전자기학 1
(Electrodynamics 1)

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Inha University

Fall 2015

Syllabus (강의 계획서)

▣ 강의 목표:

◈ 수강자 들이 전자기학과 전기동력학, 막스웰 방정식에 대한 이해와 이러한 지식을 향후 연구 개발 활동에 적용할 수 있는 능력을 키우는데 본 강의의 목표가 있음

▣ 교재:

◈"Introduction to Electrodynamics" 4th Ed., David J. Griffiths, Pearson, Boston (2013), ISBN 10: 0-321-85656-2 (ISBN 13: 978-0-321-85656-2)

▣ 평가 :

◈ 출석(10%), 숙제 (10%), 퀴즈 (20%), **토론 및 발표(20%)**, 중간고사 (20%), 기말고사 (20%)

◈ 숙제는 본인이 직접 하지 않으면 베껴 낼 필요가 없음.

◈ **사전에 e-learning 게시판에 올려진 강의록을 보고 미리 예습해 오고 수업 시간에는 문제 풀이 및 발표 위주로 운영 예정 (Flipped Learning 실시)**

▣ 참고 도서:

◈"Foundations of Electromagnetic Theory," 4th Ed. John R. Reitz, Frederick J. Milford, Robert W. Christy (차동우 역 – 한글판 "전자기학") Pearson (2009)

▣ 질문 및 면담:

5북 310호, 월 오후 3~5시 및 목 오후 1~3시,
그 외 시간도 재실 시에는 가능함.

강의 계획

1. Introduction & Vector Analysis 1 (Vector algebra, Differential calculus)
2. Vector Analysis 2 (Integral calculus, Curvilinear coordinates, The Dirac delta function)
3. Electrostatics 1 (Electric field, Divergence & curl of electrostatic fields, Electric potential)
4. Electrostatics 2 (Work and energy in electrostatics, Conductors)
5. Potentials 1 (Laplace equation, The method of images)
6. Potentials 2 (Separation of Variables, Multipole expansion)
7. Electric Fields in Matter 1 (Polarization, The field of a polarized object)
8. Mid-Term Exam

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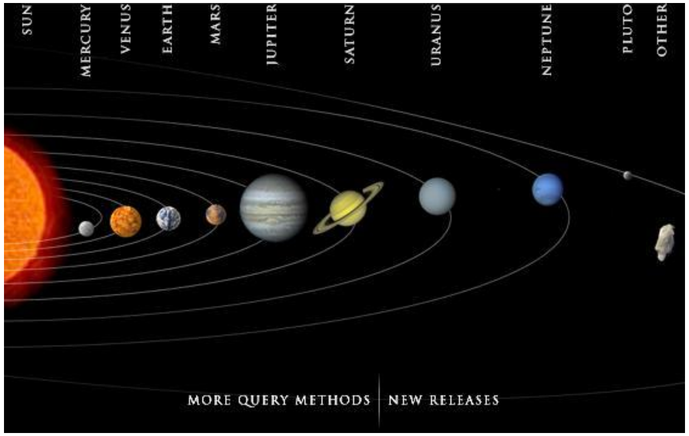
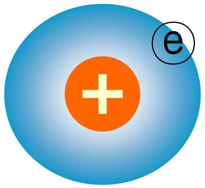
9. Electric Fields in Matter 2 (Electric displacement, Linear dielectrics)
10. Magnetostatics 1 (Lorentz force law, Biot-Savart law)
11. Magnetostatics 2 (Divergence & curl of B, Magnetic vector potential)
12. Magnetic fields in matter 1 (Magnetization, Field of a magnetized object)
13. Magnetic fields in matter 2 (Auxiliary field H, Linear & nonlinear media)
14. Electrodynamics 1 (Electromotive force, Electromagnetic induction)
15. Electrodynamics 2 (Maxwell's equations, Boundary conditions)
16. Final Exam

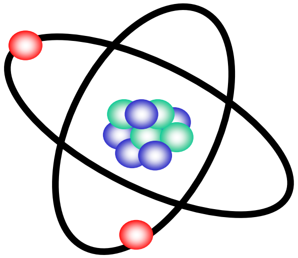
Background - 1



– 이 행성들이 태양을 중심으로 돌고 있도록 하는 힘은?



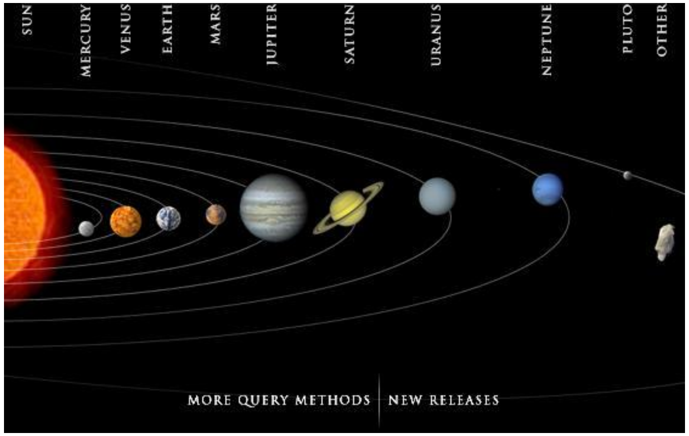
– 수소 원자내의 양성자와 전자 간의 작용하는 힘은?



– He 원자핵 내부의 **양성자**와 **중성자**간에 작용하는 힘은?

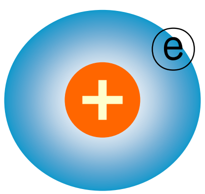
– 원자핵의 방사능 붕괴와 핵융합을 유도하는데 있어서 작용하는 힘은?

Background - 1



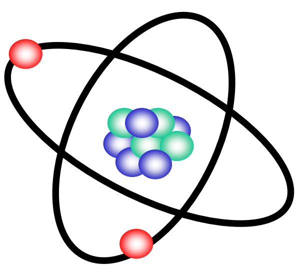
– 이 행성들이 태양을 중심으로 돌고 있도록 하는 힘은?

Gravitational force (중력)



– 수소 원자내의 양성자와 전자 간의 작용하는 힘은?

Electromagnetic force (전자기력, Coulomb force)



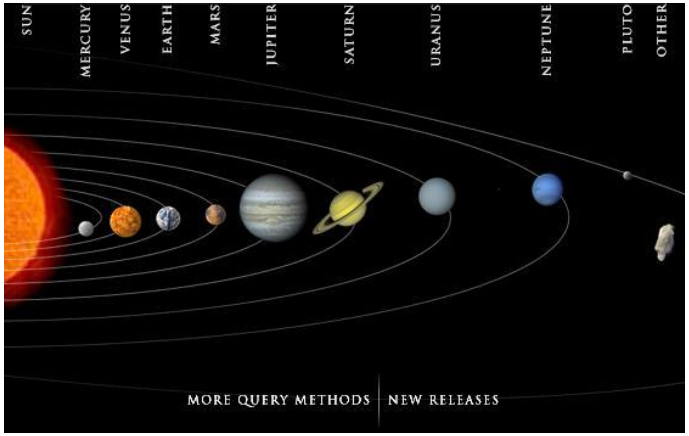
– He 원자핵 내부의 양성자와 중성자간에 작용하는 힘은?

Strong force (강력)

– 원자핵의 방사능 붕괴와 핵융합을 유도하는데 있어서 작용하는 힘은?

Weak force (약력)

Background - 1 (a)



– 이 행성들이 태양을 중심으로 돌고 있도록 하는 힘은?

(예) 지구와 태양 간에 작용하는 힘

$$F = -G\frac{mM}{r^2}$$

$$G = 6.67\times10^{-11}\ \mathrm{N\,m^2/kg^2}$$

$$m_{earth} = 6\times10^{24}\ \mathrm{kg}$$

$$M_{sun} = 2\times10^{30}\ \mathrm{kg}$$

$$r_{ave(sun-earth)} = 1.5\times10^{8}\ \mathrm{km}$$

– 태양과 지구와의 중력 (원 운동을 가정)

$$F = -G\frac{mM}{r^2} = \left(6.67\times10^{-11}\ Nm^2/kg^2\right)\frac{\left(6\times10^{24}\,kg\right)\left(2\times10^{30}\,kg\right)}{\left(1.5\times10^{11}\,m\right)^2} = 3.56\times10^{22}\ N$$

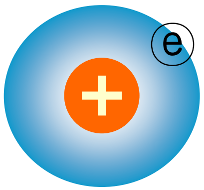
– 태양과 지구와의 중력과 동일한 원심력을 얻기 위한 원 운동의 주기는?

$$\frac{GMm}{r^2} = m\omega^2 r = mr\left(\frac{2\pi}{T}\right)^2 \longrightarrow T^2 = \left(\frac{4\pi^2}{GM}\right)r$$

$$T^2 = \left(\frac{4\pi^2}{GM}\right)r = \frac{4\pi^2}{\left(6.67\times10^{-11}\ Nm^2/kg^2\right)\left(2\times10^{30}\,kg\right)}\left(1.5\times10^{11}\,m\right)^3 = 9.97\times10^{14}\,s^2$$

$$T = \sqrt{9.97\times10^{14}\,s^2} = 31.59\times10^{6}\,s = 365.6\ days$$

Background - 1 (b)



– 수소 원자내의 양성자와 전자 간의 작용하는 전기력은?

$$F = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1 q_2}{r^2}\right)$$

$$\frac{1}{4\pi\varepsilon_o} \simeq 9\times10^9 \; N\cdot m^2/C^2$$

매개체: photon

입자	기호	전하량	질량 (kg)
양성자	p	+e	$1.6726\text{x}10^{-27}$
중성자	n	0	$1.6759\text{x}10^{-27}$
전자	e	-e	$9.1100\text{x}10^{-31}$

(e = 1.6 x 10^{-19} C)

$$\boldsymbol{r} = \frac{\varepsilon_0 \boldsymbol{h}^2}{\pi \boldsymbol{m_e e}^2}\boldsymbol{n}^2 \equiv \boldsymbol{a_H n}^2$$

$$a_H \equiv \frac{\varepsilon_0 h^2}{\pi m_e e^2} = 0.53 \text{ Angs}$$

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = \left(9\times10^9 \; Nm^2/C^2\right)\frac{\left(1.6\times10^{-19}C\right)^2}{\left(0.53\times10^{-10}m\right)^2} = 8.2\times10^{-8}N$$

– 수소 원자내의 양성자와 전자 간의 작용하는 중력은?

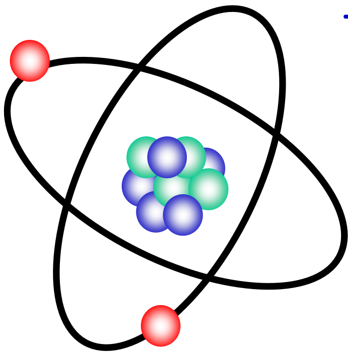
$$G = 6.67\times10^{-11} \text{ N m}^2/\text{kg}^2$$

$$F_g = -G\frac{mM}{r^2} = \left(6.67\times10^{-11} \; Nm^2/kg^2\right)\frac{\left(9.11\times10^{-31}kg\right)\left(1.67\times10^{-27}kg\right)}{\left(0.53\times10^{-10}m\right)^2} = 3.61\times10^{-47}N$$

→ 전기력/중력 비: $\frac{F_e}{F_g} = \frac{8.2\times10^{-8}N}{3.61\times10^{-47}N} = 2.27\times10^{39}$

Background - 1 (c)

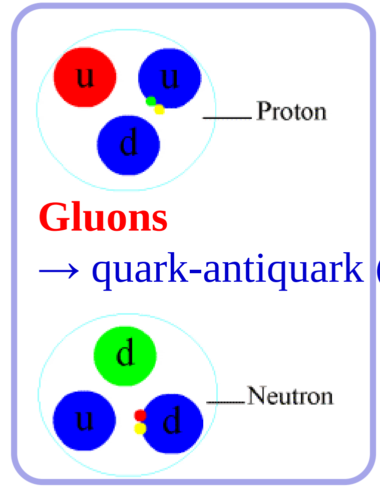
– He 원자핵 내부의 양성자와 중성자간에 작용하는 힘은?



Strong force (강력)

- Nuclear binding force
  (in a distance of 0.8 ~ 3 *fm*)

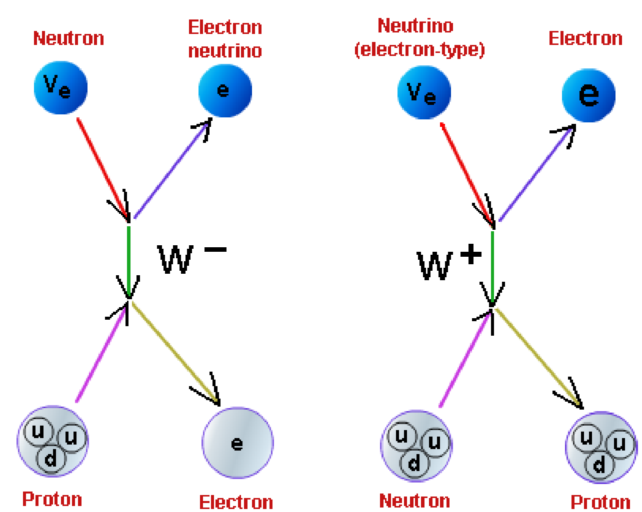
$\propto$ 100 x 전자기력
$\propto$ 10^6 x 약력
$\propto$ 10^{39} x 중력



– 원자핵의 방사능 붕괴와 핵융합을 유도하는데 있어서 작용하는 힘은?

Weak force (약력)

- 원자 핵의 존재와 구성을 유지시키는 힘으로 W, Z-bosons 교환에 의해 이루어지며, 베타 붕괴와 핵융합에 관련되어 있음

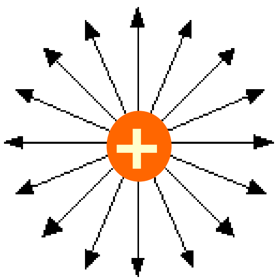


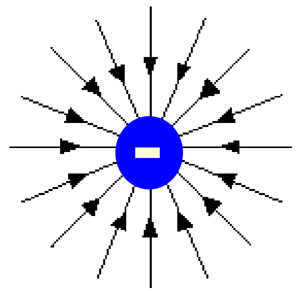
Historical List of Famous Physicists

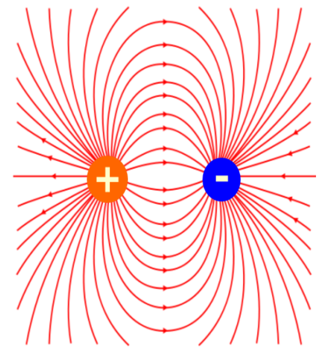
- **Classical mechanics**
  - Galileo Galilei (1564~1642, Italy)
  - Johannes Kepler (1571~1630, Germany)
  - Isaac Newton (1642~1727, England)
- **Electromagnetism**
  - Charles-Augustin de Coulomb (1736~1806, France)
  - Johann Carl Friedrich Gauss (1777~1855, Germany)
  - André Marie Ampère (177~1836, France)
  - Georg Simon Ohm (1789~1854, Germany)
  - Michael Faraday (1791~1867, England)
  - James Clerk Maxwell (1831~1879, Scottland)
- **Special relativity**
  - Albert Einstein (1879~1955, German-born Jewish)
- **Quantum mechanics**
  - Max Karl Ernst Ludwig Planck (1858~1947, Germany)
  - Niels Henrik David Bohr (1885~1962, Denmark)
  - Erwin Rudolf Josef Alexander Schrödinger (1887~1961, Austria)
  - Werner Karl Heisenberg (1901~1976, Germany)
- **Quantum field theory**
  - Wolfgang Ernst Pauli (1900~1958, Austria)
  - Paul Adrien Maurice Dirac (1902~1984, England)
  - Richard Phillips Feynman (1918~1988, USA)
  - Julian Seymour Schwinger (1918~1994, USA)

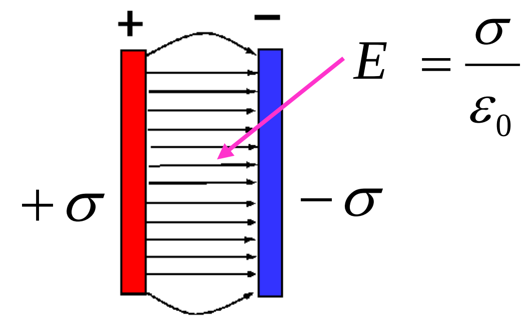
Background - 2

- **Electrostatics (정전기학)**







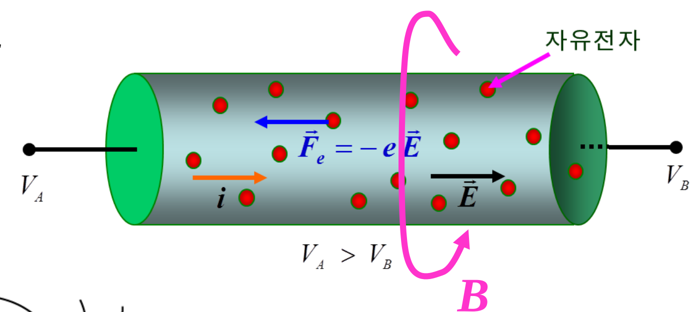


  – 전기장의 공간적인 분포도
  – 전위차

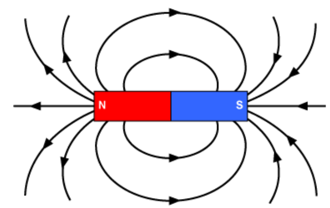
$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_o \varepsilon_r}$$

- **Electrodynamics (전기동력학)**

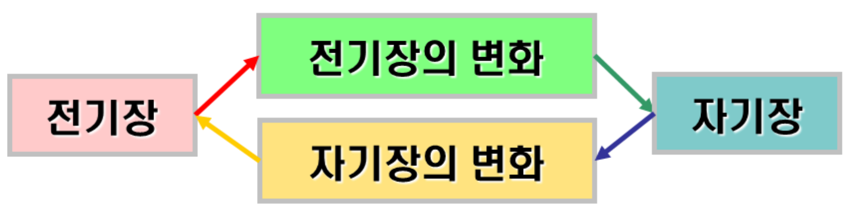
  – 전자의 이동 → 전류 → 유도 자기장



- **Magnetostatics (정자기학)**

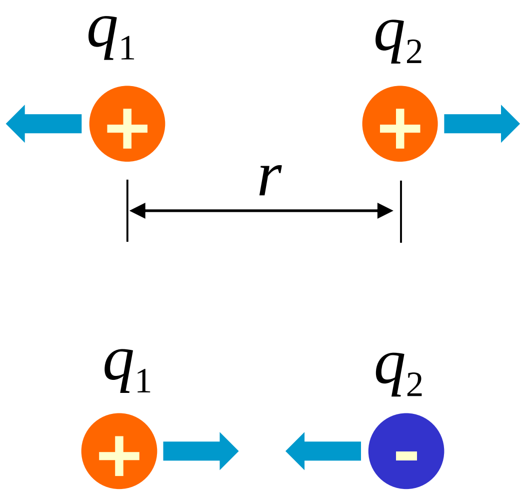


- **Electromagnetic Induction (전자기유도)**



전자기학에서의 단위 사용

전하들 간의 전기력 – Coulomb’s Law



거리 r 만큼 떨어진 두 전하 q_1과 q_2 간에 작용하는 전기력은

- In SI (MKS) unit,

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$$

· MKS: meter, kg, second

- In Gaussian (cgs) unit,

$$\vec{F} = \frac{q_1 q_2}{r^2}\hat{r}$$

· cgs: centimeter, g, second

- In HL (Heaviside-Lorentz) unit,

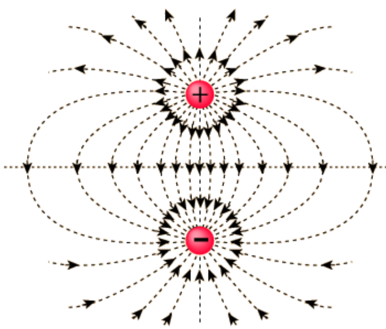
$$\vec{F} = \frac{1}{4\pi}\frac{q_1 q_2}{r^2}\hat{r}$$

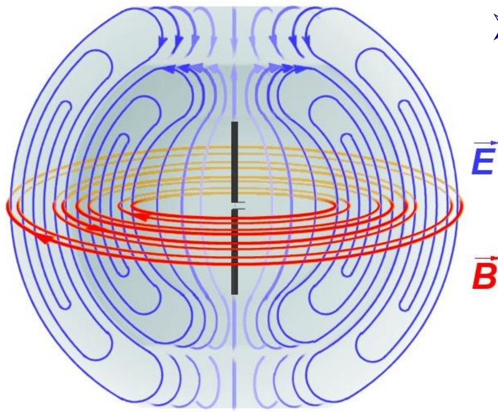
: popular in elementary particle theory (입자물리이론)

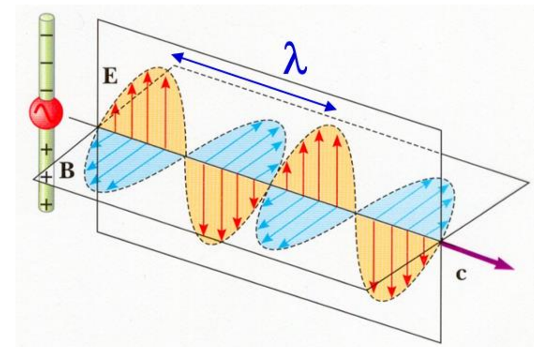
전자의 역할 - 1

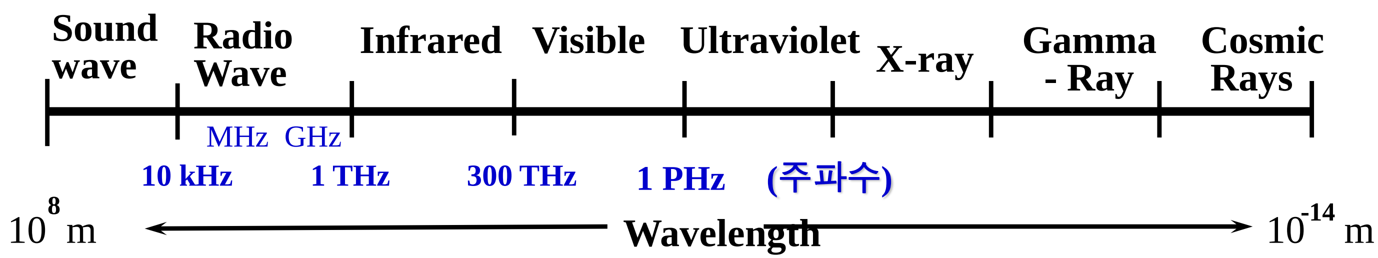
● 전자기파는 어떻게 만들어 지는가?

➢ 전기 쌍극자의 진동에 의한 전기장 및 자기장의 파동



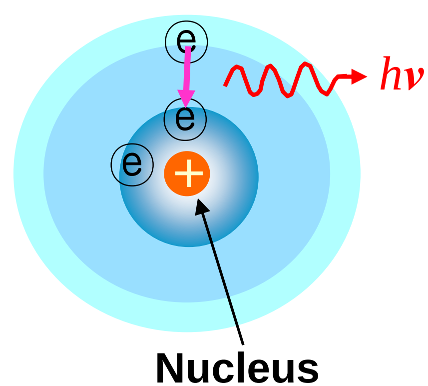




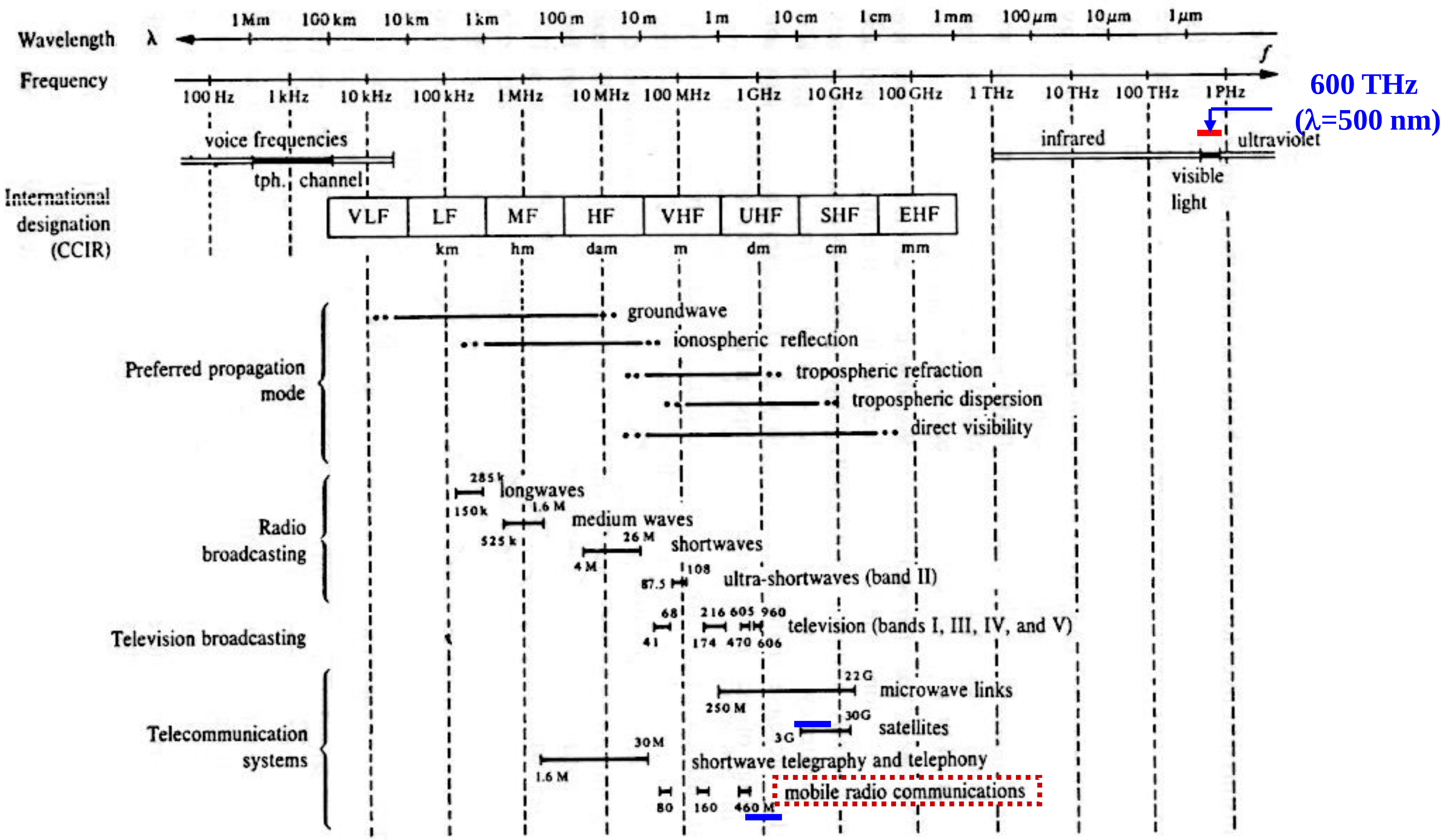


● 빛은 어떻게 만들어 지는가?

➢ 원자 (또는 분자) 내 전자의 에너지 준위 간 이동에 의한 빛의 방출

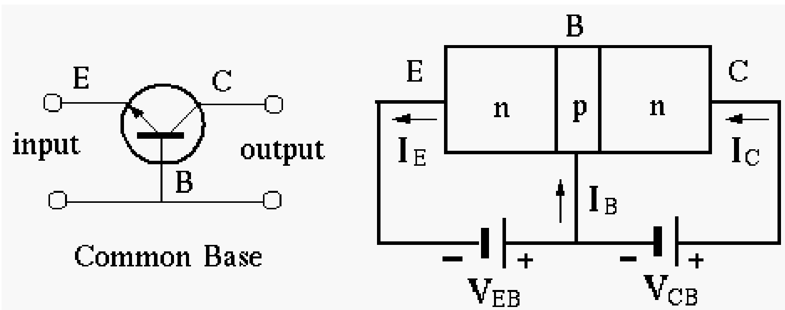


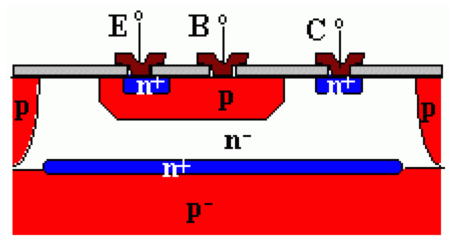
Lightwave - High Frequency Electromagnetic Waves

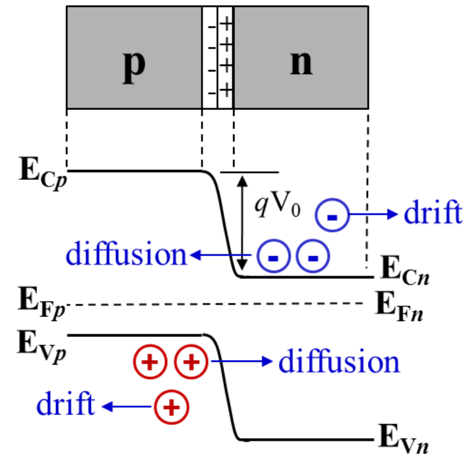


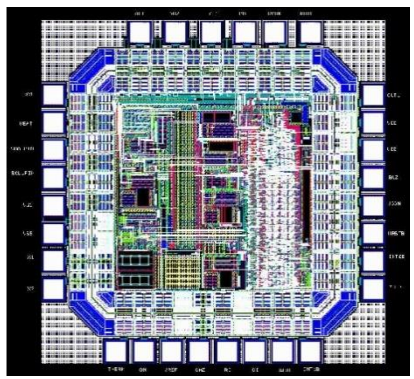
전자의 역할 - 2

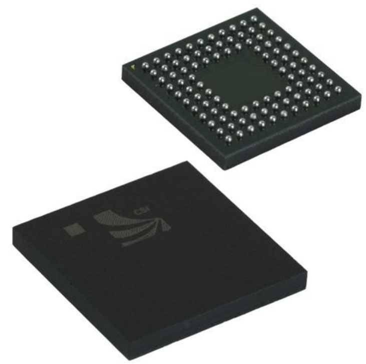
- 전자회로 & 전자집적회로소자

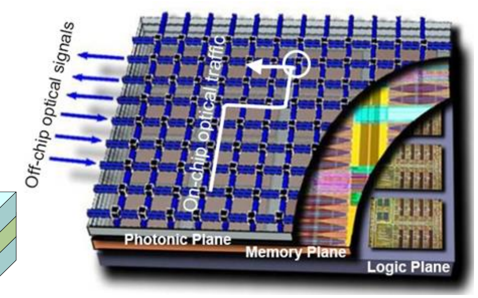




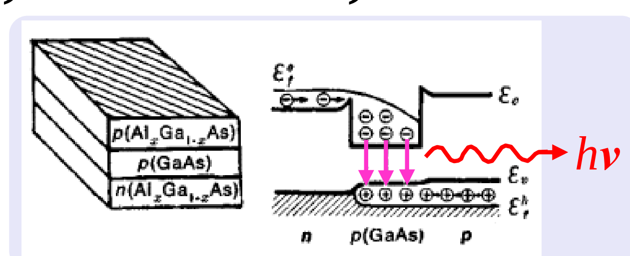


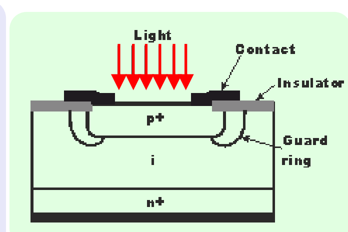


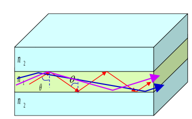


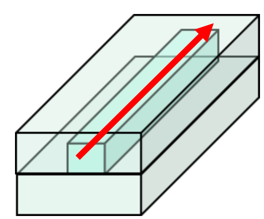


- 광원, 광검출소자, 광집적회로

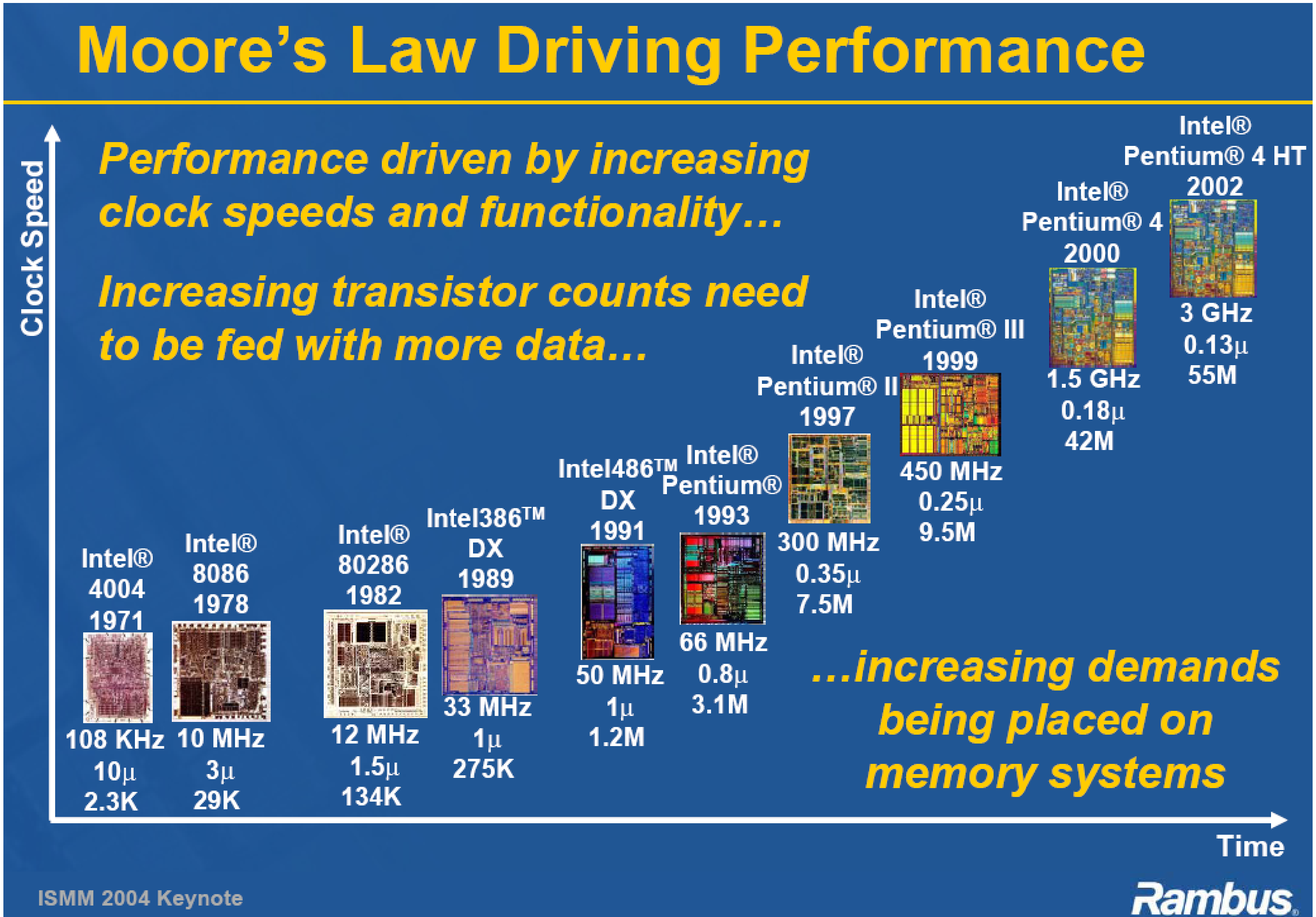


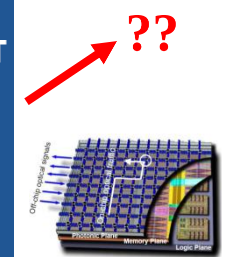






Requirement of Photonic Technology inside Chips



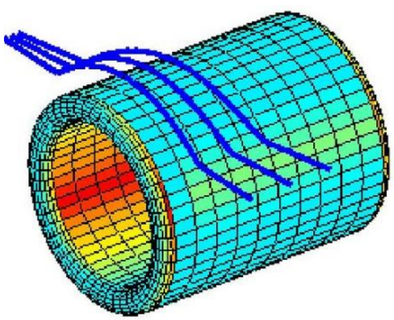


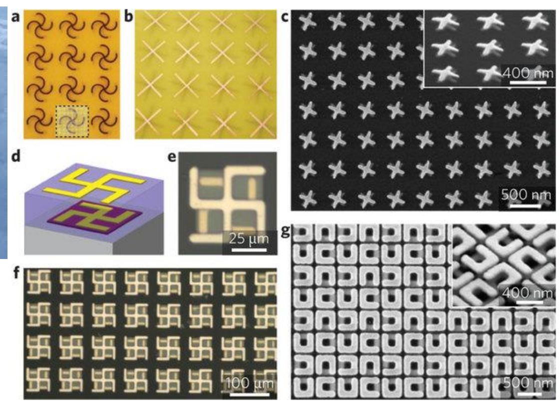
"8년뒤 가사 도우미로봇, 15년뒤 투명망토 가능"

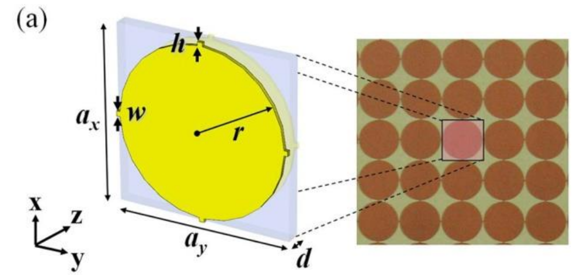
발행일 2012.03.04 윤대원기자 yun1972@etnews.com

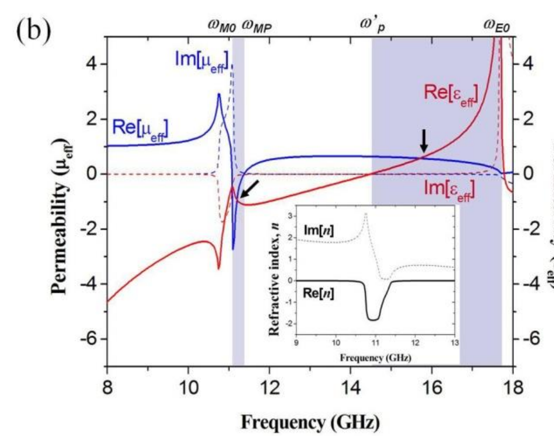
오는 2014 년이면 휴대 단말기에서도 3D 콘텐츠를 볼 수 있게 된다. 가사도우미 로봇은 10 년 뒤 선보이고, 2040 년에는 핵융합기술을 활용한 전기 생산도 가능해진다.

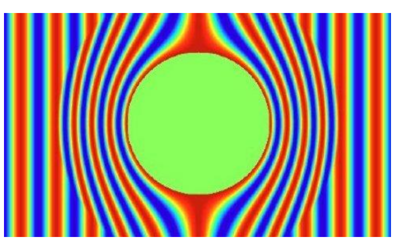
특히 2026 년에는 투명 망토기술도 개발된다. 가시광선으로부터 보이지 않도록 차폐하는 유연한 스마트 물질이 그것이다. 의료분야에서는 2027 년에 개인 기억을 컴퓨터에 스캔해 기록하는 기술이 소개된다. 원격지에서 발사되는 레이저로부터 에너지를 공급받아 위성 또는 비행체를 저궤도에 진입시키는 레이저 플라즈마 추진 비행체기술은 2039 년께에 실현될 전망이다. 핵융합실증플랜트 DEMO 건설과 운영을 통한 전기 생산 실증은 2040 년께로 가장 늦게 실용화될 기술로 꼽혔다.

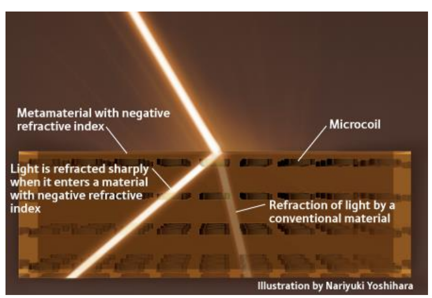


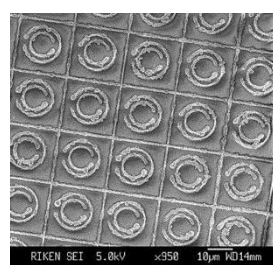










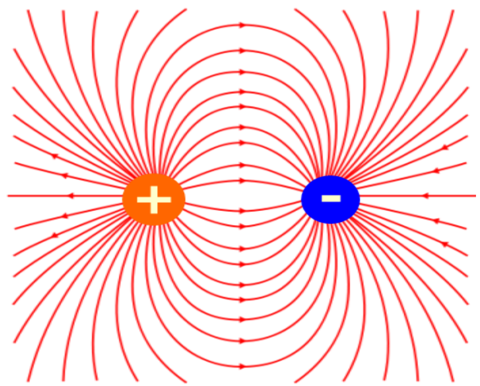


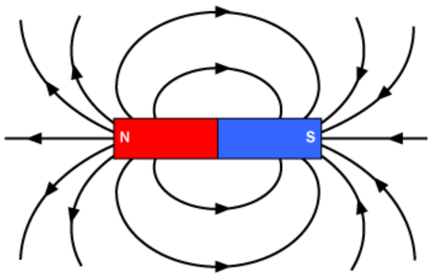
Chapter 1. Vector Analysis

- Vector Algebra
- Differential Calculus
- Integral Calculus
- Curvilinear Coordinates
- The Dirac Delta Function
- The Theory of Vector Fields

1.1 Vector Algebra (벡터 대수학)

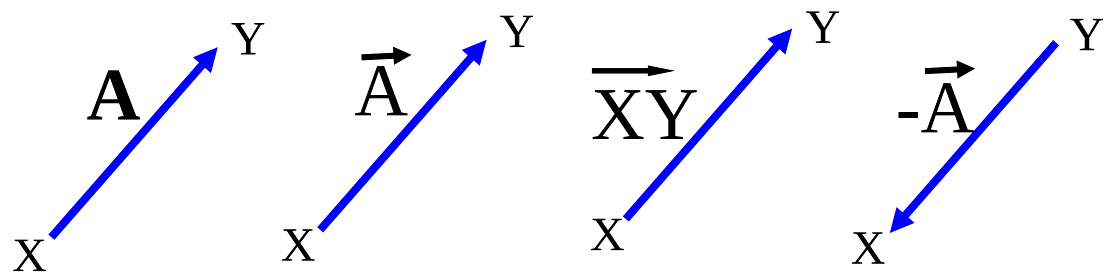
1.1.1 Vector Operations





- **Vectors** have **magnitude** and **direction**:

  velocity $\vec{v}$, acceleration $\vec{a}$, force $\vec{F}$, momentum $\vec{P}$, electric field $\vec{E}$, magnetic field $\vec{H}$, *etc*.



- **Scalars** have **magnitude** only but no direction:

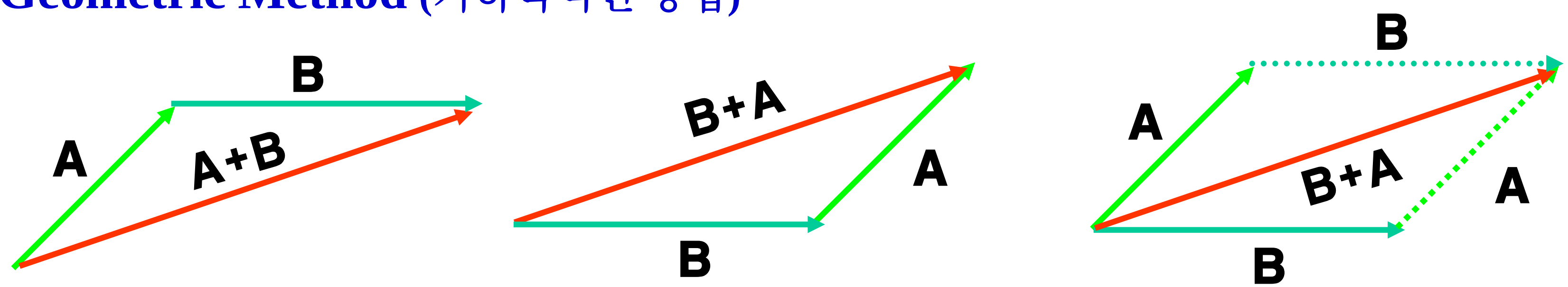
  mass m, charge q, density ρ, temperature T, electric potential V, energy U, *etc*.

1.1 Vector Algebra

(i) Addition of two vectors (벡터의 덧셈) - 1

A + B

- **Geometric Method (기하학적인 방법)**

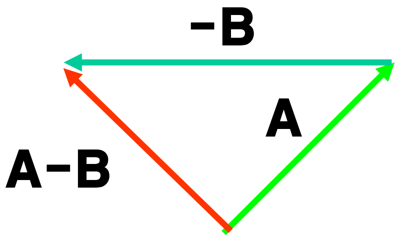


Triangular method (삼각형법)

Parallelogram (평행사변형법)

Any vectors can be expressed as sum of more than two arbitrary vectors.
모든 벡터는 임의의 둘 이상의 벡터의 합으로 표현할 수 있다.

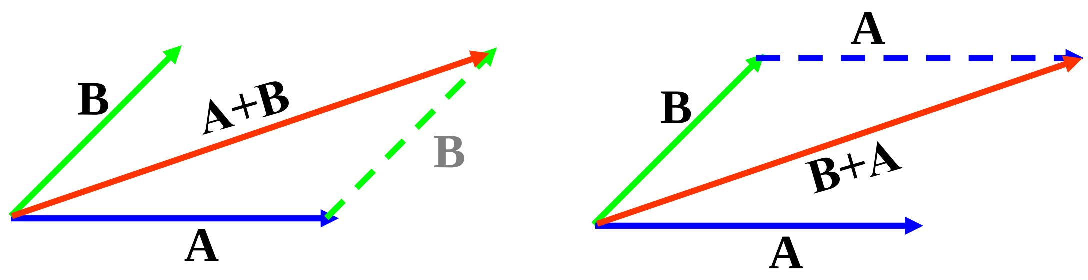
A – B=A+(–B)



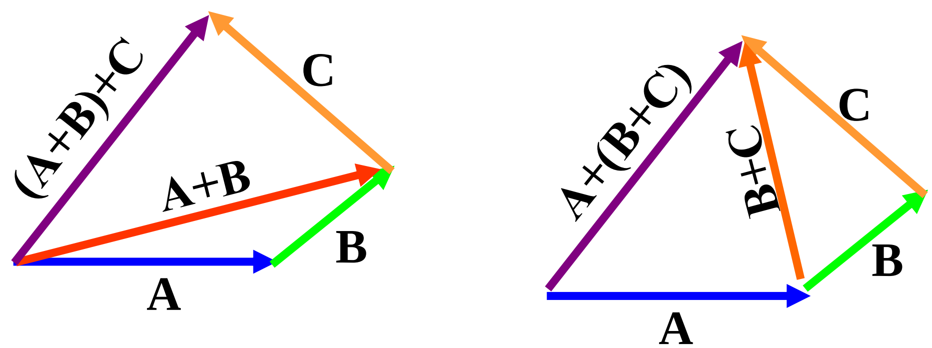
1.1 Vector Algebra

(i) Addition of two vectors (벡터의 덧셈) –2

A + B = B + A (commutative, 교환법칙)



(A+B)+C = A+(B+C) (associative, 결합법칙)



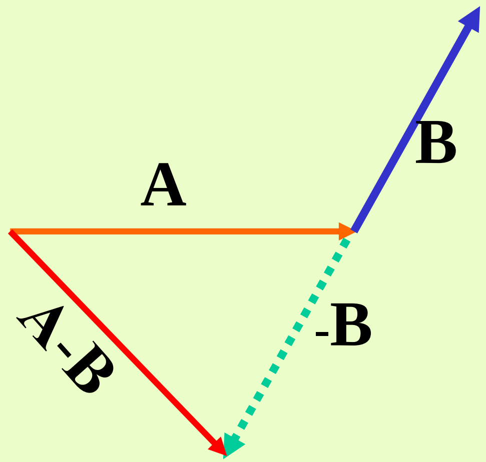
Any vectors can be expressed as sum of more than two arbitrary vectors.
모든 벡터는 임의의 둘 이상의 벡터의 합으로 표현할 수 있다.

1.1 Vector Algebra

◆ **Subtraction of a vector (벡터의 뺄셈)** $\mathbf{A}-\mathbf{B}=\mathbf{A}+(-\mathbf{B})$

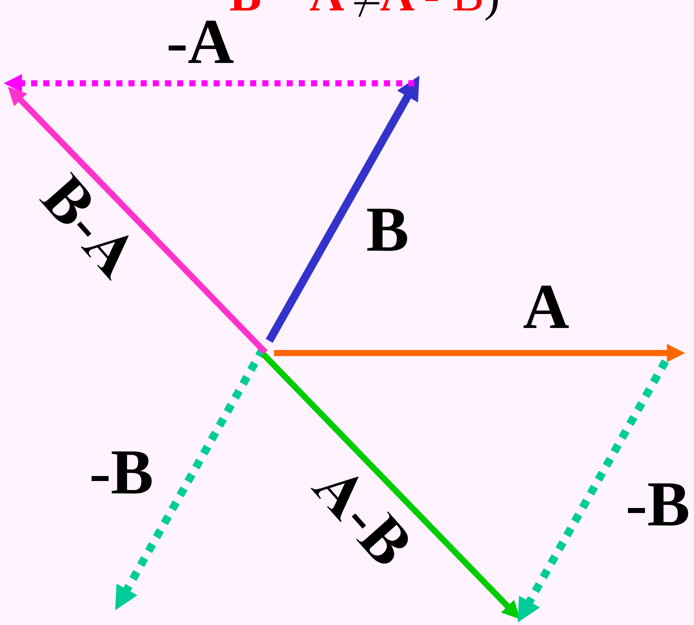
Definition of - B (a vector of the same magnitude but in opposite direction to B, B 벡터와 크기는 같고 방향이 반대인 벡터)

$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$



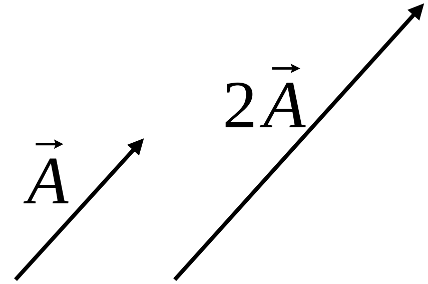
$\mathbf{B} - \mathbf{A} = -(\mathbf{A} - \mathbf{B})$

(No commutative, 교환법칙 성립 불가 : $\mathbf{B} - \mathbf{A} \neq \mathbf{A} - \mathbf{B}$)



1.1 Vector Algebra

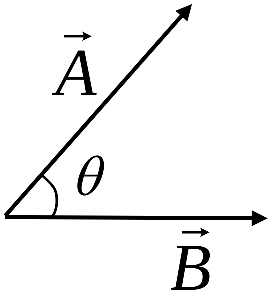
(ii) Multiplication by a scalar (스칼라 량의 곱)



$$\alpha\left(\vec{A}+\vec{B}\right)=\alpha\vec{A}+\alpha\vec{B}$$

(iii) Dot product of two vectors (2 벡터들 간의 스칼라 곱)

= "Scalar product" of two vectors



$$\vec{A}\cdot\vec{B}\equiv AB\cos\theta$$

$\vec{A}\cdot\vec{B}=\vec{B}\cdot\vec{A}$: 교환 법칙 (commutative)

$\vec{A}\cdot\left(\vec{B}+\vec{C}\right)=\vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}$: 분배 법칙 (distributive)

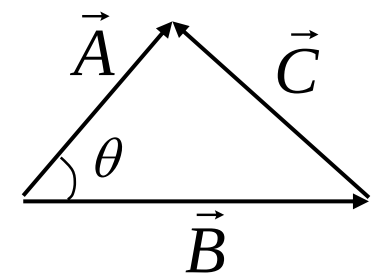
$$\vec{A}\cdot\vec{A}=A^2$$

If ***A*** and ***B*** vectors are parallel, $\vec{A}\cdot\vec{B}=AB\cos 0°=AB$

If ***A*** and ***B*** vectors are perpendicular, $\vec{A}\cdot\vec{B}=AB\cos 90°=0$

1.1 Vector Algebra

[Example of "Scalar product" of two vectors]



For $\vec{C} = \vec{A} - \vec{B}$

$$\vec{C}\cdot\vec{C} = \left(\vec{A}-\vec{B}\right)\cdot\left(\vec{A}-\vec{B}\right) = \vec{A}\cdot\vec{A} - \vec{A}\cdot\vec{B} - \vec{B}\cdot\vec{A} + \vec{B}\cdot\vec{B}$$

$$C^2 = A^2 - AB\cos\theta - BA\cos\theta + B^2$$

$$= A^2 + B^2 - 2AB\cos\theta$$

$$C^2 = A^2 + B^2 - 2AB\cos\theta$$

< Remember this !! >

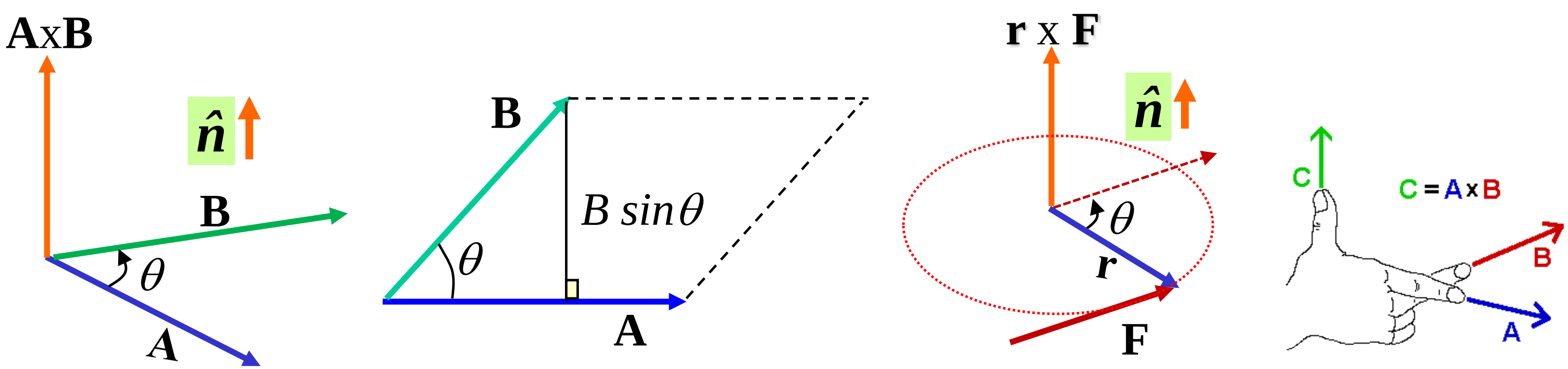
1.1 Vector Algebra

(iv) Cross product of two vectors (2 벡터들 간의 벡터 곱)

= "Vector product" of two vectors

$$\vec{A}\times\vec{B} \equiv AB\sin\theta\,\hat{n}$$

where $\hat{n}$ is a unit vector directing perpendicular to AB surface
(여기에서 $\hat{n}$ 는 AB 면에 수직 방향의 단위 벡터)



- Magnitude (크기): C = AB sin θ = area (면적) $C = \left|\vec{A}\times\vec{B}\right| = AB\sin\theta$
- Direction (방향): A에서 B로 돌릴 경우 오른나사의 진행방향: $\hat{n}$ (Right-hand rule) (오른 손 법칙)

$$\mathbf{B}\times\mathbf{A} = -\mathbf{A}\times\mathbf{B}$$

(No commutative, 교환법칙이 미성립, BxA ≠ AxB)

C = AB sin θ = |**B** x **A**| = BA sin θ **: the same magnitude (같은 크기)**

"Vector product" of two vectors

$$\vec{A}\times\left(\vec{B}+\vec{C}\right)=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$$: 분배 법칙 (distributive)

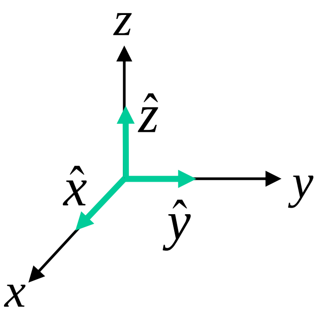
$$\vec{A}\times\vec{A}=A^2\,sin\,0°=0$$

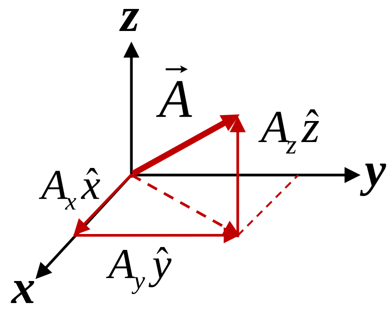
Problem 1.2 Is the cross product associative?

$$(\mathbf{A}\times\mathbf{B})\times\mathbf{C}\stackrel{?}{=}\mathbf{A}\times(\mathbf{B}\times\mathbf{C}).$$

If so, *prove* it; if not, provide a counterexample.

1.1.2 Vector Algebra: Component Form





$\hat{x}, \hat{y}, \hat{z}$: unit vectors, $|\hat{x}| = |\hat{y}| = |\hat{z}| = 1$

$$\mathbf{A} = \mathbf{A_x} + \mathbf{A_y} + \mathbf{A_z} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$$

크기 $A^2 = A_x^2 + A_y^2 + A_z^2$

$$\vec{R} = \vec{A} + \vec{B} = (A_x\hat{x} + A_y\hat{y} + A_z\hat{z}) + (B_x\hat{x} + B_y\hat{y} + B_z\hat{z})$$
$$= (A_x + B_x)\hat{x} + (A_y + B_y)\hat{y} + (A_z + B_z)\hat{z}$$
$$= R_x\hat{x} + R_y\hat{y} + R_z\hat{z}$$

1) 합 벡터의 성분은 각 벡터의 성분들끼리의 합과 동일: $R_x = A_x + B_x, \quad R_y = A_y + B_y, \quad R_z = A_z + B_z$

2) 스칼라 량의 곱은 각 벡터의 성분에 곱한 값과 동일: $a\vec{A} = (aA_x)\hat{x} + (aA_y)\hat{y} + (aA_z)\hat{z}$

3) 벡터들의 스칼라 곱은 각 벡터 성분들을 곱하여 더한 값과 동일:

$$\hat{x}\cdot\hat{x} = \hat{y}\cdot\hat{y} = \hat{z}\cdot\hat{z} = 1 \qquad \hat{x}\cdot\hat{y} = \hat{x}\cdot\hat{z} = \hat{y}\cdot\hat{z} = 0$$

$$\vec{A}\cdot\vec{B} = (A_x\hat{x} + A_y\hat{y} + A_z\hat{z})\cdot(B_x\hat{x} + B_y\hat{y} + B_z\hat{z}) = A_xB_x + A_yB_y + A_zB_z$$: 스칼라 량

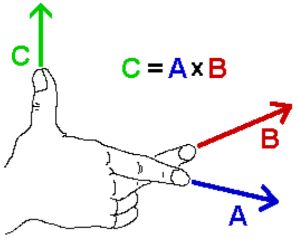
$$\vec{A}\cdot\vec{A} = A^2 = A_x^2 + A_y^2 + A_z^2 \quad \rightarrow \quad A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\vec{A}\cdot\hat{x} = A_x \qquad \vec{A}\cdot\hat{y} = A_y \qquad \vec{A}\cdot\hat{z} = A_z$$

1.1 Vector Algebra

1.1.2 Vector Algebra: Component Form

4) 벡터들의 벡터 곱은 행렬식을 이용:



$$\hat{x}\times\hat{x}=\hat{y}\times\hat{y}=\hat{z}\times\hat{z}=0$$

$$\hat{x}\times\hat{y}=-\hat{y}\times\hat{x}=\hat{z}$$

$$\hat{y}\times\hat{z}=-\hat{z}\times\hat{y}=\hat{x}$$

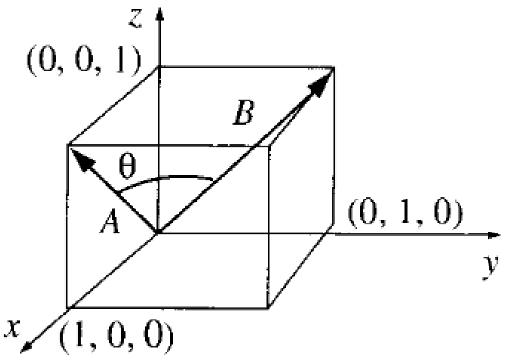
$$\hat{z}\times\hat{x}=-\hat{x}\times\hat{z}=\hat{y}$$

$$\begin{aligned}\vec{\mathrm{A}}\times\vec{\mathrm{B}}&=(A_x\hat{x}+A_y\hat{y}+A_z\hat{z})\times(B_x\hat{x}+B_y\hat{y}+B_z\hat{z})\\&=A_xB_y\hat{z}-A_xB_z\hat{y}-A_yB_x\hat{z}+A_yB_z\hat{x}+A_zB_x\hat{y}-A_zB_y\hat{x}\\&=\hat{x}(A_yB_z-A_zB_y)+\hat{y}(A_zB_x-A_xB_z)+\hat{z}(A_xB_y-A_yB_x)\end{aligned}$$

$$\vec{\mathrm{A}}\times\vec{\mathrm{B}}=\begin{vmatrix}\hat{x}&\hat{y}&\hat{z}\\A_x&A_y&A_z\\B_x&B_y&B_z\end{vmatrix}=\hat{x}\begin{vmatrix}A_y&A_z\\B_y&B_z\end{vmatrix}+\hat{y}\begin{vmatrix}A_z&A_x\\B_z&B_x\end{vmatrix}+\hat{z}\begin{vmatrix}A_x&A_y\\B_x&B_y\end{vmatrix}$$

1.1 Vector Algebra

[Example 1.2] Find the angle between the face diagonals of a cube.



$$\vec{A} = 1\hat{x} + 0\hat{y} + 1\hat{z}$$

$$\vec{B} = 0\hat{x} + 1\hat{y} + 1\hat{z}$$

(Solution)

$$A = \left|\vec{A}\right| = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{1^2 + 0^2 + 1^2} = \sqrt{2}$$

$$B = \left|\vec{B}\right| = \sqrt{B_x^2 + B_y^2 + B_z^2} = \sqrt{0^2 + 1^2 + 1^2} = \sqrt{2}$$

$$\vec{\mathrm{A}} \cdot \vec{\mathrm{B}} = A_x B_x + A_y B_y + A_z B_z = 1 \cdot 0 + 0 \cdot 1 + 1 \cdot 1 = 1$$

$$\vec{\mathrm{A}} \cdot \vec{\mathrm{B}} = AB\cos\theta = \left(\sqrt{2}\right)^2 \cos\theta = 2\cos\theta$$

$$\therefore\ 2\cos\theta = 1$$

$$\cos\theta = \frac{1}{2} \longrightarrow \theta = 60^\circ$$

1.1.3 Triple Products

(i) Scalar triple product: $\vec{A}\cdot(\vec{B}\times\vec{C})$

$$\vec{A}\cdot(\vec{B}\times\vec{C})=\vec{B}\cdot(\vec{C}\times\vec{A})=\vec{C}\cdot(\vec{A}\times\vec{B})$$

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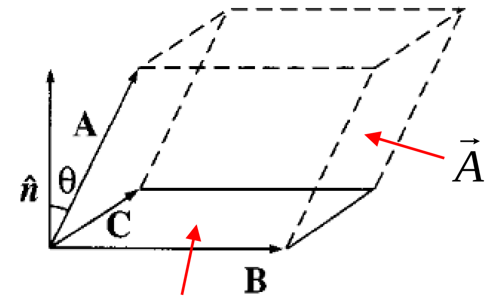


$$\vec{A}\cdot(\vec{C}\times\vec{B})=-\vec{A}\cdot(\vec{B}\times\vec{C})$$

$$\mathbf{A}\cdot(\mathbf{C}\times\mathbf{B})=\mathbf{B}\cdot(\mathbf{A}\times\mathbf{C})=\mathbf{C}\cdot(\mathbf{B}\times\mathbf{A})$$

$$\vec{A}\cdot(\vec{B}\times\vec{C})=\begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

$$\vec{A}\cdot(\vec{B}\times\vec{C})=(\vec{A}\times\vec{B})\cdot\vec{C}$$

$$(\vec{A}\cdot\vec{B})\times\vec{C}=\text{no meaningful (why?)}$$

1.1 Vector Algebra

1.1.3 Triple Products

(ii) Vector triple product: $\vec{A}\times\left(\vec{B}\times\vec{C}\right)$

$$\vec{A}\times\left(\vec{B}\times\vec{C}\right)=\vec{B}\left(\vec{A}\cdot\vec{C}\right)-\vec{C}\left(\vec{A}\cdot\vec{B}\right)$$

$$\left(\vec{A}\times\vec{B}\right)\times\vec{C}=-\vec{C}\times\left(\vec{A}\times\vec{B}\right)=-\vec{A}\left(\vec{B}\cdot\vec{C}\right)+\vec{B}\left(\vec{A}\cdot\vec{C}\right)$$

$$(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{C}\times\mathbf{D}) = (\mathbf{A}\cdot\mathbf{C})(\mathbf{B}\cdot\mathbf{D})-(\mathbf{A}\cdot\mathbf{D})(\mathbf{B}\cdot\mathbf{C})$$

$$\mathbf{A}\times(\mathbf{B}\times(\mathbf{C}\times\mathbf{D})) = \mathbf{B}(\mathbf{A}\cdot(\mathbf{C}\times\mathbf{D}))-(\mathbf{A}\cdot\mathbf{B})(\mathbf{C}\times\mathbf{D})$$

Problem 1.5 Prove the **BAC-CAB** rule by writing out both sides in component form.

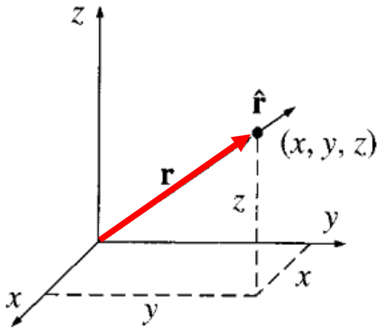
Problem 1.6 Prove that

$$[\mathbf{A}\times(\mathbf{B}\times\mathbf{C})]+[\mathbf{B}\times(\mathbf{C}\times\mathbf{A})]+[\mathbf{C}\times(\mathbf{A}\times\mathbf{B})]=0.$$

Under what conditions does $\mathbf{A}\times(\mathbf{B}\times\mathbf{C})=(\mathbf{A}\times\mathbf{B})\times\mathbf{C}$?

1.1 Vector Algebra

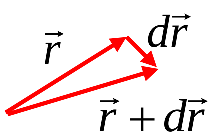
1.1.4 Position, Displacement, and Separation Vectors



위치 벡터 (**Position vector**) : $\vec{r} \equiv x\,\hat{x} + y\,\hat{y} + z\,\hat{z}$

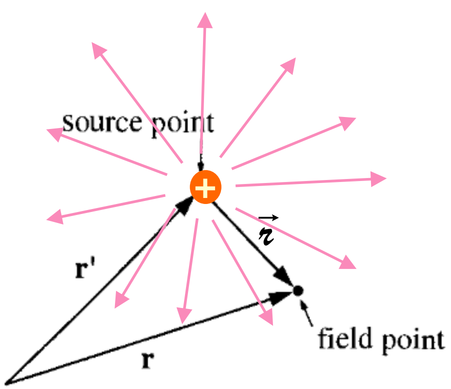
$$r = \sqrt{x^2 + y^2 + z^2} \quad \text{: 크기 (magnitude)}$$

$$\hat{r} = \frac{\vec{r}}{r} = \frac{x\,\hat{x} + y\,\hat{y} + z\,\hat{z}}{\sqrt{x^2 + y^2 + z^2}} \quad \text{: 단위 벡터 (방향)}$$



극소 변위 (infinitesimal displacement vector):

$$d\vec{r} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$$



거리 벡터 (**Separation vector**):

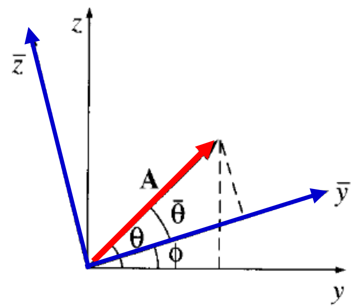
$$\vec{\mathscr{r}} = \vec{r} - \vec{r}\,' = (x - x')\hat{x} + (y - y')\hat{y} + (z - z')\hat{z}$$

$$\mathscr{r} = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$$

$$\hat{\mathscr{r}} = \frac{(x - x')\hat{x} + (y - y')\hat{y} + (z - z')\hat{z}}{\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}}$$

Problem 1.7 Find the separation vector $\vec{\mathscr{r}}$ from the source point (2,8,7) to the field point (4,6,8). Determine its magnitude ($\mathscr{r}$), and construct the unit vector $\hat{\mathscr{r}}$.

1.1.5 Rotation of a Vector



좌표의 변환과 벡터의 회전

$$A_y = A\cos\theta \qquad A_z = A\sin\theta$$

$$\bar{A}_y = A\cos\bar{\theta} = A\cos(\theta-\phi) = A(\cos\theta\cos\phi + \sin\theta\sin\phi)$$
$$= A_y\cos\phi + A_z\sin\phi$$

$$\bar{A}_z = A\sin\bar{\theta} = A\sin(\theta-\phi) = A(\sin\theta\cos\phi - \cos\theta\sin\phi)$$
$$= -A_y\sin\phi + A_z\cos\phi$$

In matrix notation,

$$\begin{pmatrix} \bar{A}_y \\ \bar{A}_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} A_y \\ A_z \end{pmatrix}$$

In 3-dimensional notations,

$$\begin{pmatrix} \bar{A}_x \\ \bar{A}_y \\ \bar{A}_z \end{pmatrix} = \begin{pmatrix} R_{xx} & R_{xy} & R_{xz} \\ R_{yx} & R_{yy} & R_{yz} \\ R_{zx} & R_{zy} & R_{zz} \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ Az \end{pmatrix}$$

A compact notation: $\bar{A}_i = \sum_{j=1}^{3} R_{ij} A_j$ where $i = x, y, z$

$1 = x,\ 2 = y,\ 3 = z$

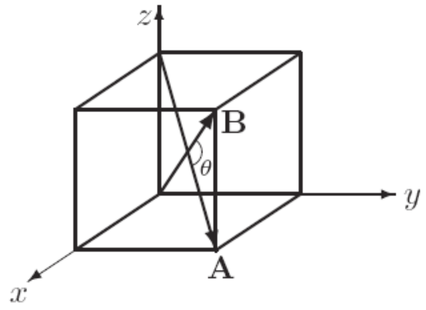
Next Class

Chapter 1. Vector Analysis

- Vector Algebra
- Differential Calculus
- Integral Calculus
- Curvilinear Coordinates
- The Dirac Delta Function
- The Theory of Vector Fields

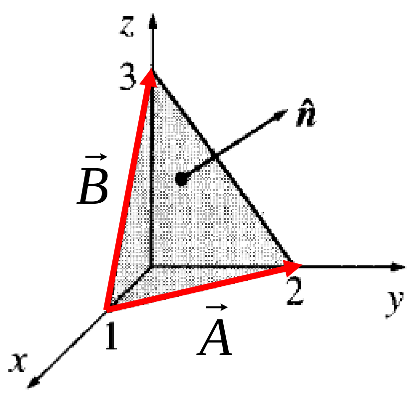
1.1 Vector Algebra

[Problem 1.3] Find the angle between the body diagonals of a cube.



[Problem 1.4] Use the cross product to find the components of the unit vector $\hat{n}$ perpendicular to the shaded plane in Fig. 1.11.

$$(\text{Hint}: \vec{R} = R\hat{n} \rightarrow \hat{n} = \frac{\vec{R}}{R})$$



Chapter 1. Vector Analysis

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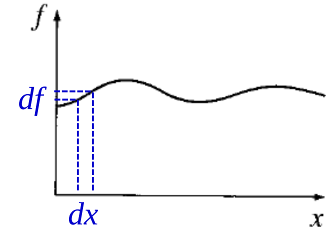
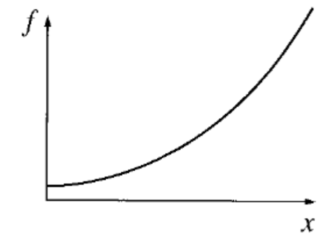
1.2 Differential Calculus

1.2.1 "Ordinary" Derivatives (상미분)





1개의 변수를 가진 함수 $f(x)$ 의 경우

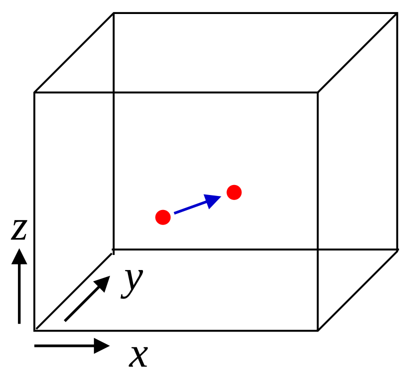
x가 dx 만큼 변할 때, $f(x)$의 변화 정도는 $$df = \underbrace{\left(\frac{df}{dx}\right)}_{\text{기울기}} dx$$

"상미분" (ordinary derivative)

1.2.2 Gradient (경사도)

3개의 변수를 가진 함수 $T(x, y, z)$ 의 경우 (예, 3차원 공간인 방 안에서의 온도 분포)

- 실내 위치에 따른 온도의 변화는? $$dT = \underbrace{\left(\frac{\partial T}{\partial x}\right)}_{\text{"편미분" (partial derivative)}} dx + \left(\frac{\partial T}{\partial y}\right) dy + \left(\frac{\partial T}{\partial z}\right) dz$$



$$dT = \left(\frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}\right) \cdot \left(dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}\right) = \left(\vec{\nabla} T\right) \cdot \left(d\vec{r}\right)$$

where $\vec{\nabla} T = \frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}$: the **gradient** of ***T***

: a **vector** → has ***magnitude*** & ***direction*** (온도 변화가 최대인 방향으로).

$$dT = \vec{\nabla} T \cdot d\vec{r} = \left|\vec{\nabla} T\right| \left|d\vec{r}\right| \cos\theta = \begin{cases} = \left|\vec{\nabla} T\right| \left|d\vec{r}\right| : \text{Maximum} \\ = 0 \qquad : \text{Minimum} \end{cases}$$

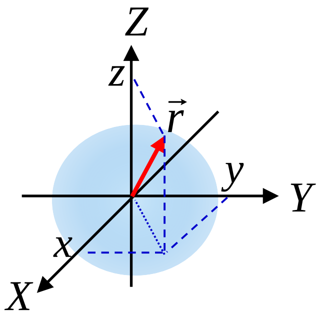
1.2 Differential Calculus

[Example 1.3] Find the gradient of $r = \sqrt{x^2 + y^2 + z^2}$ (the magnitude of the position vector).

$$\vec{\nabla} r = \frac{\partial r}{\partial x}\hat{x} + \frac{\partial r}{\partial y}\hat{y} + \frac{\partial r}{\partial z}\hat{z}$$

$$= \frac{1}{2}\frac{2x}{\sqrt{x^2+y^2+z^2}}\hat{x} + \frac{1}{2}\frac{2y}{\sqrt{x^2+y^2+z^2}}\hat{y} + \frac{1}{2}\frac{2z}{\sqrt{x^2+y^2+z^2}}\hat{z}$$

$$= \frac{x\,\hat{x} + y\,\hat{y} + z\,\hat{z}}{\sqrt{x^2+y^2+z^2}} = \frac{\vec{r}}{r} = \hat{r}$$



$$\vec{\nabla} r = \hat{r}$$

크기 = 1

방향 = "r" 방향

1.2 Differential Calculus

1.2.3 The Del Operator

Gradient of ***T*** : $\vec{\nabla} T = \frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)T$

Let us write $\vec{\nabla} = \nabla$

$$\nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

: "**del**" operator

: Not a vector but a "vector operator"

∇T (T is a scalar) : **gradient T**

$\nabla \cdot \vec{v}$: **divergence v**

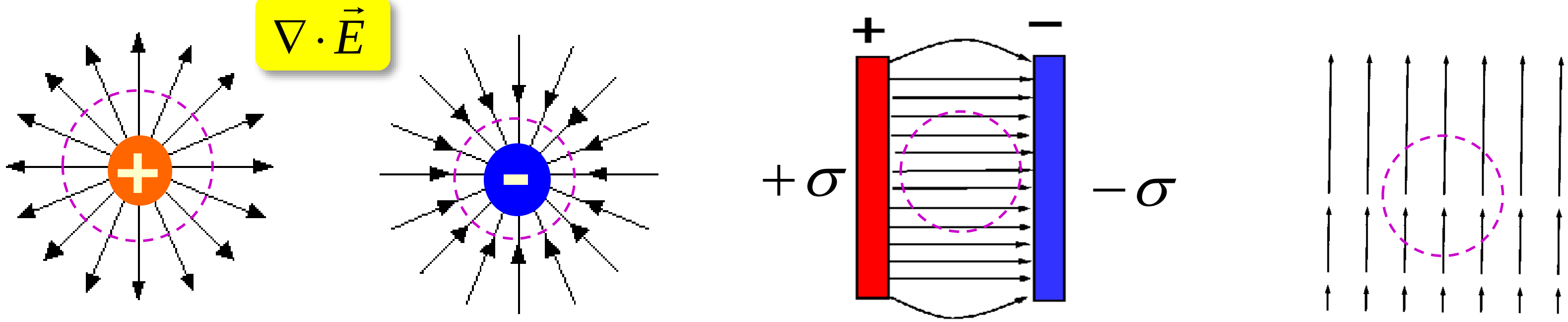
$\nabla \times \vec{v}$: **curl v**

1.2 Differential Calculus

1.2.4 The Divergence (확산)

$$\nabla\cdot\vec{\mathrm{v}}=\left(\hat{x}\frac{\partial}{\partial x}+\hat{y}\frac{\partial}{\partial y}+\hat{z}\frac{\partial}{\partial z}\right)\cdot\left(\mathrm{v}_x\hat{x}+\mathrm{v}_y\hat{y}+\mathrm{v}_z\hat{z}\right)=\frac{\partial \mathrm{v}_x}{\partial x}+\frac{\partial \mathrm{v}_y}{\partial y}+\frac{\partial \mathrm{v}_z}{\partial z}$$

$\nabla\cdot\vec{E}$



Positive divergence | Negative divergence | Zero divergence | Positive divergence

[Example 1.4] $\vec{\mathrm{v}}_a=\vec{r}=x\,\hat{x}+y\,\hat{y}+z\,\hat{z}$ → $\nabla\cdot\vec{\mathrm{v}}_a=\frac{\partial x}{\partial x}+\frac{\partial y}{\partial y}+\frac{\partial z}{\partial z}=1+1+1=3$

$\vec{\mathrm{v}}_b=\hat{z}$ → $\nabla\cdot\vec{\mathrm{v}}_b=\frac{\partial(0)}{\partial x}+\frac{\partial(0)}{\partial y}+\frac{\partial(1)}{\partial z}=0+0+0=0$

$\vec{\mathrm{v}}_c=z\,\hat{z}$ → $\nabla\cdot\vec{\mathrm{v}}_b=\frac{\partial(0)}{\partial x}+\frac{\partial(0)}{\partial y}+\frac{\partial(z)}{\partial z}=0+0+1=1$

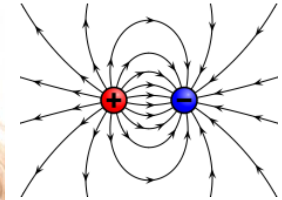
1.2 Differential Calculus

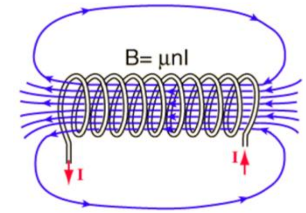
1.2.5 The Curl (컬, 곱슬)

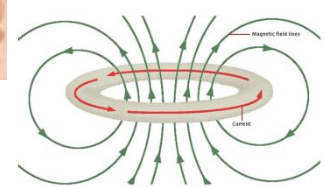
$$\nabla \times \vec{\mathrm{v}} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ \mathrm{v}_x & \mathrm{v}_y & \mathrm{v}_z \end{vmatrix}$$

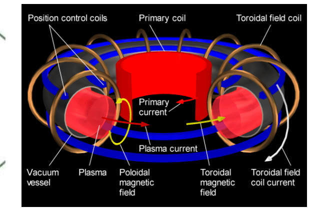
$$= \hat{x}\left(\frac{\partial \mathrm{v}_z}{\partial y} - \frac{\partial \mathrm{v}_y}{\partial z}\right) + \hat{y}\left(\frac{\partial \mathrm{v}_x}{\partial z} - \frac{\partial \mathrm{v}_z}{\partial x}\right) + \hat{z}\left(\frac{\partial \mathrm{v}_y}{\partial x} - \frac{\partial \mathrm{v}_x}{\partial y}\right)$$

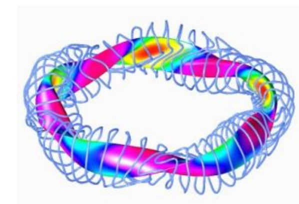
: a **vector**



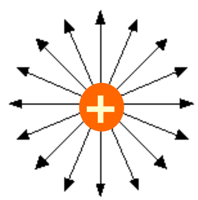


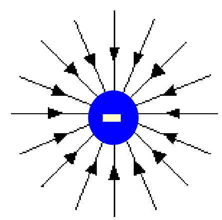


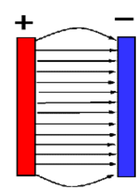


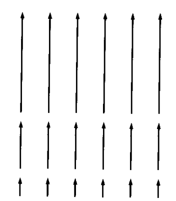


Zero curl cases →

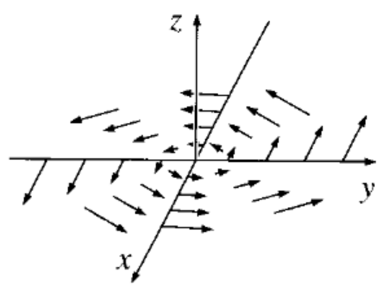


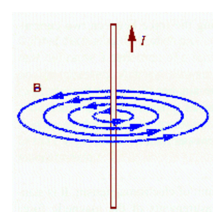


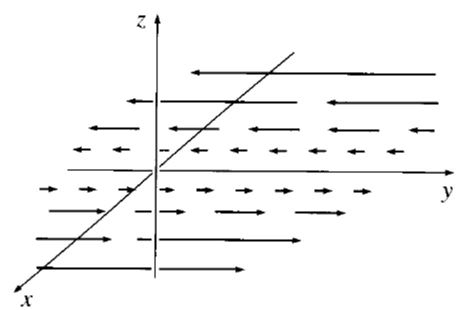




Non-zero curl cases →

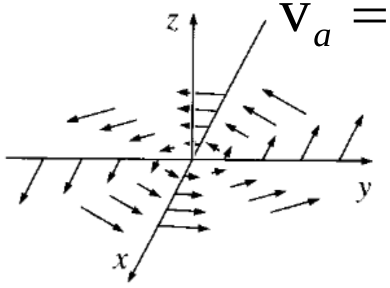






1.2 Differential Calculus

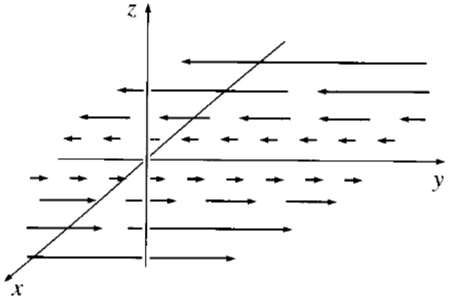
[Example 1.5] Calculate the curls of the $\mathbf{v}_a$ and $\mathbf{v}_b$



$$\vec{\mathrm{v}}_a = -y\,\hat{x} + x\,\hat{y}$$

$$\nabla\times\vec{\mathrm{v}}_a = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ -y & x & 0 \end{vmatrix}$$

$$= \hat{x}\left(\frac{\partial(0)}{\partial y} - \frac{\partial(x)}{\partial z}\right) + \hat{y}\left(\frac{\partial(-y)}{\partial z} - \frac{\partial(0)}{\partial x}\right) + \hat{z}\left(\frac{\partial(x)}{\partial x} - \frac{\partial(-y)}{\partial y}\right) = 2\hat{z}$$



$$\vec{\mathrm{v}}_b = x\,\hat{y}$$

$$\nabla\times\vec{\mathrm{v}}_b = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 0 & x & 0 \end{vmatrix}$$

$$= \hat{x}\left(\frac{\partial(0)}{\partial y} - \frac{\partial(x)}{\partial z}\right) + \hat{y}\left(\frac{\partial(0)}{\partial z} - \frac{\partial(0)}{\partial x}\right) + \hat{z}\left(\frac{\partial(x)}{\partial x} - \frac{\partial(0)}{\partial y}\right) = \hat{z}$$

1.2 Differential Calculus

1.2.6 Product Rules

- 상미분 (ordinary derivatives)

- 덧셈 법칙 : $\frac{d}{dx}(f+g)=\frac{df}{dx}+\frac{dg}{dx}$

- 상수의 곱 : $\frac{d}{dx}(kf)=k\frac{df}{dx}$

- 곱의 법칙 : $\frac{d}{dx}(fg)=f\frac{dg}{dx}+g\frac{df}{dx}$

- 분수(quotient)의 법칙 : $\frac{d}{dx}\left(\frac{f}{g}\right)=\frac{1}{g}\frac{df}{dx}-\frac{f}{g^2}\frac{dg}{dx}=\frac{1}{g^2}\left(g\frac{df}{dx}-f\frac{dg}{dx}\right)$

- 벡터 미분 (vector derivatives)

$$\nabla(f+g)=\nabla f+\nabla g$$

$$\nabla\cdot(\vec{A}+\vec{B})=\nabla\cdot\vec{A}+\nabla\cdot\vec{B}$$

$$\nabla\times(\vec{A}+\vec{B})=\nabla\times\vec{A}+\nabla\times\vec{B}$$

$$\nabla(kf)=k\nabla f$$

$$\nabla\cdot(k\vec{A})=k(\nabla\cdot\vec{A})$$

$$\nabla\times(k\vec{A})=k(\nabla\times\vec{A})$$

fg : product of two scalar functions

$\vec{A}\cdot\vec{B}$: dot product of two vector functions

$f\vec{A}$: scalar times vector

$\vec{A}\times\vec{B}$: vector product of two vector functions

1.2 Differential Calculus

- **Product rules:**

- For gradients:

$$\nabla(fg) = f\nabla g + g\nabla f \qquad \textbf{(I)}$$

$$\nabla(\vec{A}\cdot\vec{B}) = \vec{A}\times(\nabla\times\vec{B}) + \vec{B}\times(\nabla\times\vec{A}) + (\vec{A}\cdot\nabla)\vec{B} + (\vec{B}\cdot\nabla)\vec{A} \qquad \textbf{(II)}$$

- For divergences :

$$\nabla\cdot(f\vec{A}) = f(\nabla\cdot\vec{A}) + \vec{A}\cdot(\nabla f) \qquad \textbf{(III)}$$

$$\nabla\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\nabla\times\vec{A}) - \vec{A}\cdot(\nabla\times\vec{B}) \qquad \textbf{(IV)}$$

- For curls :

$$\nabla\times(f\vec{A}) = f(\nabla\times\vec{A}) - \vec{A}\times(\nabla f) \qquad \textbf{(V)}$$

$$\nabla\times(\vec{A}\times\vec{B}) = (\vec{B}\cdot\nabla)\vec{A} - (\vec{A}\cdot\nabla)\vec{B} + \vec{A}(\nabla\cdot\vec{B}) - \vec{B}(\nabla\cdot\vec{A}) \qquad \textbf{(VI)}$$

(Proof example)

$$\nabla\cdot(f\vec{A}) = \frac{\partial}{\partial x}(fA_x) + \frac{\partial}{\partial y}(fA_y) + \frac{\partial}{\partial z}(fA_z)$$

$$= \left(\frac{\partial f}{\partial x}A_x + f\frac{\partial A_x}{\partial x}\right) + \left(\frac{\partial f}{\partial y}A_y + f\frac{\partial A_y}{\partial y}\right) + \left(\frac{\partial f}{\partial z}A_z + f\frac{\partial A_z}{\partial z}\right) = \vec{A}\cdot(\nabla f) + f(\nabla\cdot\vec{A})$$

Additional rules:

$$\nabla\left(\frac{f}{g}\right) = \frac{g\nabla f - f\nabla g}{g^2}$$

$$\nabla\cdot\left(\frac{\vec{A}}{g}\right) = \frac{g(\nabla\cdot\vec{A}) - \vec{A}\cdot(\nabla g)}{g^2}$$

$$\nabla\times\left(\frac{\vec{A}}{g}\right) = \frac{g(\nabla\times\vec{A}) - \vec{A}\times(\nabla g)}{g^2}$$

1.2 Differential Calculus

1.2.7 Second Derivatives (2차 미분)

(1) Divergence of gradient: $\nabla\cdot(\nabla T)=\nabla^2 T$: the **Laplacian** of T

(2) Curl of gradient: $\nabla\times(\nabla T)=0$ ⟵ $\nabla=\hat{x}\frac{\partial}{\partial x}+\hat{y}\frac{\partial}{\partial y}+\hat{z}\frac{\partial}{\partial z}$

(3) Gradient of divergence: $\nabla(\nabla\cdot\vec{v})$: uncommon $\nabla^2\vec{v}=(\nabla\cdot\nabla)\vec{v}\neq\nabla(\nabla\cdot\vec{v})$

(4) Divergence of curl: $\nabla\cdot(\nabla\times\vec{v})=0$ ⟵ $\vec{A}\cdot(\vec{B}\times\vec{C})=(\vec{A}\times\vec{B})\cdot\vec{C}$

(5) Curl of curl: $\nabla\times(\nabla\times\vec{v})=\nabla(\nabla\cdot\vec{v})-\nabla^2\vec{v}$

(1) → $$\nabla\cdot(\nabla T)=\left(\hat{x}\frac{\partial}{\partial x}+\hat{y}\frac{\partial}{\partial y}+\hat{z}\frac{\partial}{\partial z}\right)\cdot\left(\frac{\partial T}{\partial x}\hat{x}+\frac{\partial T}{\partial y}\hat{y}+\frac{\partial T}{\partial z}\hat{z}\right)=\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=\nabla^2 T$$: scalar

$$\nabla^2\vec{v}\equiv(\nabla^2 v_x)\hat{x}+(\nabla^2 v_y)\hat{y}+(\nabla^2 v_z)\hat{z}$$: vector

(2) → $$\nabla\times(\nabla T)=0 \longleftarrow \because \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(\frac{\partial T}{\partial x}\right)$$

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