Chapter 9
Laminar Diffusion Flames

Objectives
1. Non-Reacting Laminar Jet
2. Jet Flame Physical Description
3. Flame length and parameter
   (Burner shape, flow rate, fuel type)
4. Soot formation
Nonreacting Constant-density Laminar Jet

Physical Description

- Potential core: the effect of viscous shear and diffusion
- The jet edge: the velocity and fuel concentration decrease

\[ J(x) = \rho_e v_e^2 \pi R^2 \int_0^\infty \rho(r, x) v_e^2 (r, x) r dr \]

- Momentum flow of the jet = from the nozzle, \( J_e \)

\[ J_e = \rho_e v_e^2 \pi R^2 \]

\( \rho_e \) = the density of the fuel at the nozzle exit

\( v_e \) = the velocity of the fuel at the nozzle exit

- the distribution of the fuel mass fraction is similar to the dimensionless velocity dist..

\[ Y_F(r, x) = \frac{v_F(r, x)}{v_e} \]

\[ 2\pi \int_0^\infty \rho(r, x) v_F^2 (r, x) r dr = \rho_e v_e \pi R^2 Y_F, e \]
Assumptions
1. Pressure and temp. = const.
2. B
3. Sc = v/D (Schmidt No.) = (momentum diffusivity)/(mass diffusivity) = 1.0
4. Only radial diffusion of momentum and species

Conservation Laws
Mass conservation
\[ \frac{\partial v_x}{\partial x} + \frac{1}{r} \frac{\partial (v_x r)}{\partial r} = 0 \]

Axial Momentum Conservation
\[ v_x \frac{\partial v_x}{\partial x} + v_r \frac{\partial v_x}{\partial r} = v \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_x}{\partial r} \right) \]

Species Conservation
\[ v_x \frac{\partial Y_F}{\partial x} + v_r \frac{\partial Y_F}{\partial r} = D \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial Y_F}{\partial r} \right) \]
\[ Y_{Or} = 1 - Y_F \]

Boundary Conditions
Along the jet centerline (r=0)
\[ v_x (0, x) = 0 \]
\[ \frac{\partial v_x}{\partial r} (0, x) = 0 \]
\[ \frac{\partial Y_F}{\partial r} (0, x) = 0 \]
\[ (r = \infty) \]
\[ v_x (\infty, x) = 0 \]
\[ Y_F (\infty, x) = 0 \]
At the jet exit (x=0)
\[ v_x(r \leq R, 0) = v_e \]
\[ v_x(r > R, 0) = 0 \]
\[ Y_F(r \leq R, 0) = Y_{F,e} = 1 \]
\[ Y_F(r > R, 0) = 0 \]

Solution

The axial velocity
\[ v_x = \frac{3}{8\pi \mu x} J_e \left[ 1 + \frac{\xi^2}{4} \right]^{-2} \]

The radial velocity
\[ v_r = \left( \frac{3J_e}{16\pi \rho} \right)^{1/2} \frac{1}{x} \left( \frac{\xi - \frac{\xi^3}{4}}{1 + \frac{\xi^2}{4}} \right) \]

\( J_e \): initial momentum flow
\[ J_e = \rho e v_e^2 \pi R^2 \]
\( \xi \) contains \( r/x \)

The axial velocity distribution in dimensionless.
\[ \frac{v_x}{v_e} = 0.375 \left( \rho e v_e R / \mu \right) \left( x / R \right)^{-1} \left[ 1 + \xi^2 / 4 \right]^{-2} \]

The dimensionless centerline velocity decay \((r=0, \xi=0)\)
\[ \frac{v_{x,0}}{v_e} = 0.375 \left( \rho e v_e R / \mu \right) \left( x / R \right)^{-1} \]

Centerline velocity decay for laminar jet
The solution for the concentration

\[ \frac{\partial v_r}{\partial x} + v_r \frac{\partial v_r}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_r}{\partial r} \right) \]

\[ \frac{\partial v_r}{\partial x} + v_r \frac{\partial v_r}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_r}{\partial r} \right) \]

**Sc** = \( v / D \) (Schmidt No.) = 1.0, \( v = D \)

\[ Y_r = \frac{3}{8\pi} \frac{Q_v}{D_s} \left[ 1 + \xi^2 / 4 \right]^2 \]

\( Q_v = \text{vemR}^2 \), volumetric flowrate of fuel from the nozzle

**Sc** = 1.0, \( v = D \)

\[ Y_r = 0.375 \text{Re}_j (x / R)^{-1} \left[ 1 + \xi^2 / 4 \right]^2 \]

**Solve** ex9.1, ex9.2
Jet Flame Physical Description

**Jet Flame Physical Description**

Fuel diffuses radially outward.
Oxidizer diffuses radially inward.

Flame surface ≡ Locus points where the equivalence ratio, \( \Phi \), equals unity
\[ \Phi(r = 0, x = L_f) = 1 \quad L_f : \text{flame length} \]

Jet Flame Physical Description

- **Vertical flame,**
  Hot gas → buoyant forces → acceleration of flow → narrowing of the flame → fuel concentration gradient \( \frac{dY_f}{dt} \) → enhancing diffusion

- **Soot formation for HC fuel flame**
  Soot is formed on the fuel side of the reaction zone.
Jet Flame Physical Description

Flame length

\[ Y_F = \frac{3}{8\pi} \frac{Q_F}{D_x} \left[ 1 + \frac{\xi^2}{4} \right] \]

for nonreacting flow

\[ r = 0, \text{ flame length} \]

\[ L_f \approx \frac{3}{8\pi} \frac{Q_F}{D Y_{F,\text{static}}} \]

Simplified Theoretical Descriptions

Burke and Schumann

Assumptions

1. Laminar flow, steady, axisymmetric
2. 3 species → fuel, products, oxidizer
3. At flame \( \Phi = 1.0 \)
4. Binary diffusion (Fick’s law)
5. \( Le = a/D = (\text{thermal. Diff./mass diff.}) = 1.0 \)
6. Neglect radiation
7. Only radial diffusion of momentum, thermal energy
8. Vertically upward
Basic Conservation Equations

Mass conservation (7.7)

\[
\frac{1}{r} \frac{\partial}{\partial r} (r \rho v_r) + \frac{\partial}{\partial x} (\rho v_x) = 0
\]

Axial momentum conservation (7.48)

\[
\frac{1}{r} \frac{\partial}{\partial x} (r \rho v_r v_r) + \frac{1}{r} \frac{\partial}{\partial r} (r \rho v_r v_r) - \frac{1}{r} \frac{\partial}{\partial r} \left( r \mu \frac{\partial v_r}{\partial r} \right) = (\rho_w - \rho) g
\]

Species conservation (7.20)

\[
1 \frac{\partial}{\partial x} (r \rho v_r Y_r) + \frac{1}{r} \frac{\partial}{\partial r} (r \rho v_r Y_r) - \frac{1}{r} \frac{\partial}{\partial r} \left( r \rho D \frac{Y_r}{\partial r} \right) = 0
\]

Energy conservation (Shavb-Zeldovich, 7.66)

\[
\frac{\partial}{\partial x} (r \rho v_r \int c_r dT) + \frac{\partial}{\partial r} (r \rho v_r \int c_r dT) - \frac{\partial}{\partial r} \left( r \rho D \frac{\int c_r dT}{\partial r} \right) = 0
\]

Additional relations

\[
\rho = \frac{PMW_{\text{mix}}}{RT} \quad MW_{\text{mix}} = \left( \sum Y_i / WM_i \right)^{-1}
\]

Flame Lengths for Circular-port and Slot Burners

Roper’s Correlations

Burner types \(\rightarrow\) circular, square, slot, curved slot

Flame length

\[
L_f \approx \frac{3}{8\pi} \frac{1}{D_{F,\text{stoic}}} \frac{Q_F}{Y_{F,\text{stoic}}} \quad \text{Constant density solution}
\]

\[
L_f \approx \frac{3}{8\pi} \frac{1}{\mu_{\text{Ref}}} \frac{\dot{m}_F}{\rho_{\text{Ref}}} \frac{1}{I(\rho_w / \rho_f)}
\]

\[
L_f \approx \frac{3}{8\pi} \frac{1}{D_{\text{Ref}}} \frac{Q_F}{Y_{F,\text{stoic}}} \frac{\rho_F \rho_{\text{Ref}}}{\rho_{\text{Ref}}^2} \frac{1}{I(\rho_w / \rho_f)}
\]

Variable-density solution
Circular port

\[ L_{f,\text{thy}} = \frac{Q_f T_\infty}{4\pi D_\infty \ln(1+1/S)} \left( \frac{T_\infty}{T_f} \right)^{0.67} \]

\( S \) : the molar stoichiometric oxidizer-fuel ratio

\[ L_{f,\text{expt}} = 1330 \frac{Q_f (T_\infty / T_f)}{\ln(1+1/S)} \]

Square port

\[ L_{f,\text{thy}} = \frac{Q_f (T_\infty / T_f)}{16D_\infty \text{inverf}((1+S)^{-0.5})^2} \left( \frac{T_\infty}{T_f} \right)^{0.67} \]

\[ L_{f,\text{expt}} = 1045 \frac{Q_f (T_\infty / T_f)}{\text{inverf}((1+S)^{-0.5})^2} \]

Slot-burner – Momentum controlled

\[ L_{f,\text{thy}} = \frac{b \beta^2 Q_f}{h I Y_{F,\text{stoic}}} \left( \frac{T_\infty}{T_f} \right)^2 \left( \frac{T_f}{T_\infty} \right)^{0.33} \]

\[ L_{f,\text{expt}} = 8.6 \cdot 10^4 \frac{b \beta^2 Q_f}{h I Y_{F,\text{stoic}}} \left( \frac{T_\infty}{T_f} \right)^2 \]

\[ \beta = \frac{1}{4\text{inverf}[1/(1+S)]} \]

\[ I = \frac{J_{e,\text{act}}}{\dot{m}_e v_e} \]
Flowrate and Geometry Effects

- Circular port burner:
  - Exit velocity \( \propto L_f \)

- Slot-burner port:
  - More narrow port,
  - Shorter flame length

Predicted Flame Lengths for Circular and Slot-burners

Factors Affecting Stoichiometry

The molar stoichiometric ratio

\[
S = \left( \frac{\text{moles ambient fluid}}{\text{moles nozzle fluid}} \right)_{\text{stoic}}
\]

At circular-port, same flowrate
- Flame length increases as the H/C ratio decreases.

Dependence of Flame Length on fuel stoichiometry
Factors Affecting Stoichiometry

Primary Aeration:
- At 40-60% primary aeration of Stoichi.:
  - Suppression formation of soot
  - Blue flame
  - Shorter flame length

\[ S = \frac{1 - \psi_{pri}}{\psi_{pri} + \frac{1}{S_{pure}}} \]

Effect of primary aeration on laminar jet flame lengths

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Oxygen Content of Oxidizer

Effect of O\textsubscript{2} content in the oxidizing stream on flame length
Soot Formation & Destruction

Processes
1. Formation of precursor species
   PAH: polycyclic aromatic HC
2. Particle inception
3. Surface growth and particle agglomeration
4. Particle oxidation
   ➔ near

Radial profiles of temp. and scattered light for a lamina ethylene jet diffusion flame

Smoke point: a fuel’s tendency to soot

Smoke point:
- C3H8 = 7.87
- C4H10 = 7.00
- n-C7H16 = 5.13
- i-C8H18 = 1.57

Smoke point order:
- Alkanes (paraffins) 1.57-7.87>
- Alkenes (olefins) 1.12-3.84>
- Alkynes (acetylenes) 0.51-0.8>
- Aromatics 0.22-0.28
Countflow Flames

- A one dimensional character
- Fundamental understanding of the diffusion flame

Countflow Flames

Mathematical Description

Continuity (7.7)

\[ \frac{1}{r} \frac{\partial}{\partial r} (r \rho v_r) + \frac{\partial}{\partial x} (\rho v_x) = 0 \]

Axial momentum conservation (7.43)

\[ \frac{\partial}{\partial x} (r \rho v_v v_r) = \frac{\partial}{\partial r} (r \tau_r) + \frac{\partial}{\partial x} (\tau_x) + r \frac{\partial P}{\partial x} + \rho g_r r \]

Radial momentum conservation (7.44)

\[ \frac{\partial}{\partial x} (r \rho v_v r_r) = \frac{\partial}{\partial r} (r \tau_r) + r \frac{\partial P}{\partial r} + \rho g_r r \]

Stream function is employed:

\[ \Psi \equiv r^2 F(x), \quad \begin{cases} \frac{\partial \Psi}{\partial r} = r \rho v_x = 2rF \\ - \frac{\partial \Psi}{\partial x} = r \rho v_r = -r^2 \frac{dF}{dx} \end{cases}, \quad \frac{dF}{dx} = G. \]
Countflow Flames

Ignore buoyancy

\[ \frac{\partial P}{\partial x} = f_1(x) \]
\[ \frac{1}{r} \frac{\partial P}{\partial r} = f_2(x). \]

Left-hand-sides

\[ \frac{\partial}{\partial x} \left( \frac{1}{r} \frac{\partial P}{\partial r} \right) = \frac{1}{r} \frac{\partial}{\partial x} \left( \frac{\partial P}{\partial r} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{\partial P}{\partial x} \right). \]

\( \frac{\partial}{\partial x} \left( \frac{1}{r} \frac{\partial P}{\partial r} \right) \) and \( (1/r)(\partial P/\partial r) \) are functions of \( x \) only.

\[ \frac{\partial}{\partial x} \left( \frac{1}{r} \frac{\partial P}{\partial r} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{\partial P}{\partial x} \right) = 0 \]

\[ \frac{1}{r} \frac{\partial P}{\partial r} = \text{constant } \equiv H. \]

Countflow Flames

The radial-pressure-gradient eigenvalue

\[ \frac{dH}{dx} = 0. \]

The pressure = uniform

\[ \frac{d}{dx} \left[ \mu \frac{d}{dx} \left( \frac{G}{\rho} \right) \right] - 2 \frac{d}{dx} \left( \frac{FG}{\rho} \right) + \frac{3}{\rho} G^2 + H = 0. \]

Energy and species conservation equations

\[ 2F c_p \frac{dT}{dx} - \frac{d}{dx} \left( k \frac{dT}{dx} \right) + \sum_{i=1}^{N} \rho Y_i v_i \text{eff} c_p \frac{dT}{dx} - \sum_{i=1}^{N} h_i \omega_i M W_i = 0 \]

\[ 2F \frac{dY_i}{dx} + \frac{d}{dx} \left( \rho Y_i v_i \text{eff} \right) - \omega_i M W_i = 0 \quad i = 1, 2, \ldots, N. \]
Countflow Flames

Boundary Conditions

At $x = 0$:

- $F = \rho_F v_{e_F} / 2$,
- $G = 0$,
- $T = T_F$,
- $\rho v_x Y_i + \rho Y_i v_i \text{diff} = (\rho v_x Y_i)_F$;

At $x = L$:

- $F = \rho_{Ox} v_{e_{Ox}} / 2$,
- $G = 0$,
- $T = T_{Ox}$,
- $\rho v_x Y_i + \rho Y_i v_i \text{diff} = (\rho v_x Y_i)_{Ox}$.

Countflow Flames

Equivalence ratio, temp. and velocity profiles through CH$_4$-air countflow diffusion flame

By CHEMKIN
Countflow Flames

Major species mole-fraction profiles through CH₄-air countflow diffusion flame

Homework

Chapter 9
Review Questions 5, 8

Problems 9.5, 9.9, 9.13, 9.14