[Foreign Exchange Rates and Interest Rates]

1. Interest Rate Parity Theorem

The relationship among spot rates, forward rates and domestic and foreign interest rates can be most generally represented by the important Interest Rate Parity theorem or formula.

Example for Yen/US$

S_t is the spot exchange rate in terms of Yen/HK$.

$F_{t,T}$ is the forward exchange rate in terms of Yen/HK$, known at the same time t and the contract maturity at time t+T, T = 30, 60, 90, 180 and 360 days.

i_t is the domestic, (HK), annual rate of interest.

i_t^* is the foreign, (Japan), annual rate of interest.

Step 1:
Borrow HK$1 domestically in HK and repay the amount

HK$ $[1 + i_t(T/360)]$ in period t+T

Step 2:
Use the HK$1 to buy S amount of foreign currency, i.e. Yen. Now have ¥ (S_t).

Step 3:
Then invest the yen it at the foreign (Japan) rate of interest; so that at time t+T, we will have,

¥ $(S_t)[1 + i_t^*(T/360)]$.

Step 4:
Can now buy a forward contract where the exchange rate in one years time will be F, also in terms of Yen/HK$. In one year we are guaranteed to have ¥ $(S_t)[1 + i_t^*(T/360)]$, which we now want to convert back to HK$. The forward exchange rate in terms of HK$/yen will be $1/F_{t,T}$. Hence the strategy of converting our Yen to HK$, investing in the US money market and moving the money back to US$ will be worth:

HK$ $(S_t)[1 + i_t^*(T/360)](1/F_{t,T})$.

The arbitrage condition then implies that both investment strategies must be equivalent, hence

$[1 + i_t(T/360)] = (S_t)[1 + i_t^*(T/360)](1/F_{t,T})$.

On rearranging,

$$F_{t,T}/S_t = [1 + i_t^*(T/360)] / [1 + i_t(T/360)] ,$$

which is known as the Covered Interest Rate Parity Condition. This is a very important theorem since it tells us the relationship between spot rates, forward rates and interest rates.

The implied forward rate is: $F_{t,Tt} = S_t[1 + i_t^*(T/360)] / [1 + i_t(T/360)]$.

The implied forward rate can be calculated from knowledge of the current spot rate and the domestic and foreign interest rates. The difference between the implied forward rate and the actual market determined forward rate is invariably very small, i.e. less than 0.1%.

The rule for the Covered Interest Rate Parity condition is to note that **the country with the higher rate of interest is going to have a forward rate indicating depreciation of the currency.**

But, when the exchange rates are expresses as HK$/Yen, the Covered Interest Rate Parity equation in becomes,

$$F_{t,T}/S_t = [1 + i_t(T/360)]/[1 + i_t^*(T/360)].$$

And, on subtracting 1 from both sides of equation

$$[F_{t,T}/S_t - 1] = [1 + i_t(T/360)]/[1 + i^*_t(T/360)] - 1,$$

then,

$$(F_{t,T} - S_t)/S_t = \{[1 + i_t(T/360)] - [1 + i^*_t(T/360)]\}/[1 + i^*_t(T/360)],$$

and the **Forward Premium** is then expressed as,

$$(F_{t,T} - S_t)/S_t = [i_t(T/360) - i^*_t(T/360)]/[1 + i^*_t(T/360)],$$

Approximately, the quantity of the right hand side is,

$$[i_t(T/360) - i^*_t(T/360)]$$

is known as the **Interest Rate Differential** and plays a key role in understanding investment opportunities and the likely impact of future monetary policy.

Thus, the Covered Interest Rate Parity condition also implies that **the forward premium is equal to the interest rate differential.**

Empirically the interest rate parity does not seem to be hold in the real world. There exits a small deviation from the interest rate parity; the forward premium is not exactly equal to the interest rate differential. This is the reason why the implied forward rate is not the same as the market determined forward rate. But, the deviations do not provide any arbitrage profit opportunity for the

investors since the deviations are very small.

There are several reasons why the interest rate parity may not hold exactly. The main reason for the deviations from the interest rate parity is the transaction costs (the spreads between bid and offer prices) in the foreign exchange market. Empirically, the deviations are almost equal to these transaction cost. In addition to the transaction costs, there are some other reasons such as different taxes, political risks and government's controls on the financial capital.

Arbitrage and Interest Rate Parity Theorem

If the parity condition is violated, the arbitrage activity restores the condition since the arbitrage changes the forces of supply and demand in the money markets of domestic and foreign country and spot and forward foreign exchange markets.

Example 1:

Assume that we are given the following information:

S (spot exchange rate, AU$/US$) = 1.25
AU 3-month interest rate = 12% p.a.
US 3-month interest rate = 8% p.a.

Based on the interest rate parity theorem, what do you expect the 3-month forward rate to be?
(0.03-0.02) = (F-1.25)/1.25 → 0.01 = (F-1.25)/1.25. Thus, F = [(0.01)*(1.25)]+1.25 = 1.26.

If the market forward rate is 1.26, the interest rate differential is equal to the forward premium and there is no arbitrage opportunity.

Example 2:

Using the same information, suppose the market forward premium is 1.20

→ compare the interest rate differential and forward premium:
interest rate differential: 0.03-0.02 = 0.01%
forward premium(discount): [(1.20-1.25)/1.25] = -0.04%
=> AU$ is at forward premium about 0.04% and US$ is at forward discount about 0.04%

Since the interest rate parity is not satisfied and the forward premium for AU$ is greater than the interest rate differential, there exists an arbitrage opportunity for AU$.

Arbitrage:
→ step 1: Borrow US$1million from US bank at 2% for 3 month. Then, after 3 month, repay US$1.02 m.
step 2: Convert US$ to AU$ at S = 1.25, then AUD1.25m
step 3: Invest AUD1.25m at AU bank at 3% for 3 months. Then, receive AU$1.29m after 3 months.
step 4: Sell AU$ through forward contract to buy US$ at forward rate F = 1.20. Then, after 3 months, US$1.07m will be delivered by paying AU$1.29m.

Thus, from the forward arbitrage, the net profit is US$1.07m – US$1.02m = US$0.05m.

This profit is riskless because all interest rates and exchange rates are known in advance.

Effects of arbitrage

In money markets:

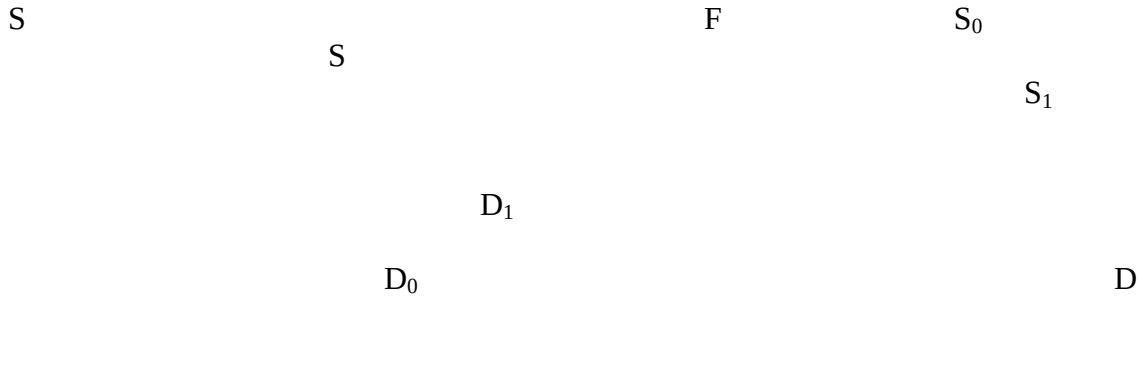
the arbitrage activity for AU$ increases the demand for AU assets (AU$) and decreases the demand for US assets (US$). Thus, the US interest rate rises but AU interest rate falls.

*In money market, the demand of money is upward sloping the against the interest rate and the supply is downward sloping since as the price level falls, the interest rate and the quantity of money demanded decreases but the quantity of money supply increases.

In foreign exchange markets,

in spot market, the demand for spot AU$ increases and causes to a rise in sot exchange rate.

in forward market, the supply for forward AU$ increases so that the forward rate decreases.



Thus, the arbitrage activity reduces the forward premium and increases the interest rate differential. So, the interest rate parity restores again eventually.

2. Exchange rates, interest rates and inflation

The Fisher effect is the expected effect of inflation on the nominal interest rate. Then, the **Fisher equation** which shows the relationship between inflation and interest rate can be,

$$i = r + \Pi,$$

where i is the nominal interest rate, r is the real rate and ï is the expected rate of inflation.

The Fisher equation implies that the increase in Π will tend to increase i. Thus, the nominal interest rate in the countries with higher inflation tend to be higher.

The linkage of exchange rates, interest rates and inflation can be presented by **international Fisher equation.** For two countries, home (HK) and foreign (US) country, the Fisher equation can be, $i_{hk} - i_{us} = \Pi_{hk} - \Pi_{us}$ assuming the real interest rate (r) is the same. By combing this Fisher equation with the covered interest rate parity, the international Fisher equation is,

$$\Pi_{hk} - \Pi_{us} = i_{hk} - i_{us} = (F - S)/S, \text{ where F and S are HK\$ in terms of US\$ (HK\$/US\$)}$$

This international Fisher equation implies that the interrelationships among exchange rates, interest rates and inflation are determined at the same time since they are affected together by new events and information and adjusted to new equilibrium.

3. Expected exchange rates and the term structure of interest rates

The term structure of interest rate refers to the structure of interest rate existing on investment opportunities over different terms to maturity. So, if the interest rate decreases as the terms to maturity becomes longer such as 1-month, 3-momth, …12-month, the term structure would be falling.

There are three theories to explain the term structure:

- Expectations theory: the bond with differing maturity are perfectly substitute, causing their interest rates to be different only because the investors expected that short-term interest rates will rise or fall.
- Liquidity premium theory: unlikely to the short term bonds, the long term bonds contain the risk premium and the risk would result in the interest rate increase with the holding period of the bond.
- Preferred habitat theory: bonds bearing interest rates with different maturity terms are nonsubstituable so that the interest rates are different since they are determined in the segmented and separate markets.

In international finance, the term structures for different currencies are being used in order to expect the exchange rate changes.

As presented in the Covered interest rate parity, when one country has higher interest rate than another, the high interest rate currency is expected to depreciate relative to the low

interest rate currency. Thus, at each point in the term structure, the difference between the interest rates should reflect the expected changes in the exchange rates for two currencies since the forward rate can be considered as the expected future spot rate **(Uncovered interest rate parity)**

This term structure is very useful especially when the forward exchange market is not available because the interest rate differential from the term structures between two currencies provides the information about the expected future exchange rates.

If the differential between two interest rates is constant, it implies the future changes in the exchange rate are expected to be at some constant rate.

If the differential is smaller (the two lines of the term structures converges), the high interest rate currency is expected to depreciate and the low interest rate currency to appreciate.

If the two line are diverging, the high interest rate currency is expected to appreciate and the low interest rate currency to depreciate.