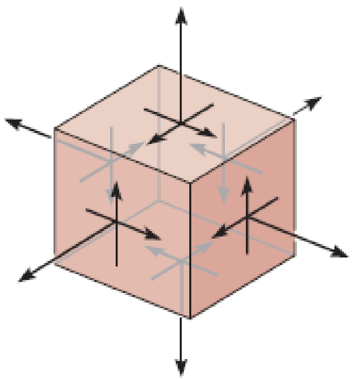
9.5 ABSOLUTE MAXIMUM SHEAR STRESS

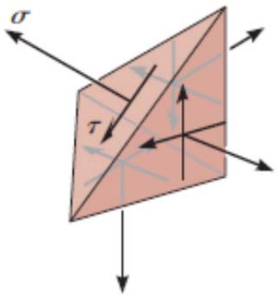
- State of stress in 3-dimensional space:

$$\tau_{\substack{\text{abs}\\ \text{max}}} = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

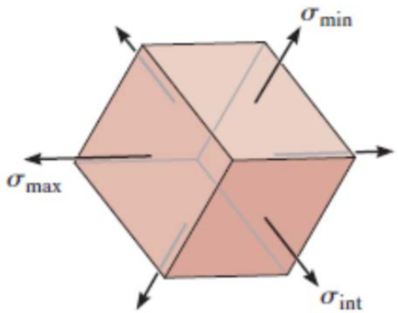
$$\sigma_{\text{avg}} = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$



(a)



(b)



triaxial stress

(c)

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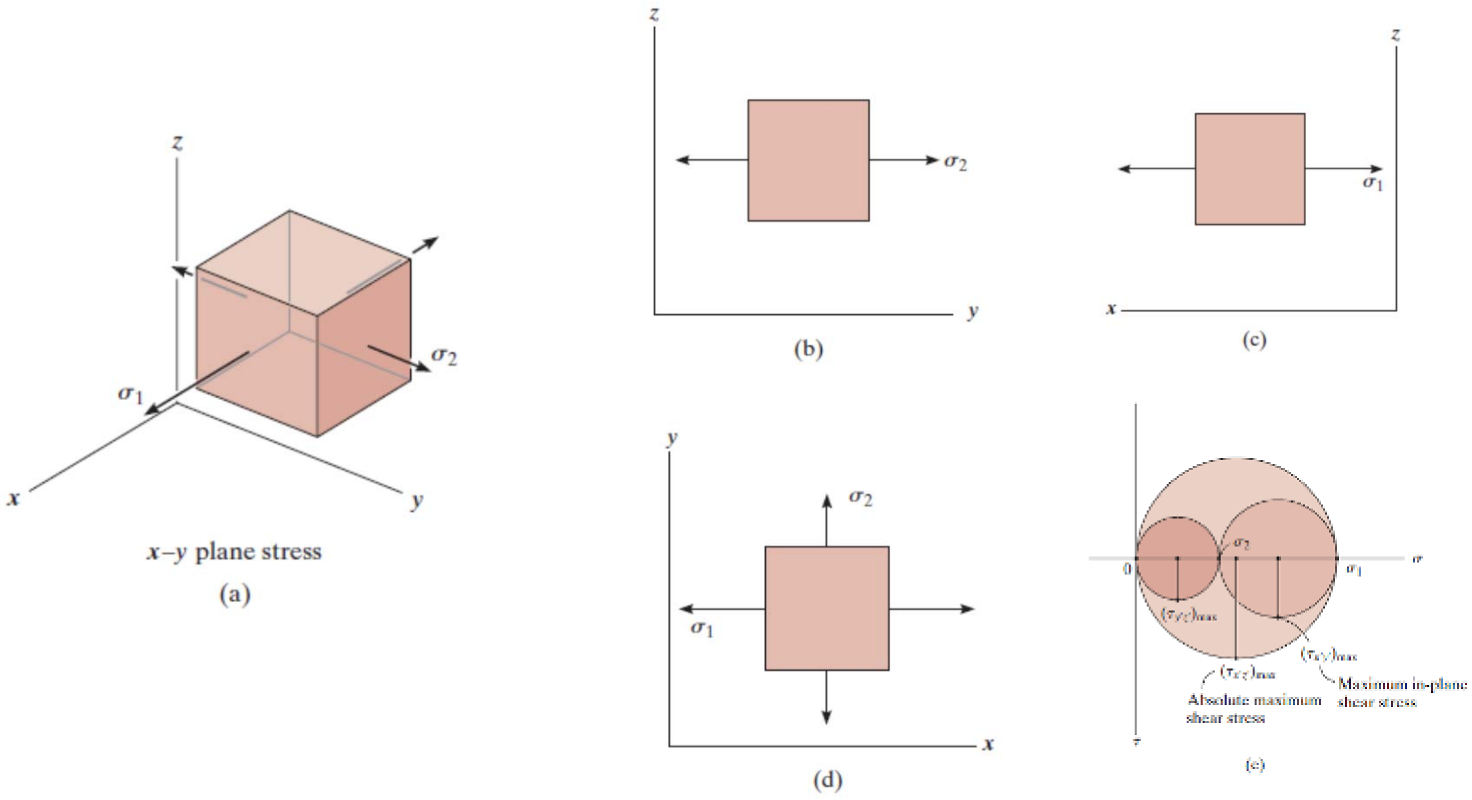
ABSOLUTE MAXIMUM SHEAR STRESS (cont)

- State of stress in 3-dimensional space:



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ABSOLUTE MAXIMUM SHEAR STRESS (cont)

- State of stress in 3-dimensional space:

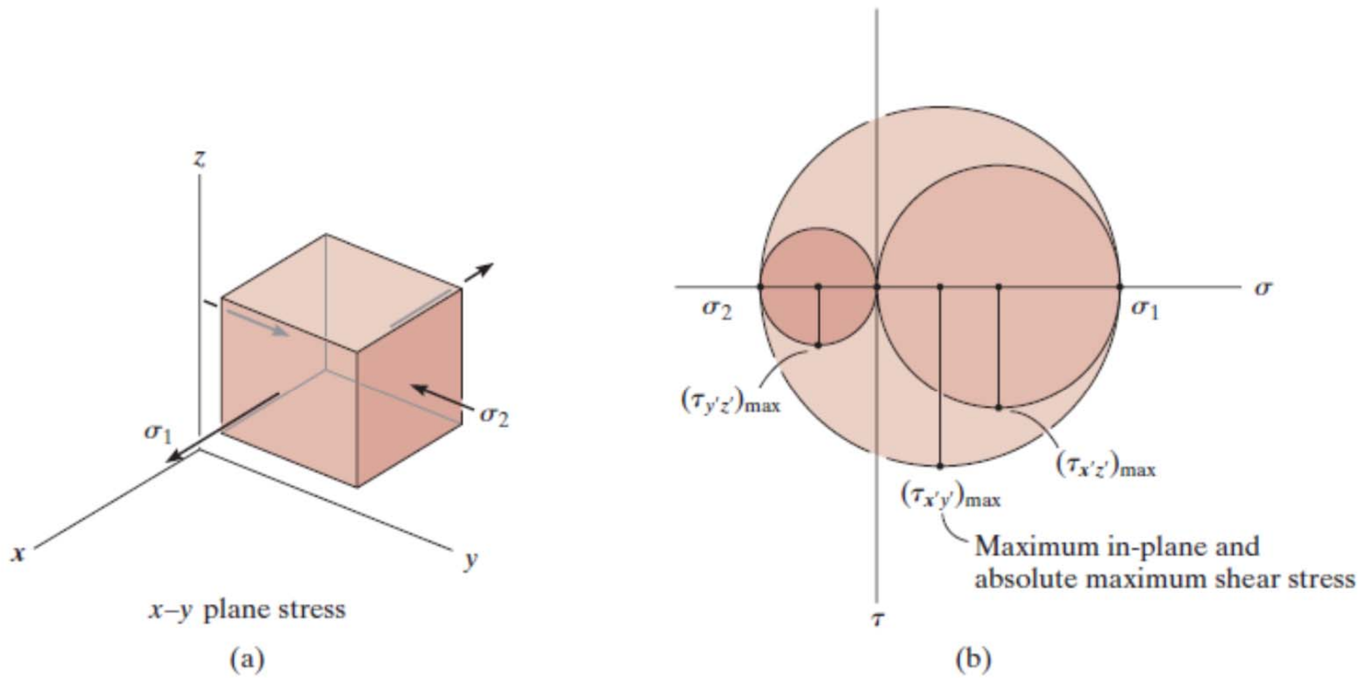

Fig. 9–23

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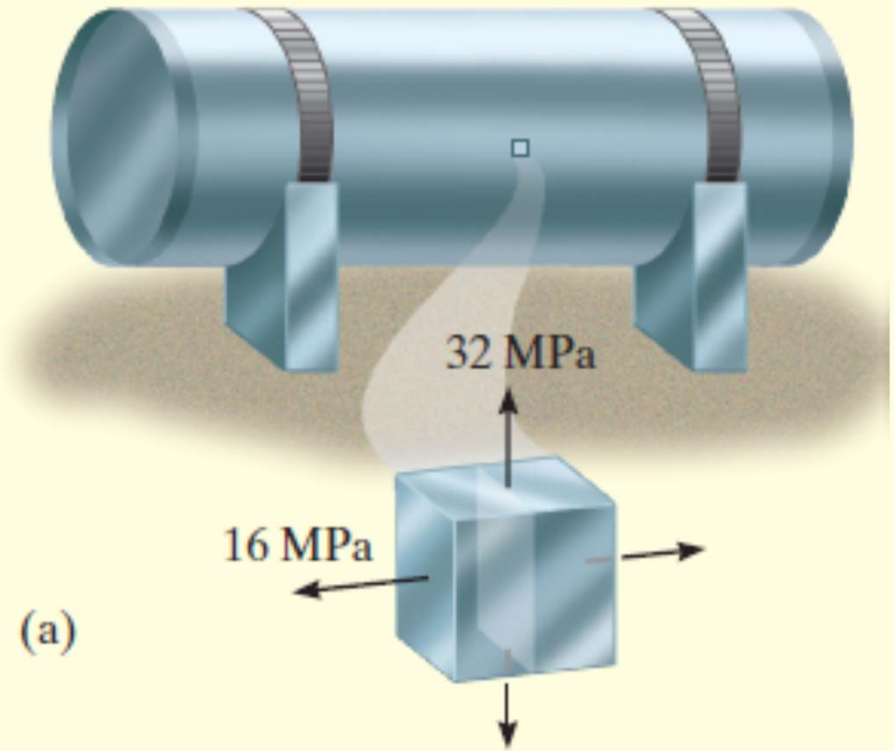
EXAMPLE 9.10

The point on the surface of the cylindrical pressure vessel in Fig. 9–24*a* is subjected to the state of plane stress. Determine the absolute maximum shear stress at this point.



(a)

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EXAMPLE 9.10 (cont)

Solutions

An orientation of an element 45° within this plane yields the state of absolute maximum shear stress and the associated average normal stress, namely,

$$\tau_{\substack{\text{abs}\\\text{max}}} = 16\text{ MPa} \quad , \quad \sigma_{\text{avg}} = 16\text{ MPa} \qquad \text{(Ans)}$$

Same result for can be obtained from direct application of Mohr's circle.

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{\sigma_1}{2} = \frac{32}{2} = 16\text{ MPa}$$

$$\sigma_{\text{avg}} = \frac{32+0}{2} = 16\text{ MPa} \qquad \text{(Ans)}$$

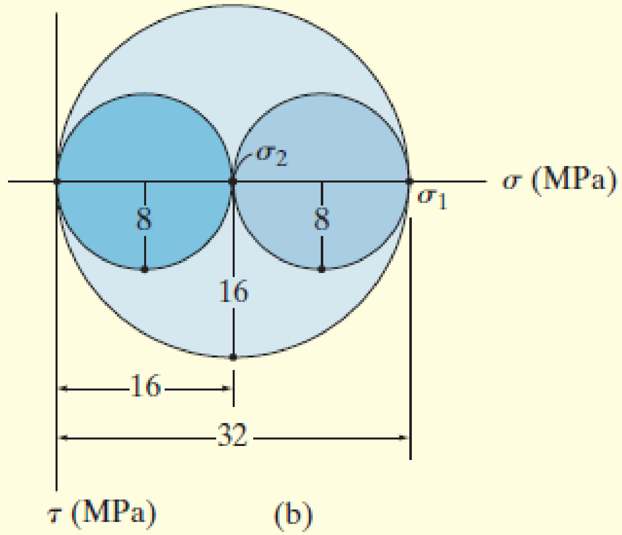

Fig. 9–24

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EXAMPLE 9.10 (cont)

Solutions

By comparison, the maximum in-plane shear stress can be determined from the Mohr's circle,

$$\tau_{\substack{\max \\ \text{in-plane}}} = \frac{32-16}{2} = 8\ \text{MPa}$$

$$\sigma_{avg} = \frac{32+16}{2} = 24\ \text{MPa} \qquad \text{(Ans)}$$

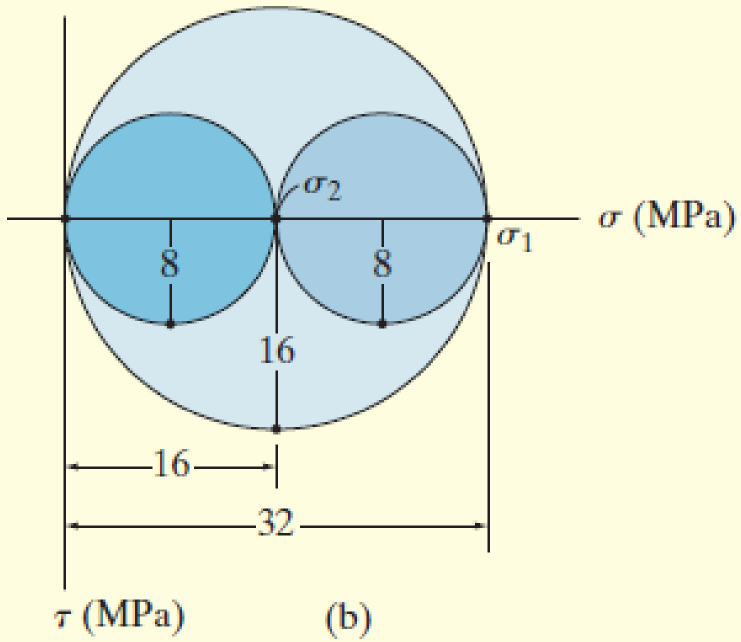

Fig. 9–24

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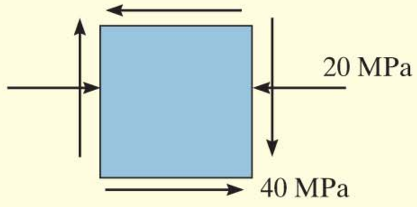
EXAMPLE 9.11

Due to an applied loading, an element at the point on a machine shaft is subjected to the state of plane stress shown in Fig. 9–25*a*. Determine the principal stresses and the absolute maximum shear stress at the point.

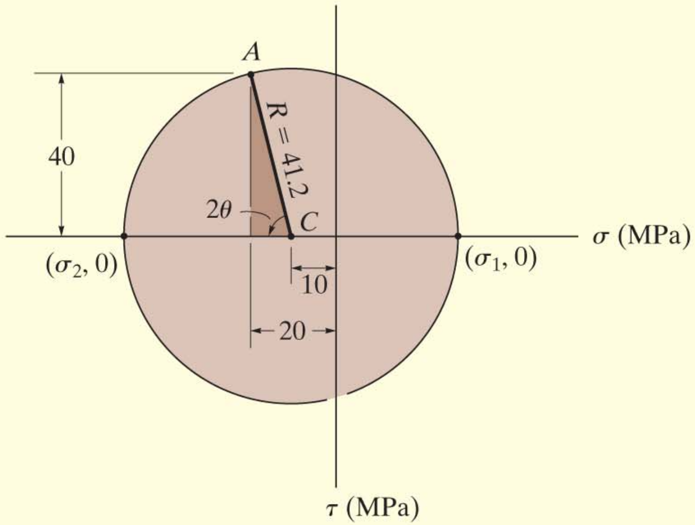


(a)

SOLUTION

Principal Stresses. The in-plane principal stresses can be determined from Mohr's circle. The center of the circle is on the σ axis at $\sigma_{avg} = (-20 + 0)/2 = -10$ MPa. Plotting the reference point $A(-20, -40)$, the radius CA is established and the circle is drawn as shown in Fig. 9–25*b*. The radius is

$$R = \sqrt{(20 - 10)^2 + (40)^2} = 41.2 \text{ MPa}$$



(b)

The principal stresses are at the points where the circle intersects the σ axis; i.e.,

$$\sigma_1 = -10 + 41.2 = 31.2 \text{ MPa}$$

$$\sigma_2 = -10 - 41.2 = -51.2 \text{ MPa}$$

From the circle, the *counterclockwise* angle 2θ, measured from CA to the $-\sigma$ axis, is

$$2\theta = \tan^{-1}\left(\frac{40}{20 - 10}\right) = 76.0^\circ$$

Thus,

$$\theta = 38.0^\circ$$

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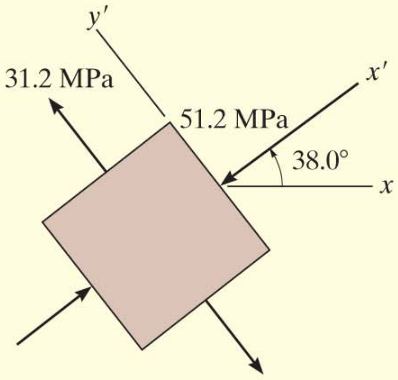
EXAMPLE 9.11



(c)

This *counterclockwise* rotation defines the direction of the x' axis and σ_2 and its associated principal plane, Fig. 9–25*c*. We have

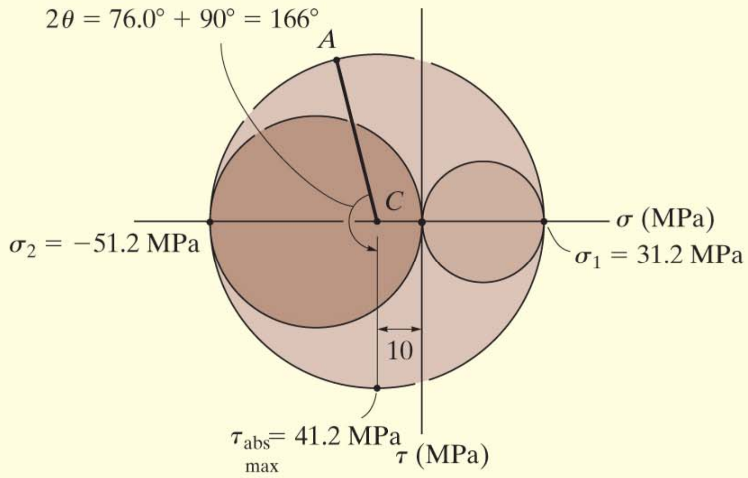
$$\sigma_1 = 31.2 \text{ MPa} \quad \sigma_2 = -51.2 \text{ MPa} \qquad \textit{Ans.}$$

Absolute Maximum Shear Stress. Since these stresses have opposite signs, applying Eq. 9–14 we have

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{\sigma_1 - \sigma_2}{2} = \frac{31.2 - (-51.2)}{2} = 41.2 \text{ MPa} \qquad \textit{Ans.}$$

$$\sigma_{avg} = \frac{31.2 - 51.2}{2} = -10 \text{ MPa}$$

NOTE: These same results can also be obtained by drawing Mohr's circle for each orientation of an element about the x, y, and z axes, Fig. 9–25*d*. Since σ_1 and σ_2 are of *opposite signs*, then the absolute maximum shear stress equals the maximum in-plane shear stress.



(d)

Fig. 9–25

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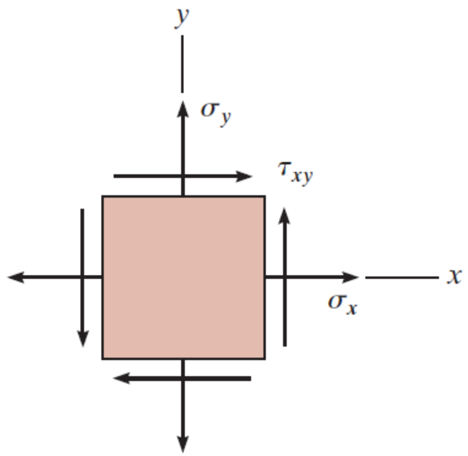
CHAPTER REVIEW

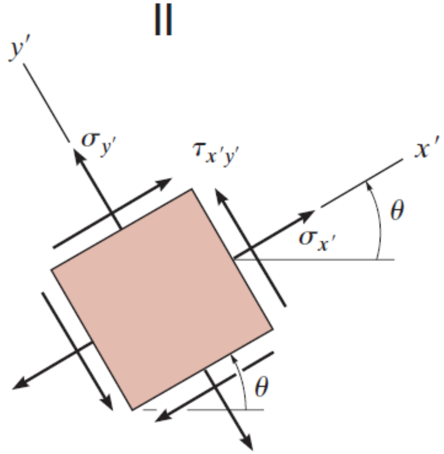
Plane stress occurs when the material at a point is subjected to two normal stress components σ_x and σ_y and a shear stress τ_{xy}. Provided these components are known, then the stress components acting on an element having a different orientation θ can be determined using the two force equations of equilibrium or the equations of stress transformation.

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$



(a)

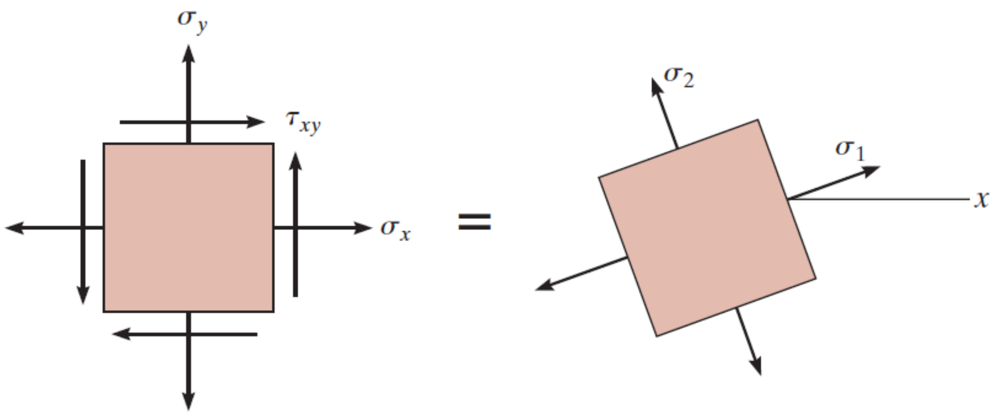


(b)

CHAPTER REVIEW

For design, it is important to determine the orientation of the element that produces the maximum principal normal stresses and the maximum in-plane shear stress. Using the stress transformation equations, it is found that no shear stress acts on the planes of principal stress. The principal stresses are

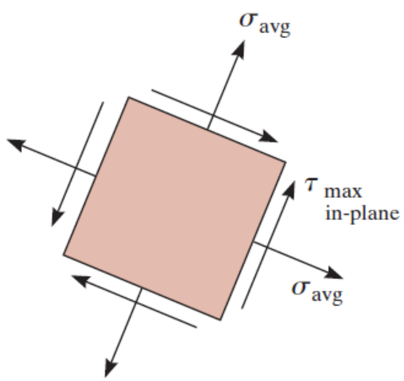
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



The planes of maximum in-plane shear stress are oriented 45° from this orientation, and on these shear planes there is an associated average normal stress.

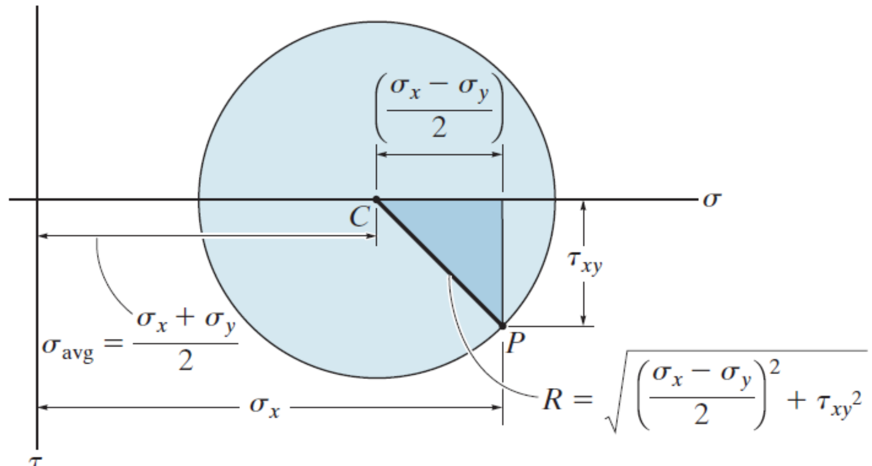
$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$



CHAPTER REVIEW

Mohr's circle provides a semi-graphical method for finding the stresses on any plane, the principal normal stresses, and the maximum in-plane shear stress. To draw the circle, the σ and τ axes are established, the center of the circle $C[(\sigma_x + \sigma_y)/2, 0]$ and the reference point $A(\sigma_x, \tau_{xy})$ are plotted. The radius R of the circle extends between these two points and is determined from trigonometry.



(b)

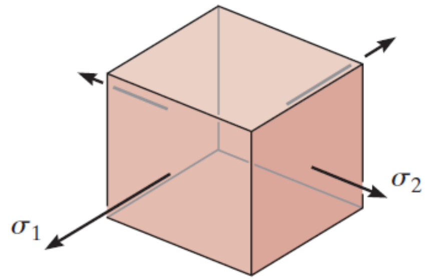
CHAPTER REVIEW

If σ_1 and σ_2 are of the same sign, then the absolute maximum shear stress will lie out of plane.

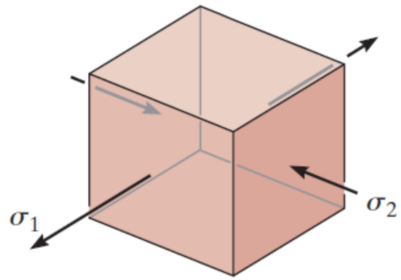
$$\tau_{\substack{\text{abs}\\ \text{max}}} = \frac{\sigma_1}{2}$$

In the case of plane stress, the absolute maximum shear stress will be equal to the maximum in-plane shear provided the principal stresses σ_1 and σ_2 have the opposite sign.

$$\tau_{\substack{\text{abs}\\ \text{max}}} = \frac{\sigma_1 - \sigma_2}{2}$$



x–y plane stress



x–y plane stress

Strain Transformation

10

Chapter Objectives

- ✓ Navigate between rectilinear co-ordinate system for strain components.
- ✓ Determine principal strains and maximum in-plane shear strain.
- ✓ Determine the absolute maximum shear strain in 2D and 3D cases.
- ✓ Know ways of measuring strains.
- ✓ Define stress-strain relationship.
- ✓ Predict failure of material.

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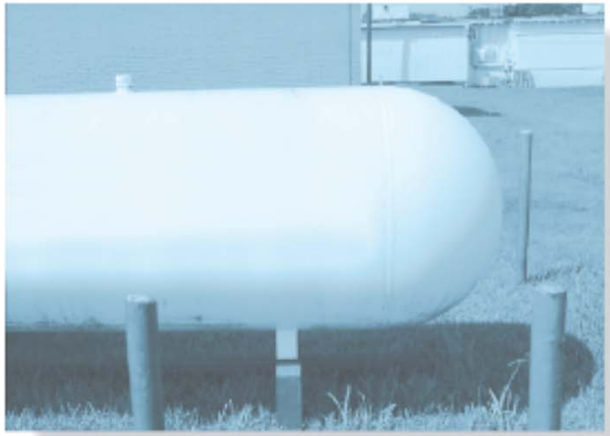
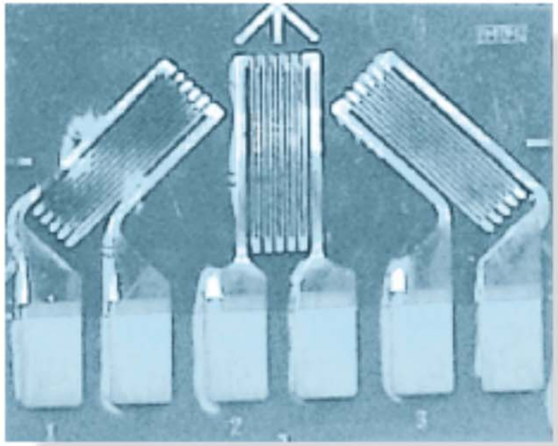
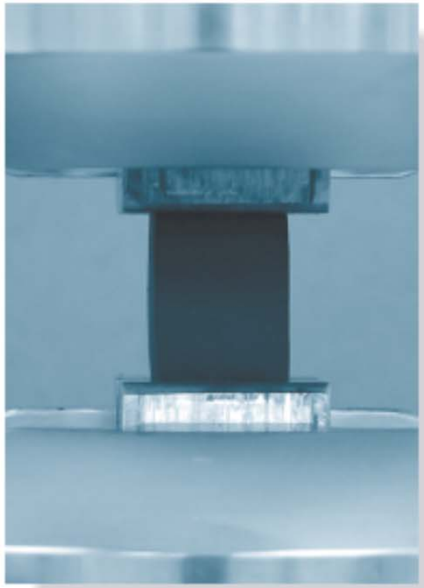
APPLICATIONS







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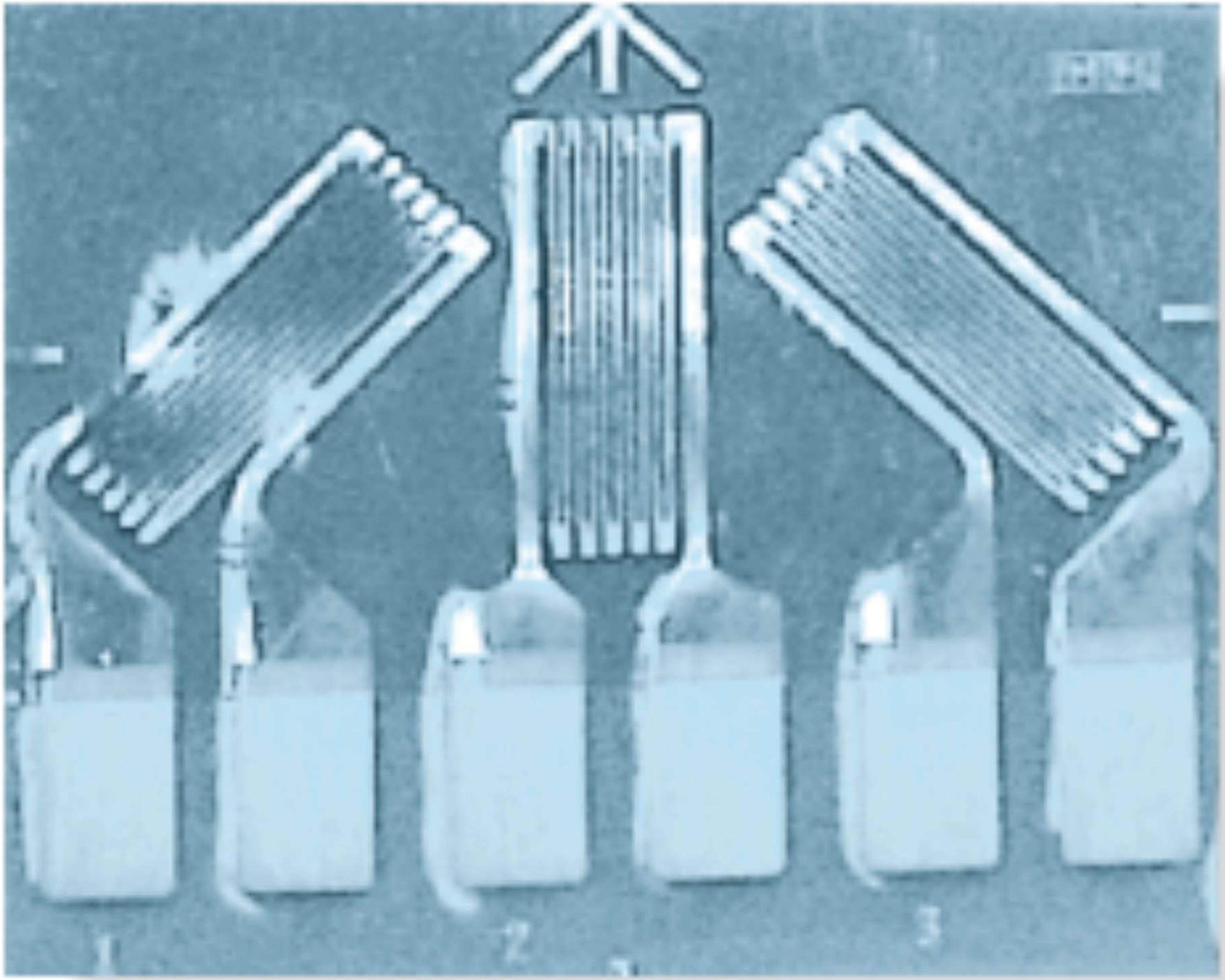

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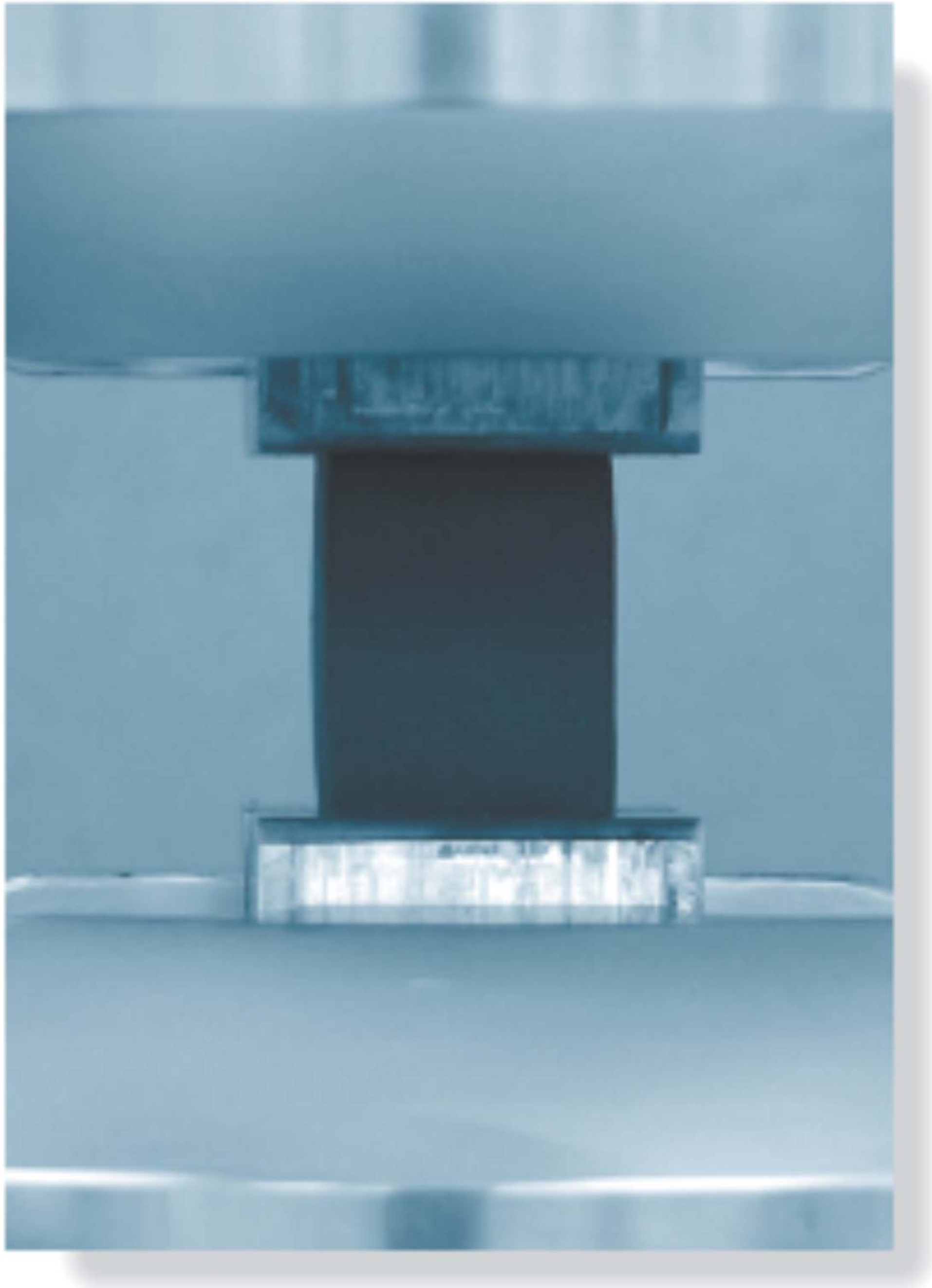

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10. 1 STATE OF PLANE-STRAIN

- In 3D, the general state of strain at a point is represented by a combination of 3 components of normal strain ε_x, ε_y, ε_z and 3 components of shear strain γ_{xy} , γ_{yz} , γ_{xz}.

- In plane-strain cases, ε_z, γ_{xz} and γ_{yz} are zero.

- The state of plane strain at a point is uniquely represented by 3 components (ε_x, ε_y and γ_{xy}) acting on an element that has a specific orientation at the point.

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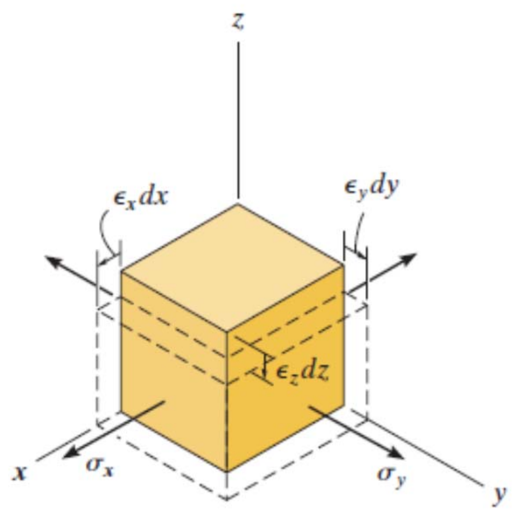
10.2 EQUATIONS OF PLANE-STRAIN TRANSFORMATION



Plane stress, σ_x, σ_y, does not cause plane strain in the x–y plane since $\epsilon_z \neq 0$.

Fig. 10–1

Note: *Plane-stress case ≠ plane-strain case*

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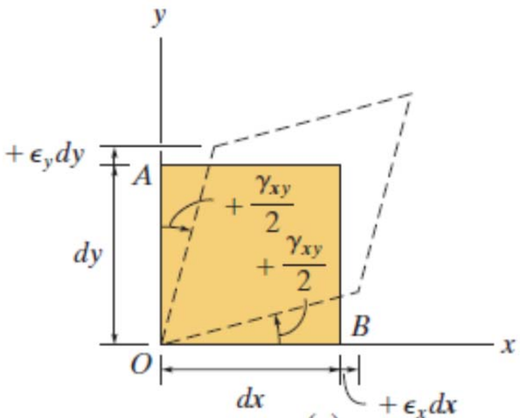
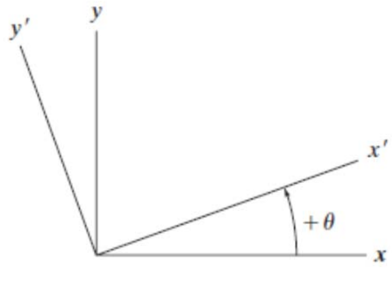
EQUATIONS OF PLANE-STRAIN TRANSFORMATION (cont)

- Positive normal strain ε_x and ε_y cause elongation
- Positive shear strain γ_{xy} causes small angle AOB
- Both the x-y and x'-y' system follow the right-hand rule
- The orientation of an inclined plane (on which the normal and shear strain components are to be determined) will be defined using the angle θ. The angle is measured from the positive x- to positive x'-axis. It is positive if it follows the curl of the right-hand fingers.



(a)



(b)

Positive sign convention

Fig. 10–2

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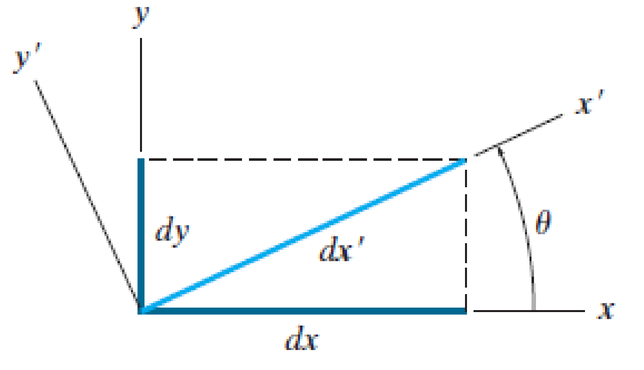
EQUATIONS OF PLANE-STRAIN TRANSFORMATION (cont)

- Normal and shear strains
  - Consider the line segment dx'

$$dx = dx'\cos\theta$$

$$dy = dx'\sin\theta$$



Before deformation

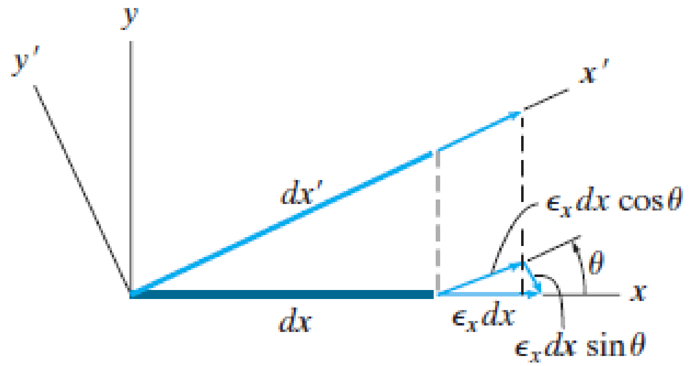
(a)

$$\delta x' = \varepsilon_x dx\cos\theta + \varepsilon_y dy\sin\theta + \gamma_{xy} dy\cos\theta$$

$$\varepsilon_{x'} = \frac{\delta x'}{dx'} = \varepsilon_x \cos^2\theta + \varepsilon_x \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$



Normal strain ϵ_x

(b)

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EQUATIONS OF PLANE-STRAIN TRANSFORMATION (cont)

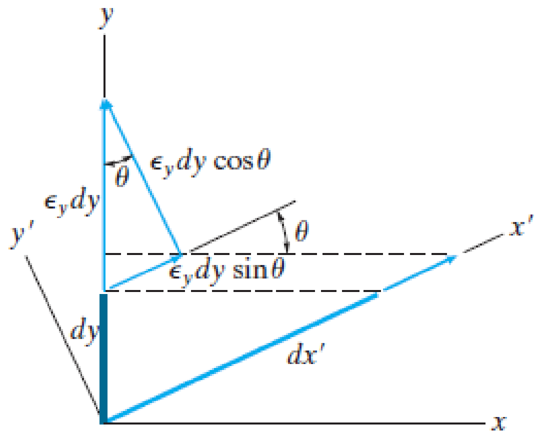
$$\delta y' = -\varepsilon_x dx \sin\theta + \varepsilon_y dy \cos\theta - \gamma_{xy} dy \sin\theta$$

$$\alpha = \frac{\delta y'}{dx'} = \left(-\varepsilon_x + \varepsilon_y\right)\sin\theta\cos\theta - \gamma_{xy}\sin^2\theta$$

- Similarly,

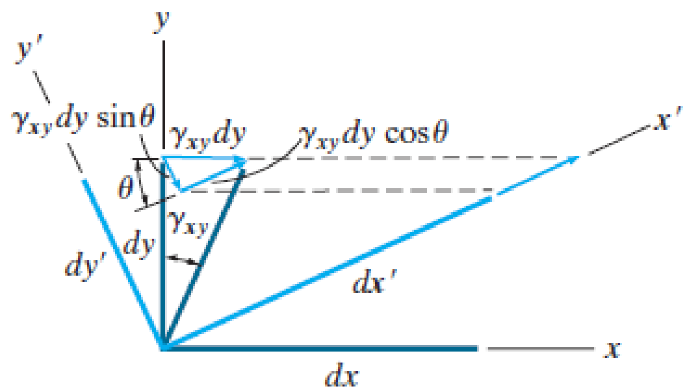
$$\beta = \alpha(\theta + 90°) = \left(-\varepsilon_x + \varepsilon_y\right)\sin(\theta + 90°)\cos(\theta + 90°) - \gamma_{xy}\sin^2(\theta + 90°)$$

$$= -\left(-\varepsilon_x + \varepsilon_y\right)\cos\theta\sin\theta - \gamma_{xy}\cos^2(\theta + 90°)$$



Normal strain ϵ_y

(c)



Shear strain γ_{xy}

(d)

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EQUATIONS OF PLANE-STRAIN TRANSFORMATION (cont)

$$\gamma_{x'y'} = \alpha - \beta = -(\varepsilon_x - \varepsilon_y)\sin\theta\cos\theta + \gamma_{xy}(\cos^2\theta - \sin^2\theta)$$

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

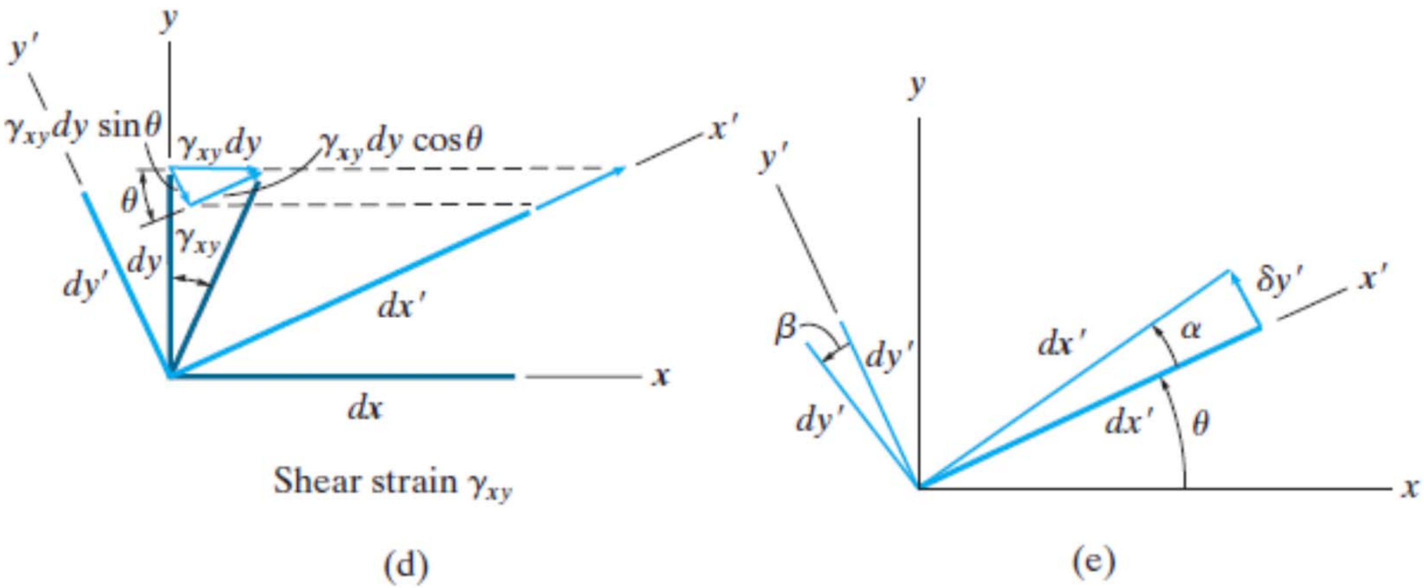

Fig. 10–3 (cont.)

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PRINCIPAL AND MAXIMUM IN-PLANE SHEAR STRAIN

- Similar to the deviations for principal stresses and the maximum in-plane shear stress, we have

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}\,, \qquad \varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

- And,

$$\tan 2\theta_S = -\left(\frac{\varepsilon_x - \varepsilon_y}{\gamma_{xy}}\right)$$

$$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}\,, \quad \varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2}$$

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PRINCIPAL AND MAXIMUM IN-PLANE SHEAR STRAIN (cont)

- When the state of strain is represented by the principal strains, no shear strain will act on the element.

- The state of strain at a point can also be represented in terms of the maximum in-plane shear strain. In this case an average normal strain will also act on the element.

- The element representing the maximum in-plane shear strain and its associated average normal strain is 45° from the element representing the principal strains.

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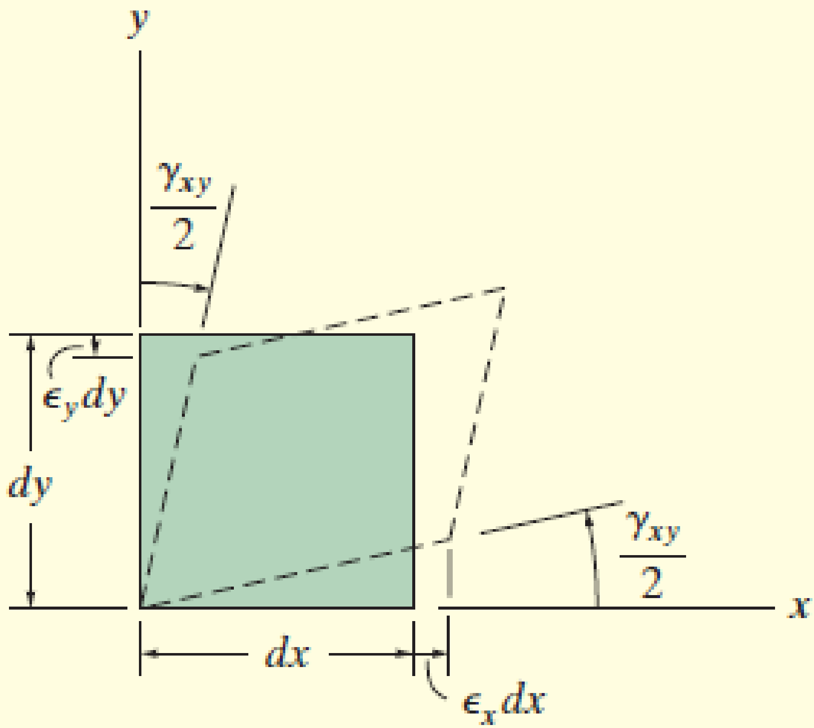
EXAMPLE 10.1

A differential element of material at a point is subjected to a state of plane strain $\varepsilon_x = 500\left(10^{-6}\right), \varepsilon_y = -300\left(10^{-6}\right), \gamma_{xy} = 200\left(10^{-6}\right)$ which tends to distort the element as shown in Fig. 10–5*a*. Determine the equivalent strains acting on an element of the material oriented at the point, *clockwise* 30° from the original position.



(a)

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EXAMPLE 10.1 (cont)

Solutions

- Since θ is *positive counter-clockwise,*

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{500 + (-300)}{2}\right](10^{-6}) + \left[\frac{500 - (-300)}{2}\right](10^{-6})\cos(2(-30°)) + \left[\frac{200(10^{-6})}{2}\right]\sin(2(-30°))$$

$$\Rightarrow \varepsilon_{x'} = 213(10^{-6}) \quad \text{(Ans)}$$

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$= -\left[\frac{500 - (-300)}{2}\right](10^{-6})\sin(2(-30°)) + \left[\frac{200(10^{-6})}{2}\right]\cos(2(-30°))$$

$$\Rightarrow \gamma_{x'y'} = 793(10^{-6}) \quad \text{(Ans)}$$

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EXAMPLE 10.1 (cont)

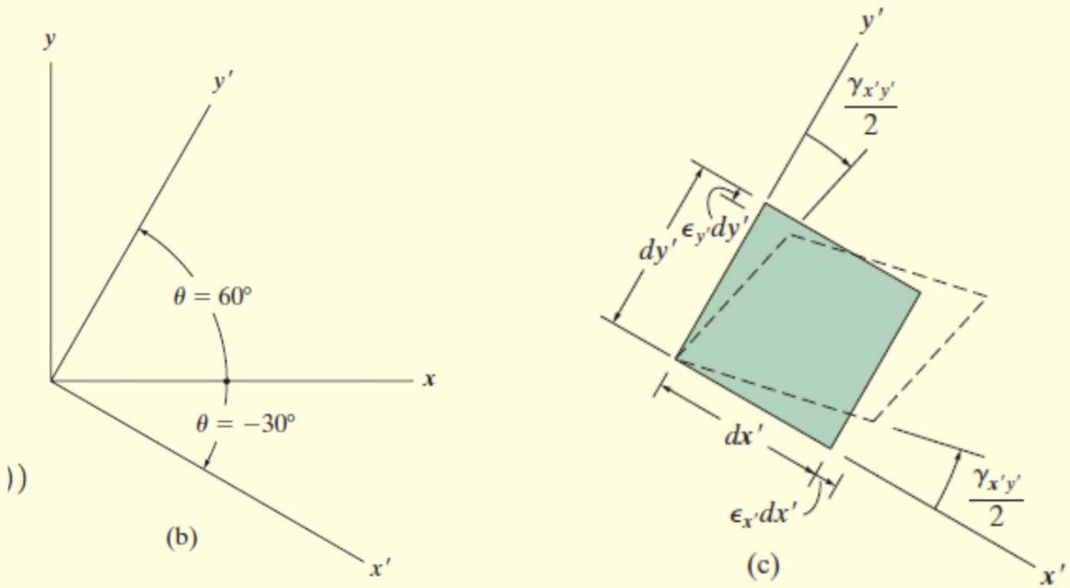
Solutions

- By replacement,

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{500 + (-300)}{2}\right](10^{-6}) + \left[\frac{500 - (-300)}{2}\right](10^{-6})\cos(2(60°)) + \left[\frac{200(10^{-6})}{2}\right]\sin(2(60°))$$

$$\Rightarrow \varepsilon_{x'} = -13.4(10^{-6}) \quad \text{(Ans)}$$



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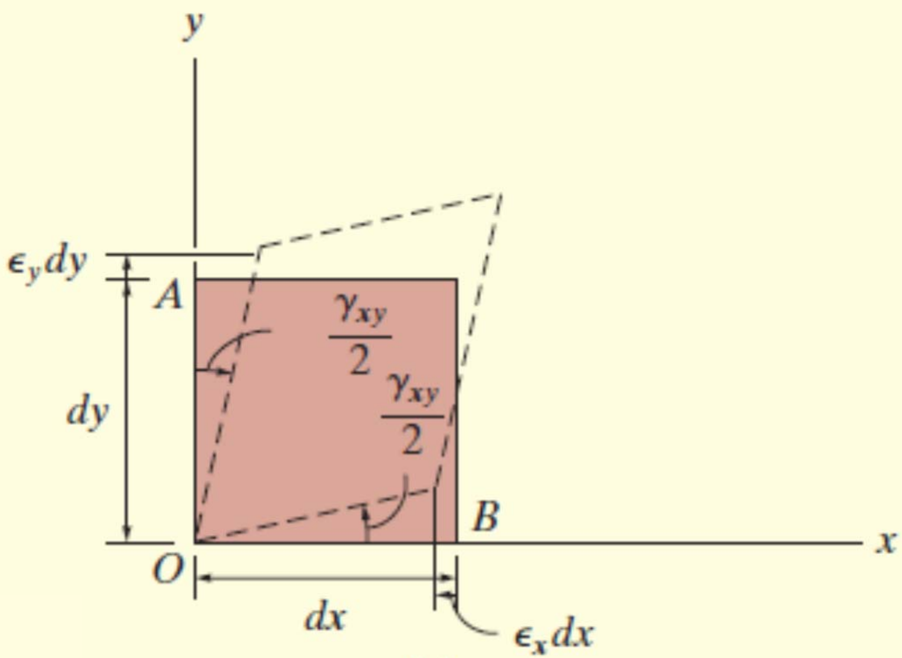
EXAMPLE 10.2

A differential element of material at a point is subjected to a state of plane strain defined by $\varepsilon_x = -350(10^{-6}), \varepsilon_y = 200(10^{-6}), \gamma_{xy} = 80(10^{-6})$ which tends to distort the element as shown in Fig. 10–7*a*. Determine the maximum in-plane shear strain at the point and the associated orientation of the element.



(a)

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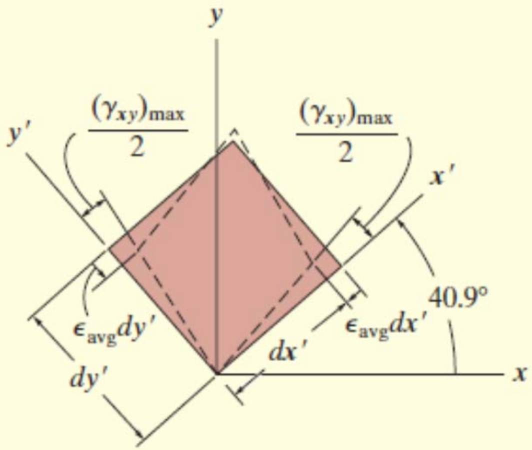
EXAMPLE 10.2 (cont)

Solutions

- Looking at the orientation of the element,

$$\tan 2\theta_s = -\left(\frac{\varepsilon_x - \varepsilon_y}{\gamma_{xy}}\right) = -\left(\frac{-350-200}{80}\right)$$

$$\Rightarrow \theta_s = 40.9^\circ \text{ and } 131^\circ$$



(b)

- For maximum in-plane shear strain,

$$\frac{\gamma_{\substack{\max \\ \text{in plane}}}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\Rightarrow \gamma_{\substack{\max \\ \text{in plane}}} = 556\left(10^{-6}\right) \quad \text{(Ans)}$$

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EXAMPLE 10.3

A differential element of material at a point is subjected to a state of plane strain defined by $\epsilon_x = -350(10^{-6})$, $\epsilon_y = 200(10^{-6})$, $\gamma_{xy} = 80(10^{-6})$, which tends to distort the element as shown in Fig. 10–7*a*. Determine the maximum in-plane shear strain at the point and the associated orientation of the element.

SOLUTION

Orientation of the Element. From Eq. 10–10 we have

$$\tan 2\theta_s = -\left(\frac{\epsilon_x - \epsilon_y}{\gamma_{xy}}\right) = -\frac{(-350 - 200)(10^{-6})}{80(10^{-6})}$$

Thus, $2\theta_s = 81.72°$ and $81.72° + 180° = 261.72°$, so that

$$\theta_s = 40.9° \text{ and } 131°$$

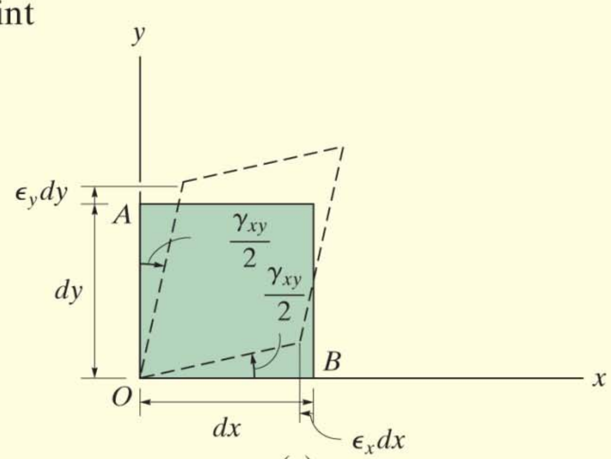
Note that this orientation is 45° from that shown in Fig. 10–6*b* in Example 10.2 as expected.

Maximum In-Plane Shear Strain. Applying Eq. 10–11 gives

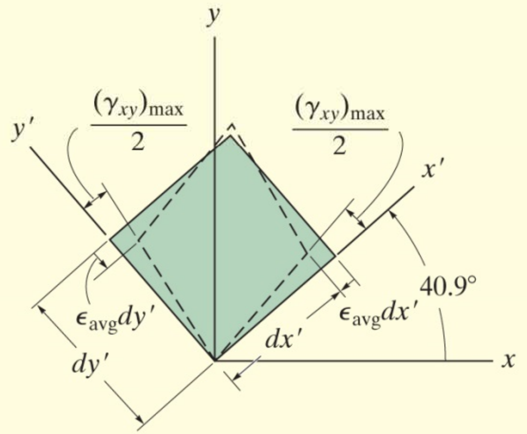
$$\frac{\gamma^{\max}_{\text{in-plane}}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\sqrt{\left(\frac{-350 - 200}{2}\right)^2 + \left(\frac{80}{2}\right)^2}\right](10^{-6})$$

$$\gamma^{\max}_{\text{in-plane}} = 556(10^{-6}) \qquad \textit{Ans.}$$



(a)



(b)

Fig. 10–7

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EXAMPLE 10.3 CONTINUED

Due to the square root, the proper sign of $\gamma_{\text{in-plane}}^{\text{max}}$ can be obtained by applying Eq. 10–6 with $\theta_s = 40.9°$. We have

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$= -\left(\frac{-350 - 200}{2}\right)(10^{-6})\sin 2(40.9°) + \frac{80(10^{-6})}{2}\cos 2(40.9°)$$

$$\gamma_{x'y'} = 556(10^{-6})$$

This result is positive and so $\gamma_{\text{in-plane}}^{\text{max}}$ tends to distort the element so that the right angle between dx' and dy' is *decreased* (positive sign convention), Fig. 10–7*b*.

Also, there are associated average normal strains imposed on the element that are determined from Eq. 10–12:

$$\epsilon_{\text{avg}} = \frac{\epsilon_x + \epsilon_y}{2} = \frac{-350 + 200}{2}(10^{-6}) = -75(10^{-6})$$

These strains tend to cause the element to contract, Fig. 10–7*b*.

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10.3 MOHR'S CIRCLE FOR PLANE STRAIN

- A geometrical representation of Equations 10-5 and 10-6; i.e.

$$\left(\varepsilon_{x'} - \varepsilon_{avg}\right)^2 + \left(\frac{\gamma_{x'y'}}{2}\right)^2 = R^2$$

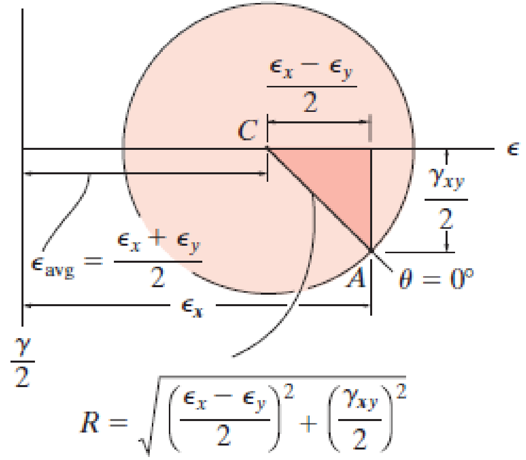

Fig. 10–8

where

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} \quad \text{and} \quad R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

- Sign convention: ε is positive to the right, and γ/2 is positive downwards.

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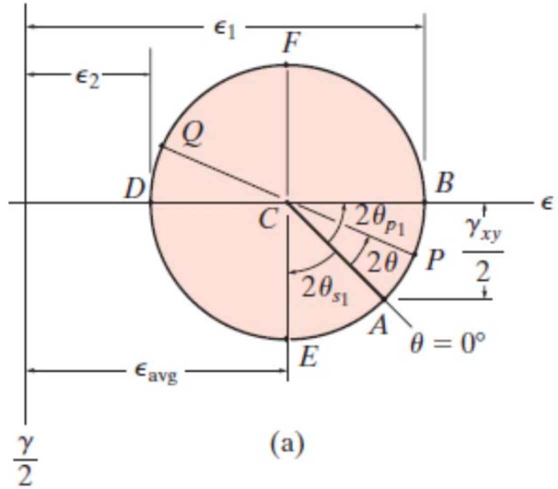
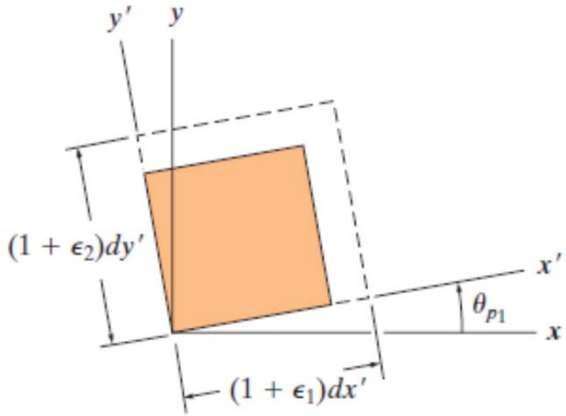
MOHR’S CIRCLE FOR PLANE STRAIN (cont)



(a)



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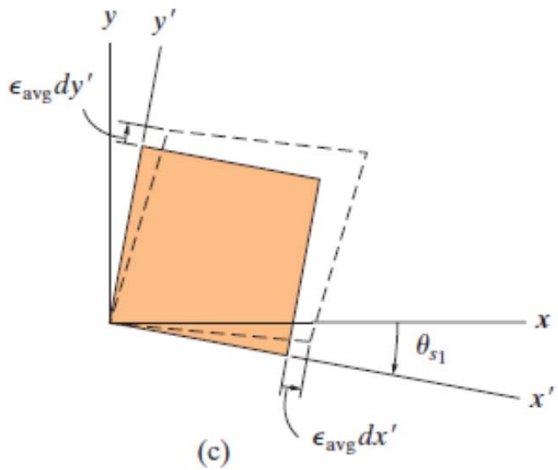

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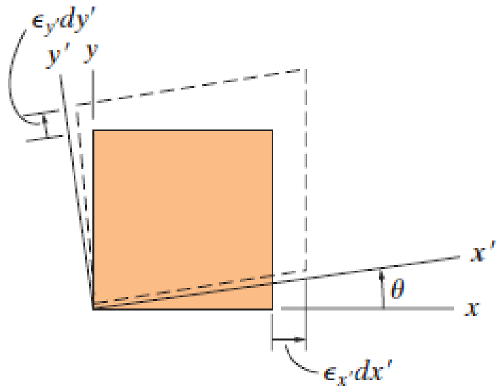

(d)

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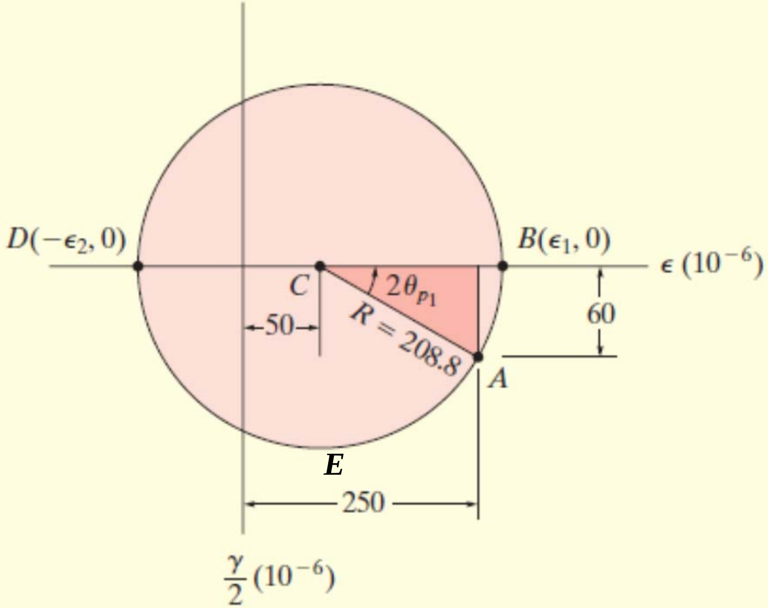
EXAMPLE 10.4

The state of plane strain at a point is represented by the components:

$$\varepsilon_x = 250\left(10^{-6}\right), \varepsilon_y = -150\left(10^{-6}\right), \gamma_{xy} = 120\left(10^{-6}\right)$$

Determine the principal strains and the orientation of the element.



(a)

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EXAMPLE 10.4 (cont)

Solutions

- From the coordinates of point B and D, we have

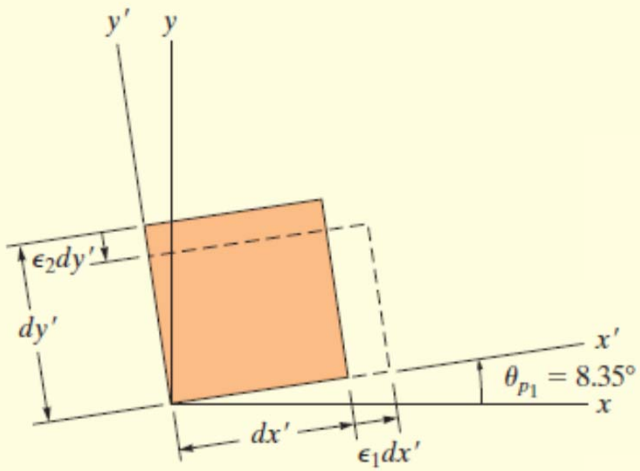
$$\varepsilon_1 = \varepsilon_{ave} + R = 50 + 208.8 = 258.8\left(10^{-6}\right)$$

$$\varepsilon_2 = \varepsilon_{ave} - R = 50 - 208.8 = -158.8\left(10^{-6}\right)$$

- To orient the element, we can determine the clockwise angle.

$$\tan 2\theta_{p1} = \frac{60}{200}$$

$$\theta_{p1} = 8.35° \quad \text{(Ans)}$$



(b)

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EXAMPLE 10.4 (cont)

Solutions

- From the coordinates of point *E*, we have

$$\frac{\left(\gamma_{x'y'}\right)_{\substack{\max \\ \text{in plane}}}}{2} = 208.8\left(10^{-6}\right)$$

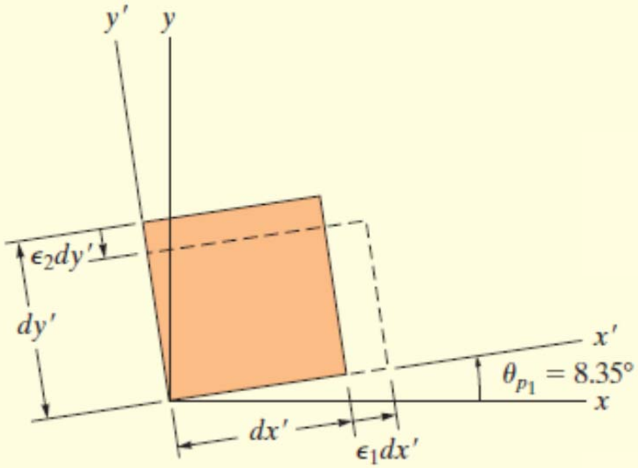
$$\left(\gamma_{x'y'}\right)_{\substack{\max \\ \text{in plane}}} = 418\left(10^{-6}\right)$$

$$\varepsilon_{avg} = 50\left(10^{-6}\right)$$

- To orient the element, we can determine the clockwise angle.

$$2\theta_{s_1} = 90° - 2(8.35°)$$

$$\theta_{s_1} = 36.7° \quad \text{(Ans)}$$



(b)

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EXAMPLE 10.5

The state of plane strain at a point is represented by the components $\epsilon_x = 250(10^{-6})$, $\epsilon_y = -150(10^{-6})$, and $\gamma_{xy} = 120(10^{-6})$. Determine the maximum in-plane shear strains and the orientation of an element.

SOLUTION

The circle has been established in the previous example and is shown in Fig. 10–11*a*.

Maximum In-Plane Shear Strain. Half the maximum in-plane shear strain and average normal strain are represented by the coordinates of point E or F on the circle. From the coordinates of point E,

$$\frac{(\gamma_{x'y'})_{\substack{\max \\ \text{in-plane}}}}{2} = 208.8(10^{-6})$$

$$(\gamma_{x'y'})_{\substack{\max \\ \text{in-plane}}} = 418(10^{-6}) \qquad \textit{Ans.}$$

$$\epsilon_{\text{avg}} = 50(10^{-6})$$

To orient the element, we can determine the clockwise angle $2\theta_{s_1}$ measured from CA ($\theta = 0°$) to CE.

$$2\theta_{s_1} = 90° - 2(8.35°)$$

$$\theta_{s_1} = 36.7° \qquad \textit{Ans.}$$

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EXAMPLE 10.5 CONTINUED

This angle is shown in Fig. 10–11b. Since the shear strain defined from point E on the circle has a positive value and the average normal strain is also positive, these strains deform the element into the dashed shape shown in the figure.

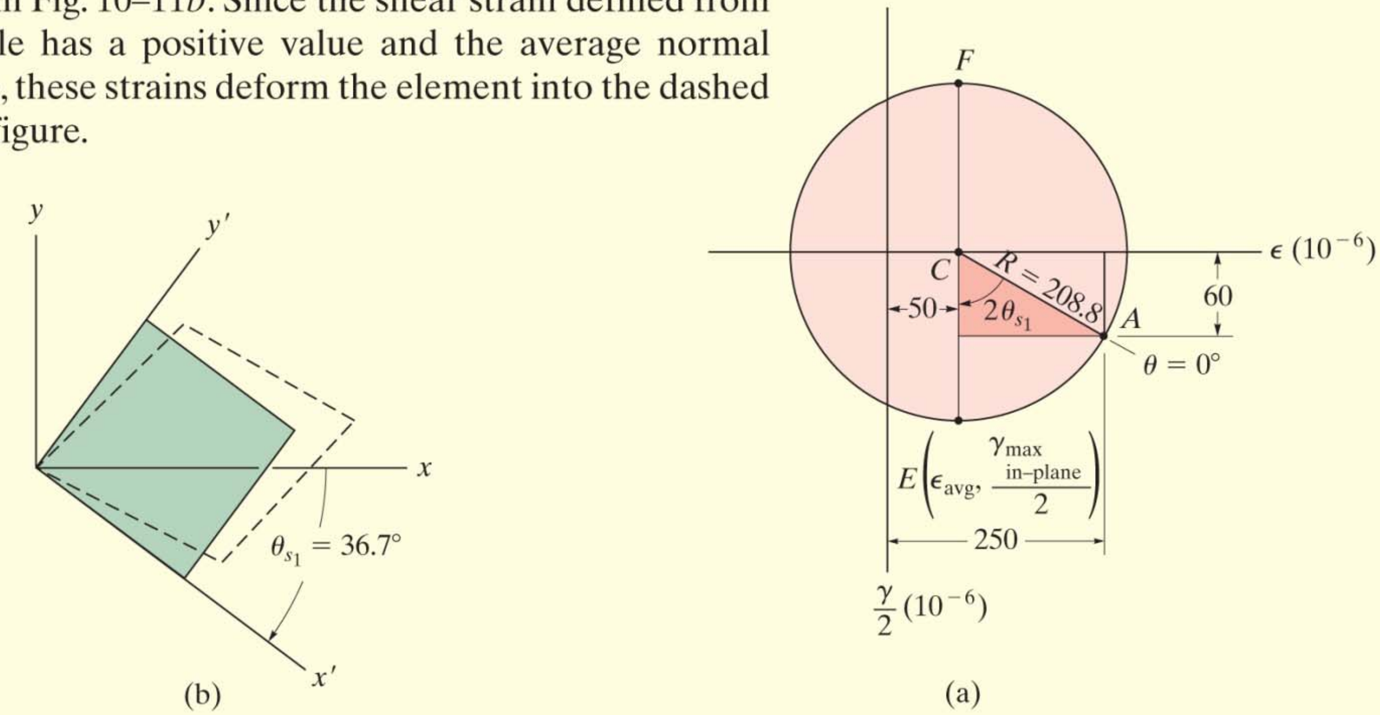

Fig. 10–11

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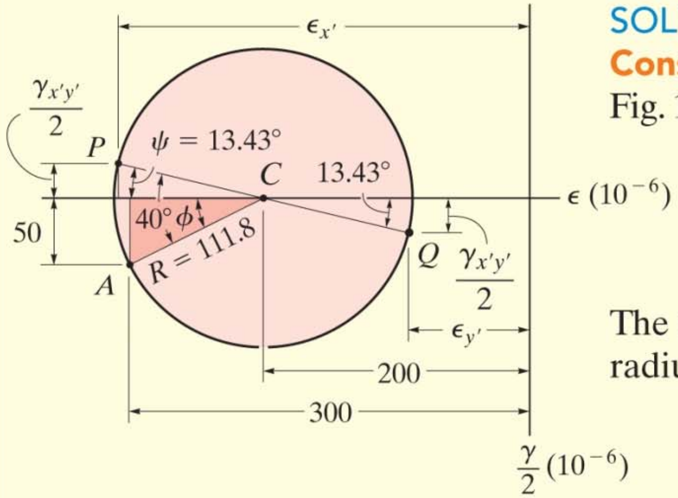
EXAMPLE 10.6

The state of plane strain at a point is represented on an element having components $\epsilon_x = -300(10^{-6})$, $\epsilon_y = -100(10^{-6})$, and $\gamma_{xy} = 100(10^{-6})$. Determine the state of strain on an element oriented 20° clockwise from this reported position.



(a)

SOLUTION

Construction of the Circle. The ϵ and $\gamma/2$ axes are established in Fig. 10–12*a*. The center of the circle is on the ϵ axis at

$$\epsilon_{avg} = \left(\frac{-300 - 100}{2}\right)(10^{-6}) = -200(10^{-6})$$

The reference point A has coordinates $A(-300(10^{-6}), 50(10^{-6}))$. The radius CA determined from the shaded triangle is therefore

$$R = \left[\sqrt{(300 - 200)^2 + (50)^2}\right](10^{-6}) = 111.8(10^{-6})$$

Strains on Inclined Element. Since the element is to be oriented 20° *clockwise*, we must establish a radial line CP, $2(20°) = 40°$ *clockwise*, measured from CA ($\theta = 0°$), Fig. 10–12*a*. The coordinates of point P $(\epsilon_{x'}, \gamma_{x'y'}/2)$ are obtained from the geometry of the circle. Note that

$$\phi = \tan^{-1}\left(\frac{50}{(300 - 200)}\right) = 26.57°, \qquad \psi = 40° - 26.57° = 13.43°$$

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EXAMPLE 10.6 CONTINUED

Thus,

$$\epsilon_{x'} = -(200 + 111.8 \cos 13.43^\circ)(10^{-6})$$
$$= -309(10^{-6}) \qquad \textit{Ans.}$$

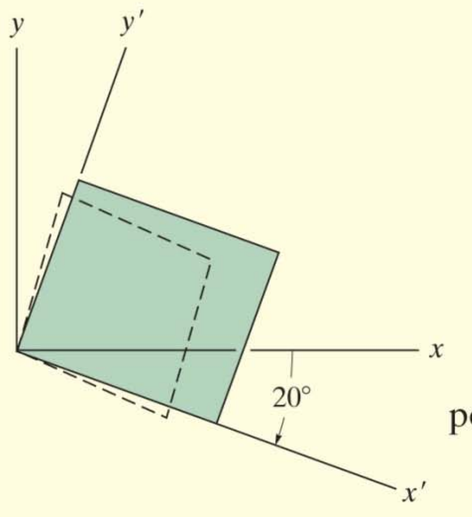
$$\frac{\gamma_{x'y'}}{2} = -(111.8 \sin 13.43^\circ)(10^{-6})$$

$$\gamma_{x'y'} = -52.0(10^{-6}) \qquad \textit{Ans.}$$

The normal strain $\epsilon_{y'}$ can be determined from the ϵ coordinate of point Q on the circle, Fig. 10–12*a*. Why?

$$\epsilon_{y'} = -(200 - 111.8 \cos 13.43^\circ)(10^{-6}) = -91.3(10^{-6}) \qquad \textit{Ans.}$$

As a result of these strains, the element deforms relative to the x', y' axes as shown in Fig. 10–12*b*.



(b)

Fig. 10–12

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10.4 ABSOLUTE MAXIMUM SHEAR STRAIN

- State of strain in 3-dimensional space:

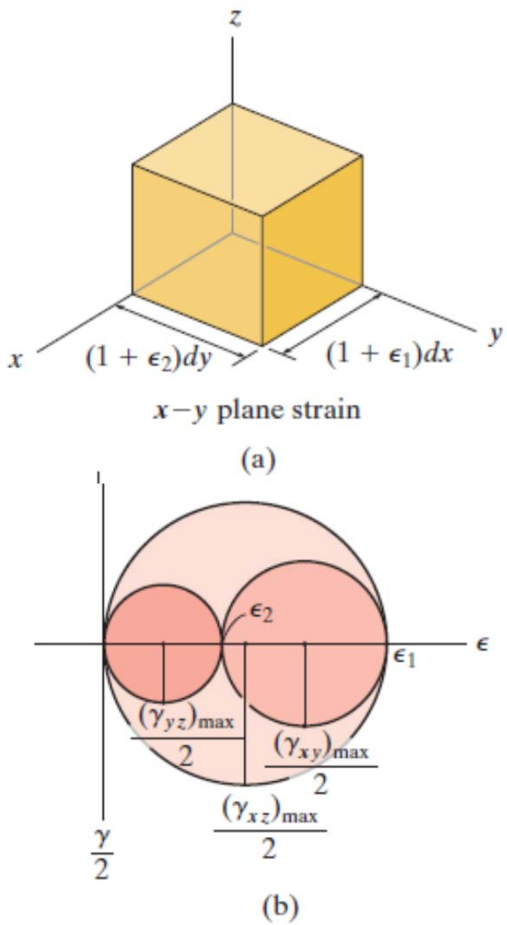

Fig. 10–13

$$\gamma_{\substack{abs\\max}} = \varepsilon_{max} - \varepsilon_{min}$$

$$\varepsilon_{avg} = \frac{\varepsilon_{max} + \varepsilon_{min}}{2}$$

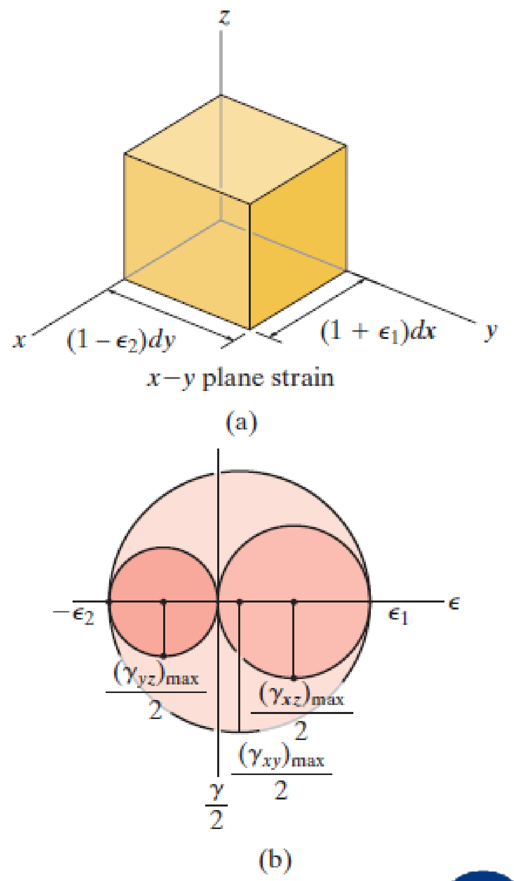

Fig. 10–14

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EXAMPLE 10.7

The state of plane strain at a point is represented by the components:

$$\varepsilon_x = -400\left(10^{-6}\right), \varepsilon_y = 200\left(10^{-6}\right), \gamma_{xy} = 150\left(10^{-6}\right)$$

Determine the maximum in-plane shear strain and the absolute maximum shear strain.

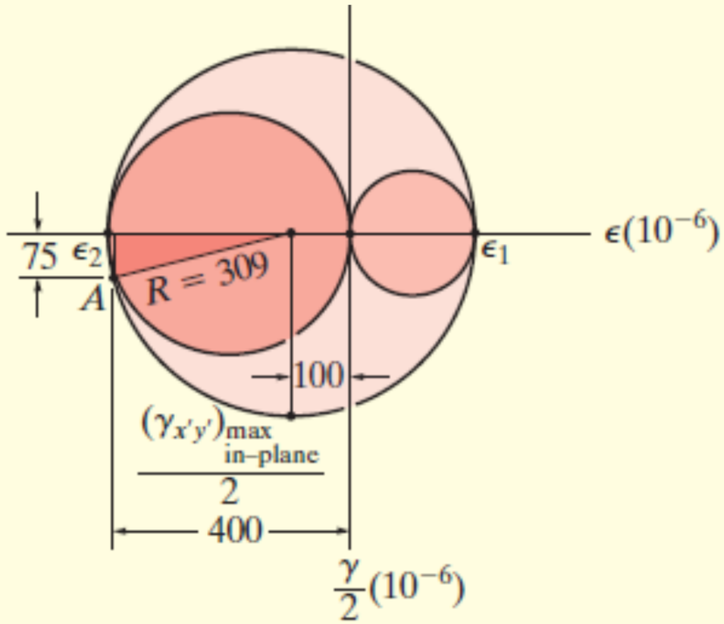

Fig. 10–15

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EXAMPLE 10.7 (cont)

Solutions

- From the strain components, the centre of the circle is on the ε axis at

$$\varepsilon_{avg} = \frac{-400+200}{2}\left(10^{-6}\right) = -100\left(10^{-6}\right)$$

- Since $\frac{\gamma_{xy}}{2} = 75\left(10^{-6}\right)$, the reference point has coordinates

$$A\left(-400\left(10^{-6}\right), 75\left(10^{-6}\right)\right)$$

- Thus the radius of the circle is

$$R = \left[\sqrt{(400-100)^2 + 75^2}\right]\left(10^{-6}\right) = 309\left(10^{-9}\right)$$

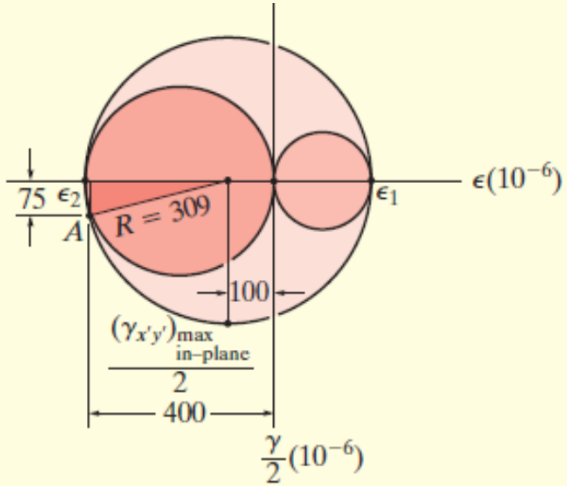

Fig. 10–15

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EXAMPLE 10.7 (cont)

Solutions

- Computing the in-plane principal strains, we have

$$\varepsilon_{max} = (-100 + 309)(10^{-6}) = 209(10^{-6})$$
$$\varepsilon_{min} = (-100 - 309)(10^{-6}) = -409(10^{-6})$$

- From the circle, the maximum in-plane shear strain is

$$\gamma_{\substack{max \\ \text{in plane}}} = \varepsilon_{max} - \varepsilon_{min} = \left[209 - (-409)\right](10^{-6}) = 618(10^{-6}) \quad \text{(Ans)}$$

- From the above results, we have

$$\varepsilon_{max} = 209(10^{-6}),\ \varepsilon_{int} = 0,\ \varepsilon_{min} = -409(10^{-6})$$

- Thus the Mohr's circle is as follow,

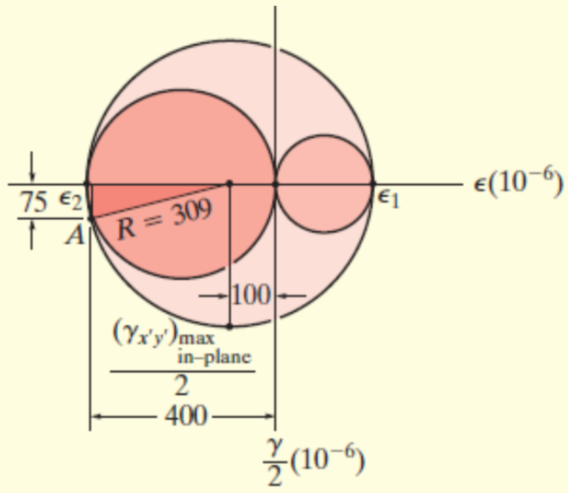

Fig. 10–15

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10.5 MEASUREMENT OF STRAINS BY STRAIN ROSETTES

- Ways of arranging 3 electrical-resistance strain gauges

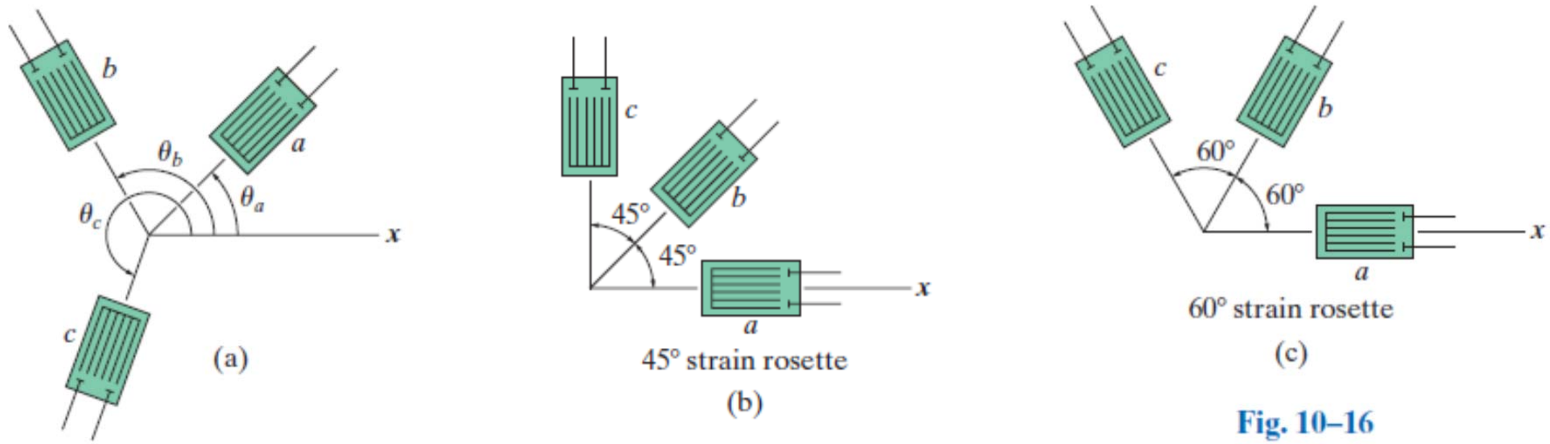

Fig. 10–16

- In general case (a):

$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy} \sin\theta_a \cos\theta_a$$

$$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy} \sin\theta_b \cos\theta_b$$

$$\varepsilon_c = \varepsilon_x \cos^2\theta_c + \varepsilon_y \sin^2\theta_c + \gamma_{xy} \sin\theta_c \cos\theta_c$$

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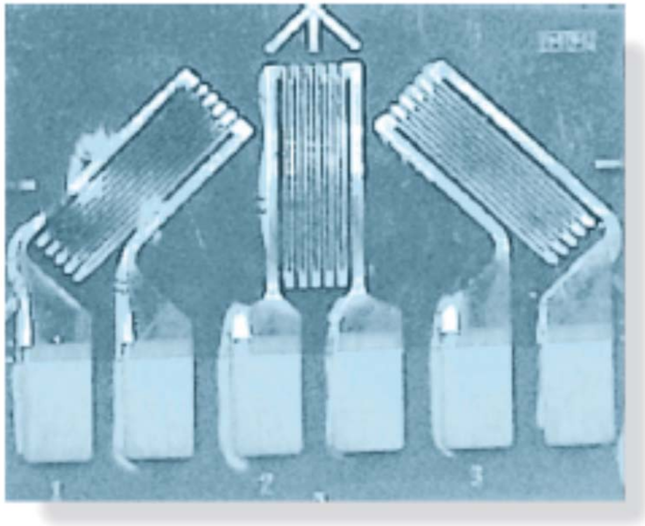
MEASUREMENT OF STRAINS BY STRAIN ROSETTES (cont)

- In 45° strain rosette [case (b)],

$$\varepsilon_x = \varepsilon_a$$

$$\varepsilon_y = \varepsilon_c$$

$$\gamma_{xy} = 2\varepsilon_b - (\varepsilon_a + \varepsilon_c)$$



- In 60° strain rosette [case (c)],

$$\varepsilon_x = \varepsilon_a$$

$$\varepsilon_y = \frac{1}{2}(2\varepsilon_b + 2\varepsilon_c - \varepsilon_a)$$

$$\gamma_{xy} = \frac{2}{\sqrt{3}}(\varepsilon_b + \varepsilon_c)$$

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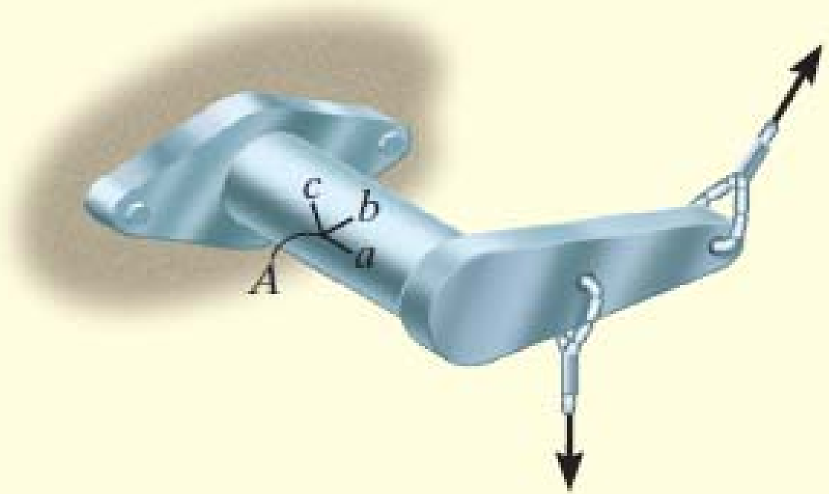
EXAMPLE 10.8

The state of strain at point *A* on the bracket in Fig. 10–17*a* is measured using the strain rosette shown in Fig. 10–17*b*. Due to the loadings, the readings from the gauges give

$$\varepsilon_a = 60(10^{-6}),\ \varepsilon_b = 135(10^{-6}),\ \varepsilon_c = 264(10^{-6})$$

Determine the in-plane principal strains at the point and the directions in which they act.



(a)

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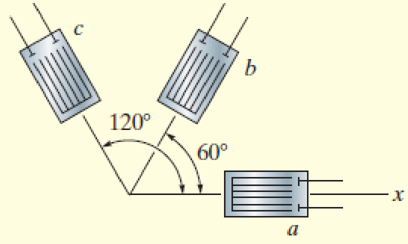
EXAMPLE 10.8 (cont)

Solutions

- Measuring the angles counter-clockwise,



(b)

$$\theta_a = 0°, \theta_b = 60° \text{ and } \theta_c = 120°$$

- By substituting the values into the 3 strain-transformation equations, we have

$$\varepsilon_x = 60\left(10^{-6}\right) \;,\; \varepsilon_y = 246\left(10^{-6}\right) \;,\; \gamma_{xy} = -149\left(10^{-6}\right)$$

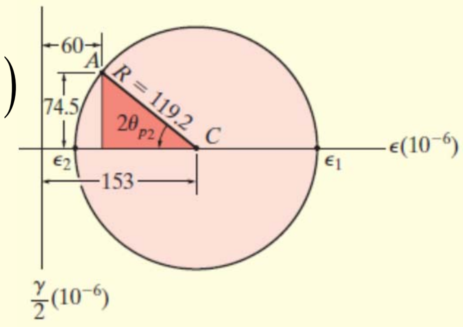
- Using Mohr's circle, we have A(60(10^{-6}), 60(10^{-6})) and center C (153(10^{-6}), 0).

$$R = \left[\sqrt{(153-60)^2 + 74.5^2}\right]\left(10^{-6}\right) = 119.1\left(10^{-6}\right)$$

$$\varepsilon_1 = 272\left(10^{-6}\right) \;,$$

$$\varepsilon_2 = 33.9\left(10^{-6}\right) \;,$$

$$\theta_{p2} = 19.3° \quad \text{(Ans)}$$



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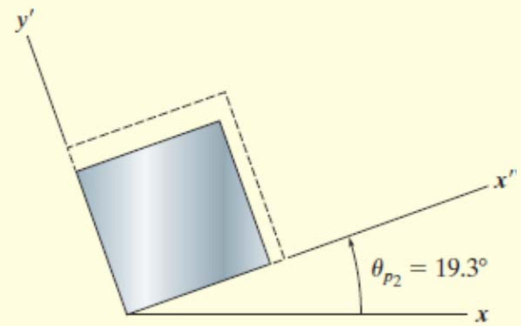

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10.6 STRESS-STRAIN RELATIONSHIP

- Use the principle of superposition

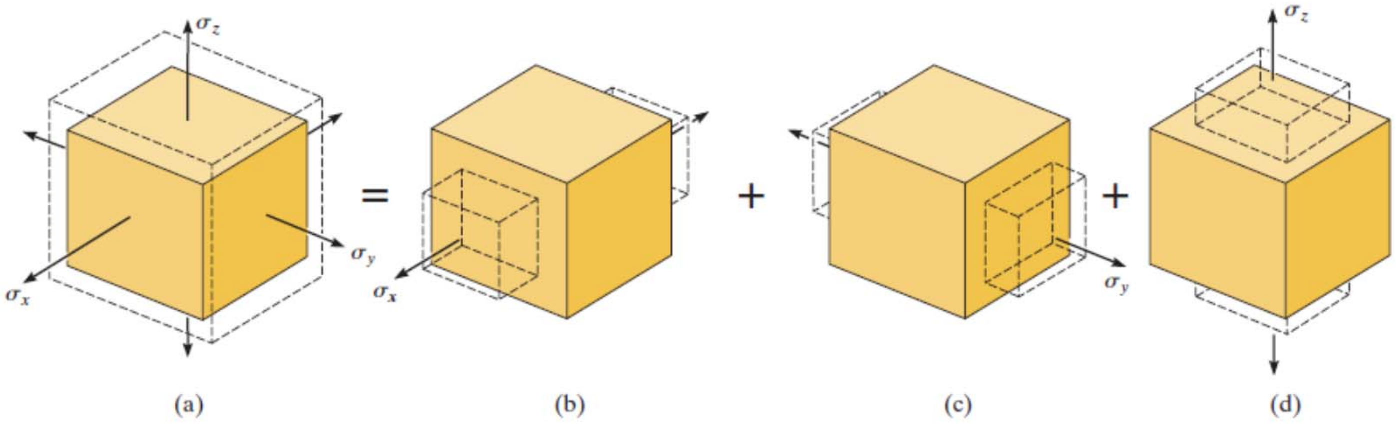

Fig. 10–18

- Use Poisson's ratio, $\varepsilon_{\text{lateral}} = -\nu\varepsilon_{\text{longitudinal}}$

- Use Hooke's Law (as it applies in the uniaxial direction),

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] \quad , \quad \varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] \quad , \quad \varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right]$$

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