[ \frac{d[c'(1 - S')]}{dS'} = \frac{dc_f}{dS'} - (1 - R) c' \]  

(8-19)

✓ Since \( dc/dS' = 0 \) ⇒

\[ (1 - S') \frac{dc'}{dS'} + c' \frac{d(1 - S')}{dS'} = - (1 - R) c' \]  

(8-20)

or

\[ \frac{dc'}{dS'} = \frac{R \, c'}{(1 - S')} \]  

⇒

\[ \int \frac{dc'}{c'} = - \int \frac{R \, d(1 - S')}{(1 - S')} \]  

(8-21) & (8-22)
8.6 Reverse Osmosis

Integration over the whole system between the boundaries 0 to S and c_f to c_r
- \( c_r = c_f (1 - S)^{-R} \)  
- \( c_p = c_f (1 - R)(1 - S)^{-R} \)

Permeate concentration \( \neq \) constant \( \Rightarrow \) use an average permeate concentration \( c_p \)
- Eq(8-15) \( c_r = \frac{(c_f - S \cdot c_p)}{1 - S} \) \( \Rightarrow \) \( c_r = \frac{(c_f - S \bar{c}_p)}{1 - S} \)
- Combining Eq(8-26) with Eq(8-23) \( \Rightarrow \) \( \bar{c}_p = \frac{c_f}{S} \left[ 1 - (1 - S)^{1-R} \right] \)

Estimation example for RO with 95% retention
- NaCl concentration in feed = 2,000 ppm
- If zero recovery (S = 0) \( \Rightarrow \) permeate salt concentration (\( c_p \)) = 100 ppm from Eq(8-4)
- If 80% recovery (S = 0.8) \( \Rightarrow \) permeate salt concentration (\( c_p \)) = 193 ppm from Eq(8-27)
8.7 Diafiltration

- **Purpose of diafiltration**
  - For complete separation between high MW and low MW solutes
  - Used in biotechnology or the pharmaceutical and food industries

- **Process (dilution mode)**
  1. pre-concentration
  2. Dilute retentate with solvent (water)
  3. Filtration again ⇒ low MW solutes are washed out to permeate

- **Characteristics**
  - Same with other membrane process or membrane operation
  - Design simply to obtain a better purification or fractionation
  - Use UF units

<Figure 8-31> Schematic of diafiltration arrangement.
8.7 Diafiltration

- Comparison of diafiltration with CSTR
  - **CSTR**: all the solutes present (low and high MW) are washed out.
  - **Diafiltration**
    - High MW solutes: retained by membrane
    - Low MW solutes: permeates through the membrane

<Figure 8-32> Schematics of a continuous stirred tank reactor, CSTR (left) and a diafiltration system (right).
<Assume>

- Retain high MW solutes (macro molecule) by UF membrane completely \( R = 1 \)
- Pass low MW solutes (salts) through the membrane \( R = 0 \)
- Add water at a rate equal to the permeation rate \( \Rightarrow \) Volume in feed tank = constant
- Macromolecules remain in the feed tank

Mass balance

- Amount of solute in the feed tank per unit time = permeation rate of the salt
- Water: \( q_w = q_p \) \hspace{1cm} (8-28)
- Solute: \( \frac{q_p c_p}{V_o} = - \frac{dc_r}{dt} \) \hspace{1cm} (8-29)
  where \( c_p = (1 - R) c_r \) \hspace{1cm} (8-30)
  \( R \) = membrane retention for the low MW solute

Integration of Eq(8-29) with boundary conditions

BC 1: \( c_r = c_r^0 \) at \( t = 0 \)
BC 2: \( c_r = c_r^\dagger \) at \( t = t \)

\[
\frac{c_r^\dagger}{c_r^0} = \exp \left[ - \frac{q_w t (1 - R)}{V_o} \right] \hspace{1cm} (8-31)
\]
8.7 Diafiltration

✓ Total volume of water at time $t$: $w = q_w \cdot t$ \hspace{1cm} (8-32)

✓ Substitution of Eq(8-32) into Eq(8-31) ⇒

\[
\frac{c_f^t}{c_f^0} = \exp \left[ - \frac{V_w}{V_o} \left( 1 - R \right) \right] \hspace{1cm} (8-33)
\]

✓ For low MW, $R = 0$ ⇒ Eq(8-33) ⇒ 37% of low MW solute with water (volume = $V_o$)

⇒ need at least 5 times of initial volume $V_o$ for > 99% removal of the low MW solute

(or to reduce the ratio $c_r^t/c_r^0$ to less than 0.01)

✓ In real, $R \neq 0$ ⇒ need more water

✓ CSTR(no membrane) by setting $R=0$

• Eq(8-33) ⇒

\[
\frac{c_f^t}{c_f^0} = \exp \left[ - \frac{V_w}{V_o} \right] \hspace{1cm} \text{: Equation for CSTR} \hspace{1cm} (8-34)
\]

• No fractionation is obtained with a CSTR
8.8 Gas Separation and Vapor Permeation

- <Assume>
  - Permeability coefficients = constant
  - Separation occurs under isothermal condition
  - Complete mixing both in feed and permeate ⇒ most simple equations
    - Reasonable only systems operated at low recovery
    - Calculations are dependent on the flow pattern in the module.

- Gas separation systems: resemble with cross-flow conditions
  - Plug flow at the feed side
  - Complete mixing at the permeate side

- Vapor permeation
  - Same approach with gas separation
  - Permeability coefficient = \( f(\text{vapor activity}) \neq \text{constant} \)
8.8 Gas Separation and Vapor Permeation

8.8.1 Gas Separation under Complete Mixing Conditions

- **Complete mixing**
  - Concentrations at feed side = same at any point in the module = retentate concentrations
  - Concentration of permeate side = same at any point (see <Figure 8-33>)

- **Overall mass balance:**
  \[ q_f = q_p + q_r \] (8-35)

- **Mass balance for component \( i \)**
  - \( q_{f,i} = q_{p,i} + q_{r,i} \) (8-36)
  - or

- **Recovery (※ use 'cut' or 'stage cut' in gas separation)**
  - \[ S = \frac{q_p}{q_f} \] (8-38)
  - Dividing Eq(8-38) by \( q_f \) ⇒ permeate concentration
    \[ x_{p,i} = \frac{x_{f,i} - x_{r,i}(1 - S)}{S} \] (8-39)
    and
    \[ x_{r,i} = \frac{x_{f,i} - S x_{p,i}}{(1 - S)} \] (8-40)
8.8 Gas Separation and Vapor Permeation

8.8.1 Gas Separation under Complete Mixing Conditions

**Flux of a gas i through a membrane assuming perfect mixing**

\[
J_i = \frac{P_i}{\ell} \Delta p_i = \frac{P_i}{\ell} \left( x_{r,i} p_h - x_{p,i} p_\ell \right)
\]  

(8-41)

where \( P_i \) = permeability coefficient of component i

\( \ell \) = membrane thickness

\( p_h \) = pressure on feed side (high-pressure side)

\( p_\ell \) = pressure on permeate side (low pressure side)

\( x_{r,i} \) = constant mole fractions of component i in feed

\( x_{p,i} \) = constant mole fractions of component i in permeate

**Permeate flow rate of component i (q_{p,i})**

\[
q_{p,i} = q_p \times x_{p,i} = J_i A = \frac{A P_i}{\ell} \left( x_{r,i} p_h - x_{p,i} p_\ell \right)
\]  

(8-42)

and for component j

\[
q_{p,j} = q_p (1 - x_{p,i}) = J_j A = \frac{A P_j}{\ell} \left[ (1 - x_{r,i}) p_h - (1 - x_{p,i}) p_\ell \right]
\]  

(8-43)
8.8 Gas Separation and Vapor Permeation

8.8.1 Gas Separation under Complete Mixing Conditions

Dividing Eq(8-42) by Eq(8-43) ⇒

\[
\frac{x_{p,i}}{1 - x_{p,i}} = \frac{P_i}{P_j} \left[ \frac{x_{r,i} - \frac{P_h}{P_\ell} x_{p,i}}{(1 - x_{r,i}) - \frac{P_h}{P_\ell} (1 - x_{p,i})} \right] \quad (8-44)
\]

where

\[
x_{p,i} = B - \left[ B^2 - \frac{\alpha}{(\alpha - 1) P_r} \right] x_{r,i}^{0.5} \quad (8-46)
\]

\[
P_r = \frac{P_i}{P_h} \text{ and } \alpha = \frac{P_i}{P_j} \quad (8-47)
\]

In order to relate \(x_{p,i}\) to \(x_{f,i}\), substitute Eq(8-40) into Eq(8-44) ⇒

\[
\left[ P_r (1 - S) (1 - \alpha) + S (\alpha - 1) \right] x_{p,i}^2 + (S - 1) (P_r (\alpha - 1) + 1) + x_{f,i} (1 - \alpha - \alpha \cdot S) x_{p,i} + \alpha \cdot x_{f,i} = 0 \quad (8-48)
\]
8.8 Gas Separation and Vapor Permeation

**8.8.2 Gas Separation under Cross-flow Conditions**

- **Mass balance over dA for cross-flow model**
  
  \[
  - d \left( q' \ x'_i \right) = \frac{P_i}{\ell} \left( x'_i \ p_h - x_{p,i} \ p_l \right) dA \quad (8-49)
  \]

  \[
  - dq' \ (1 - x'_i) = \frac{P_i}{\ell} \left[ (1 - x'_i) \ p_h - (1 - x_{p,i}) \ p_l \right] dA \quad (8-50)
  \]

  Where superscript ' = high pressure feed side

- **Permeate concentration (x_{p,i})**
  
  \[
  x_{p,i} = \frac{d (x'_i \ q')}{d (x'_i \ q') + d ([1 - x'_i] \ q')} = \frac{d (x'_i \ q')}{dq'}
  \]
  
  \[
  (8-51)
  \]

- **For component j**

  \[
  1 - x_{p,i} = \frac{d ([1 - x'_i] \ q')}{d (x'_i \ q') + d ([1 - x'_i] \ q')} = \frac{d ([1 - x'_i] \ q')}{dq'}
  \]

  \[
  (8-52)
  \]

- **Substitution of Eq (8-49) and (8-50) in (8-51) and (8-52)**

  \[
  x_{p,i} = \frac{P_i}{P_j} \left[ \frac{x_i}{P_i} - \frac{P_h}{P_l} x_{p,i} \right]
  \]

  \[
  \frac{1 - x_{p,i}}{P_j} \left[ (1 - x_i) - \frac{P_h}{P_l} (1 - x_{p,i}) \right]
  \]

  \[
  (8-53)
  \]

  \[
  x_{p,i} = \frac{P_i}{P_j} \left[ \frac{x_{r,i}}{P_i} - \frac{P_h}{P_l} x_{p,i} \right]
  \]

  \[
  \frac{1 - x_{p,i}}{P_j} \left[ (1 - x_{r,i}) - \frac{P_h}{P_l} (1 - x_{p,i}) \right]
  \]

  \[
  (8-44)
  \]

**<Figure 8-34> Flow diagram of a cross-flow design.**
8.8 Gas Separation and Vapor Permeation

8.8.2 Gas Separation under Cross-flow Conditions

- **Cross-flow model**
  - Feed concentration = dependent on the place in the module
  - Module to be divided in an infinite number of small modules where complete mixing occurs. ⇒ Cross-flow model = small complete mixing segments are continuously connected.
  - When the feed and retentate concentrations differ quite considerably ($x_r/x_f < 0.5$), the system may be divided into a number of steps with $x_r/x_f = 0.5$ because otherwise the error in the calculations will become too large.

- Log mean average feed concentration $x$:
  \[ \overline{x}_i = \frac{x_{f,i} - x_{r,i}}{\ln\left(\frac{x_{f,i}}{x_{r,i}}\right)} \] \hspace{1cm} (8-54)

- Solving Eq(8-53) ⇒ Composition of the permeate
  \[ x_{p,i} = B - \left[ B^2 - \frac{\alpha}{(\alpha - 1) P_r} \overline{x}_i \right]^{0.5} \] \hspace{1cm} (8-55)

  where
  \[ B = 0.5 \left[ 1 + \frac{1}{(\alpha - 1) P_r} + \frac{\overline{x}_i}{P_r} \right] \] \hspace{1cm} (8-56)

- Membrane area:
  \[ A = \frac{q_{p,i}}{J_i} = \frac{q_p x_{p,i}}{J_i} \] \hspace{1cm} (8-57)
8.9 Pervaporation

- Driving force and permeability coefficient for pervaporation = \( f(\text{temperature and concentration}) \)
- Two process configurations
  - Complete mixing at feed & permeate \( \Rightarrow \) easy calculation
  - Cross-flow at feed & complete mixing at vapor phase
    (More similar to commercial pervaporation systems)

\[ \text{Figure 8-35} \] Pervaporation with perfect mixing at feed side and permeate side.
8.9 Pervaporation

- Mass balances

  - Retentate composition from Eq(8-40): 
  \[ x_{r,i} = \frac{x_{f,i} - S \cdot x_{p,i}}{1 - S} \]  
  where recovery \( S = \frac{q_p}{q_r} \)  

  - Selectivity for a perfect mixed system with components \( i \) and \( j \)  
  \[ \alpha_{i/j} = \frac{x_{p,i}}{x_{r,i}} \frac{x_{p,j}}{x_{r,j}} = \frac{x_{p,i}}{x_{r,i}} \cdot \frac{x_{r,j}}{1 - x_{r,j}} \]  
  where \( i = \) more permeable component  

  - Combination of Eq(8-40) with (8-59)  
  \[ S(\alpha_{i/j} - 1) x_{p,i}^2 - [x_{f,i} (\alpha_{i/j} - 1) + (1 - S) + \alpha_{i/j} \cdot S] x_{p,i} + \alpha_{i/j} \cdot x_{f,i} = 0 \]  
  quadratic in \( x_{p,i} \) and linear in \( x_{f,i} \) and \( S \)  

  - \( x_{p,i} \) and \( x_{f,i} \) ⇒ Calculate recovery,  
  \[ S = \frac{\alpha_{i/j} \cdot x_{p,i} - (\alpha_{i/j} - 1) \cdot x_{f,i}}{x_{r,i} \cdot (x_{p,i}^2 - x_{p,i})} \]  
  (8-61)  

  - \( x_{p,i} \) and \( S \) ⇒ Calculate \( x_{f,i} \),  
  \[ x_{f,i} = \frac{(1 - \alpha_{i/j}) \cdot x_{p,i} - (\alpha_{i/j} - 1) \cdot S \cdot x_{p,i}^2 + \alpha_{i/j} \cdot S \cdot x_{p,i}}{\alpha_{i/j} - (\alpha_{i/j} - 1) \cdot x_{p,i}} \]  
  (8-62)
**8.9 Pervaporation**

## 8.9.1 Complete Mixing in Pervaporation

- **Energy balance**
  - Temperature at feed side = constant for complete mixing.
  - \( q_f c_{p,f} (T_f - T') = q_r c_{p,r} (T_r - T') + q_f c_{p,p} (T_p - T') + \Delta H_{vap} \cdot q_p \) (8-63)
    - Where \( \Delta H_{vap} \) = heat of vaporization and \( c \) = heat capacity
  - Thermal equilibrium between feed & permeate \( \Rightarrow T_r = T_p = T' \)
    - Where \( T' \) = reference temperature (arbitrarily chosen)
  - Eq(8-63) \( \Rightarrow q_f c_{p,f} (T_f - T_r) = \Delta H_{vap} \cdot q_p \) (8-64)

- Flux of component \( i \):
  - \( J_i = \frac{P_i}{\ell} \Delta p_i \) (8-66)

- Liquid feed ≠ ideal generally, but gas phase = assumed to be ideal
  - \( J_i = \frac{P_i}{\ell} \left( x_{r,i} \gamma_i p_i^0 - x_{p,i} p_\ell \right) \) (8-67)
    - Where \( \gamma_i \) = activity coefficient of component \( i \) in the mixture
    - \( P_i^0 \) = saturation pressure of the pure component \( i \) at temperature \( T \)

- From \( x_{r,i} \) and \( x_{p,i} \) \( \Rightarrow \) Calculate flux \( \Rightarrow \) Calculate membrane area from Eq(8-57)
8.9 Pervaporation

- Cross-flow at feed side
  - More realistic pattern
  - Perfect mixing at permeate side

- Difference with complete mixing case
  - Concentration of $i$ at feed = change gradually from feed inlet $x_{r,i}$ to retentate $x_{r,i}$
  - Temperature $\downarrow$ as well across the feed side
  - Permeability coefficient ($P_i$) = $f$(concentration & temperature)
  - Divided into a number of segments (permeability coefficient = constant in every segment)

  $\Rightarrow$ Determine $P_i$ at every segment ($\because P_i = f(c, T)$)

- Flux in a certain stage

$$J_i = \left( \frac{\overline{P}_i}{\ell} \right) \left( \overline{x}_i \, \gamma_i \, P_i^0 - x_{p,i} \, P_\ell \right)$$  \hspace{1cm} (8-68)

where $x_i = \text{log mean concentration}$

$P_i = \text{average permeability coefficient}$
8.10 Electro-dialysis (ED)

- Basic requirements of a system
  - Membrane stack (200 ~ 600 cell pair) in series
  - Pumps for feed, dilute and concentrate

- Equations to calculate membrane area required
  - Limiting current density $i_{\text{lim}} \Rightarrow$ main process parameter
  - Operating current density ($i_{\text{actual}} \leq i_{\text{lim}}$
  - Basic equation to remove ions
    \[
    I = \frac{z \, \mathcal{F} \, q \, \Delta c}{\xi} \quad \text{(8-69)}
    \]
    Where \( \mathcal{F} = \text{Faraday const.} (96500 \text{ Coulomb/eq}) \)
    - \( q = \text{flow rate (L/sec)} \)
    - \( \Delta c = \text{concentration difference} \)
      (feed ↔ product stream, eq/L)
    - \( \xi = \text{current utilization} \)

*Figure 8-36* Flow diagram for a single-stage ED process.
Current utilization ($\bar{\xi}$) = Number of cell(n) × electrical efficiency

Electrical efficiency ("how efficient the current is used to achieve the separation") depend on

- Efficiency of membrane ($\eta_s$) which is determined by membrane selectivity
- Efficiency due to water transport ($\eta_w$)
- Efficiency due to leakage of current through the manifold clamping membranes ($\eta_m$)
- Electrical efficiency < 1.0 ⇒ a value of 0.9 as a realistic estimate

$$\bar{\xi} = n \cdot \text{(electrical efficiency)} = n \cdot \eta_s \cdot \eta_w \cdot \eta_m$$  \hspace{1cm} (8-70)

Current density, $i = I / A_m$  \hspace{1cm} (8-71)

where $A_m = \text{area of a cation or anion exchange membrane}$

Total area, $A = n \cdot A_m$  \hspace{1cm} (8-72)

Substitution of Eq (8-71) and (8-72) into (8-69) ⇒ Total membrane area required

$$A = \frac{zF q n (c_{\text{feed}} - c_{\text{product}})}{i \cdot \bar{\xi}}$$  \hspace{1cm} (8-73)

Electrical efficiency

$$\text{Electrical efficiency} = \frac{F q \Delta c}{n I}$$  \hspace{1cm} (8-74)
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8.10 Electro-dialysis (ED)

✓ Total number of cells, \[ n = \frac{F \Delta c}{\text{electrical efficiency} \cdot I} \]  (8-75)

✓ Energy consumption, \[ E = n I^2 R_{cp} \eta \]  (8-76) where \( R_{cp} \) = resistance of a cell pair
  \[ n = \text{number of cell pairs in a stack} \]

✓ Resistance of a cell(\( R_{cp} \)) = resistance of membrane + solution resistance
  - Solution resistance \( \propto 1/(\text{salt concentration}) \)
  - Low salt concentration in dilute compartment \( \Rightarrow \) determine solution resistance

✓ Combination of Eq(8-69) and (8-76) \( \Rightarrow \) energy consumption
  \[ E = \frac{n I z F R_{cp} \Delta c q t}{\xi} \]  (8-77)

as a function of
  - Applied current
  - Current utilization
  - Electrical resistance
  - Amount of salt removed

✓ Total energy consumption = ionic transfer + pumps for
  - concentrated stream
  - depleted streams
  - anode- and cathode-rinse solutions

✓ Energy consumption, \[ E_p = \frac{q_v \Delta P}{\eta} \]  (8-78)

where \( E_p \) = energy requirement of the pump, \( q_v \) = flow rate
\[ \Delta P = \text{pressure drop} \quad \eta = \text{pump efficiency} \]
Hemodialysis in artificial kidney

- Remove toxic small MW metabolites (urea, creatinine, uric acid and others)
- Flow
  - Feed stream: blood
  - Solvent stream (Dialysate): contains vital salts (Na, K, Ca, Mg)

Diffusion dialysis

- Remove protons (H\(^+\)) or hydroxyl ion (OH\(^-\)) from an aqueous stream
- Flux of solute \(i\) through the membrane, \(J_i = k_{i,ov} (c_{f,i} - c_{d,i})\) (8-79)
  
  where \(c_{f,i}\) and \(c_{d,i}\) as the average feed and dialysate concentration
  
  \[ k_{i,ov} = \text{overall mass transfer coefficient from a resistance model} \]

- Resistance model: 
  \[
  \frac{1}{k_{i,ov}} = \frac{1}{k_{i,feed}} + \frac{1}{k_{i,dial}} + \frac{\ell}{P_i} \]
  (8-80)
  where \(k_{i,feed}\) = mass transfer coefficient of solute in feed boundary layer
  
  \(k_{i,dial}\) = mass transfer coefficient of solute in dialysate boundary layer
  
  \(\ell\) = membrane thickness

\(P_i\) = permeability coefficient of solute \(i\)

*Figure 8-37* Schematic of a counter-current flow.
8.11 Dialysis

- **Transfer rate of component** \( i \) **through membrane**:
  \[ q_i = k_{i,ov} A (c_{f,i} - c_{d,i}) \]  
  (8-81)

- **Logarithmic mean \( \Delta c \) and for a counter-current flow**
  \[
  \frac{(c_{f,i} - c_{d,i})}{c_{i}^{\text{in}} - c_{i}^{\text{out}}} = \frac{\ln \left( \frac{c_{f,i} - c_{d,i}}{c_{f,i} - c_{d,i}} \right)}{\ln \left( \frac{c_{f,i}^{\text{out}} - c_{f,i}^{\text{in}}} {c_{f,i}^{\text{out}} - c_{f,i}^{\text{in}}} \right)}
  \]
  (8-82)

  Furthermore
  \[ q_i = q_f (c_{f,i}^{\text{in}} - c_{f,i}^{\text{out}}) = q_d (c_{d,i}^{\text{out}} - c_{d,i}^{\text{in}}) \]
  (8-83)

- **Removal efficiency**:
  \[
  \frac{(c_{f,i}^{\text{in}} - c_{f,i}^{\text{out}})}{(c_{f,i}^{\text{in}} - c_{f,i}^{\text{in}})} = \frac{1 - \exp \left[ \frac{k_{i,ov} A}{q_f} \left( 1 - \frac{q_f}{q_d} \right) \right]}{q_f - \exp \left[ \frac{k_{i,ov} A}{q_f} \left( 1 - \frac{q_f}{q_d} \right) \right]}
  \]
  (8-84)

- **For co-current flow, Eq(8-82)**
  \[
  \frac{(c_{f,i} - c_{d,i})}{c_{f,i}^{\text{in}} - c_{f,i}^{\text{out}}} = \frac{(c_{f,i}^{\text{in}} - c_{d,i}^{\text{in}})}{(c_{f,i}^{\text{out}} - c_{d,i}^{\text{out}})}
  \]
  (8-85)

- **Fractional solute removal**:
  \[
  \frac{(c_{f,i} - c_{f,i}^{\text{out}})}{(c_{f,i}^{\text{in}} - c_{f,i}^{\text{in}})} = 1 - \exp \left[ - \frac{k_{i,ov} A}{q_f} \left( 1 + \frac{q_f}{q_d} \right) \right]
  \]
  (8-86)
8.12 Energy Requirements

- Driving force $\uparrow \Rightarrow$ energy consumption $\uparrow \Rightarrow$ flux $\uparrow \Rightarrow$ membrane area $\downarrow \Rightarrow$ investment cost $\downarrow$
- 2nd law of thermodynamics $\Rightarrow$ determine minimum amount of work of separation process
- Reversible separation process $\Rightarrow$ requires as much as work as a reversible mixing process
- Actual energy consumption $\Rightarrow$ this minimum energy for separation

<Figure 8-38> Schematic of capital cost (investment) and energy cost as a function of driving force.
8.12 Energy Requirements

**Low pressure application (MF/UF)**

- Severe concentration polarization and fouling ⇒ Need high mass transfer in boundary layer
  - High cross flow velocity ↑ ⇒ Use 2 pumps (one for pressurizing and other for circulation)
- Low hydrodynamic pressures ⇒ Use circulation pump = major energy consumption
- Energy consumption to pressurize a liquid from $P_1$ to $P_2$:
  \[ E_p = \frac{q_v \Delta P}{\eta} \]  
  (8-87)
  where $q_v$ = flow rate (m³/sec), $\Delta P$ = pressure drop (N/m²), $\eta$ = pump efficiency ($0.5 \leq \eta \leq 0.8$)

**High pressure applications (RO/NF, especially SWRO)**

- Need turbine to recover energy
- $Et = -\eta q_v \Delta P$  
  (8-88)
  $\eta$ = Turbine efficiency ($0.5 \leq \eta \leq 0.8$)

*Figure 8-39* Schematic of power devices applied in pressure driven membrane processes
8.12 Energy Requirements

- Processes using partial pressure difference as the driving force
  - Application process: Gas separation, Pervaporation, Vapor permeation
  - Compressor: pressurize the feed to a few bar
  - Vacuum pump at the permeate side: adjust the partial pressure difference

- Compression
  - <Assume> 1. Ideal gas (※ in real, gas = non-ideal)
  - 2. Isothermal compression (※ in real, gas = adiabatic compression)

- Power:
  \[ E = - \frac{1}{\eta} \int n \, RT \, dP = - \frac{n \, RT}{\eta} \ln \left( \frac{P_2}{P_1} \right) \]  
  \[ (8-89) \]
  where \( n = \text{mol/s} \)

- Vacuum pump
  - Same equations apply as for the compressor

<Figure 8-40> Schematic of gas separation
Dialysis and diffusion dialysis

✓ Low energy consuming membrane processes
✓ Pumps to circulate feed and dialysate ⇒ Energy consumption ($E_p$) [Eq(8-87)]