경제수학 제 1 장

함수 (Functions)

경제수학에서 쓰이는 수학적 개념

- Apply mathematics to economic variables
- Understand the relationship between total and average revenue
- Obtain and plot various cost functions
- Write an expression for profit
- Use Excel to plot functions and perform calculations

그래프 그리기

- 좌표(coordinates) : a pair of numbers (*x,y*) that represent the position of a point
  - ➤ x 좌표는 원점에서 수평축으로 떨어진 거리 (+ 이면 오른쪽, - 이면 왼쪽으로)
  - ➤ y 좌표는 원점에서 수직축으로 떨어진 거리 (+ 이면 위쪽, - 이면 아래쪽으로)
- 사분면(quadrant): x 축과 y 축을 기준으로 나뉘는 4개의 영역. 우측 상단(x>0, y>0) 으로부터 시계반대 방향으로 1사분면, 2사분면, 3사분면, 4사분면으로 나눔.

변수와 상수 그리고 함수

- 변수(Variable): a quantity represented by a symbol that can take different possible values

- 상수(Constant): a quantity whose value is fixed, even if we do not know its numerical amount

- 함수(Function): a systematic relationship between pairs of values of the variables, written

$$y = f(x)$$

알아둘 것들

- If y is a function of x, $y = f(x)$
- A function is a rule telling us how to obtain y values from x values
- x: independent (explanatory) variable (독립/설명변수) → 그래프의 가로축
- y: dependent (explained) variable (종속/피설명변수) → 그래프의 세로축

비례적 관계(Proportional Relationship)

- Each *y* value is the same amount times the corresponding *x* value

- All points lie on a *straight line* through the origin

- Example:
  $y = 6x$
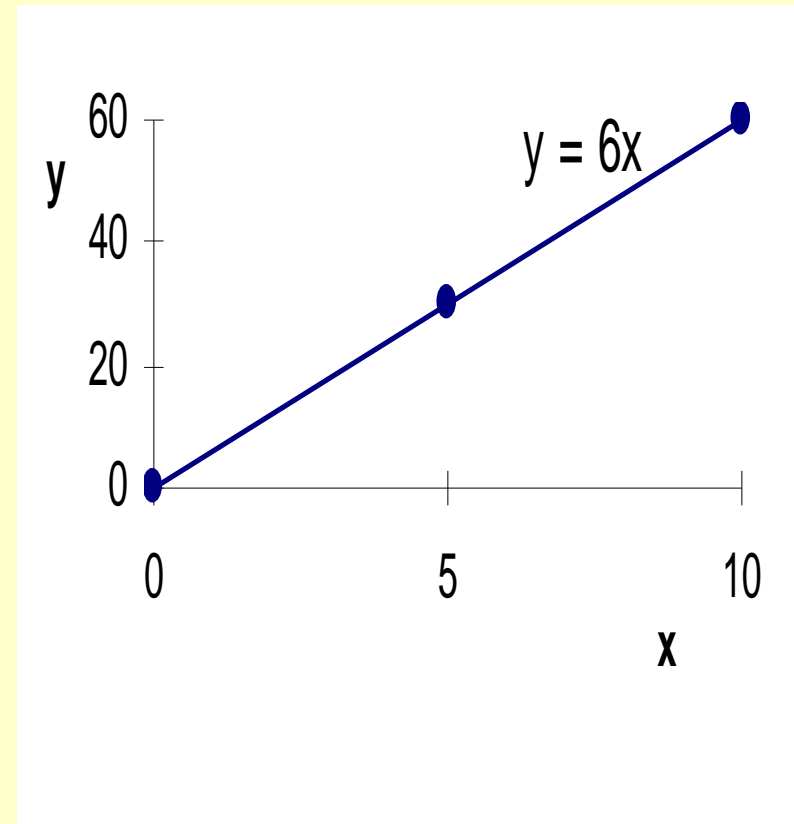
선형관계(Linear Relationship)

- Linear function: a relationship in which all the pairs of values form points on a straight line
- Shift: a vertical movement upwards or downwards of a line or curve
- y-절편(y-intercept): the value at which a function cuts the *y* axis
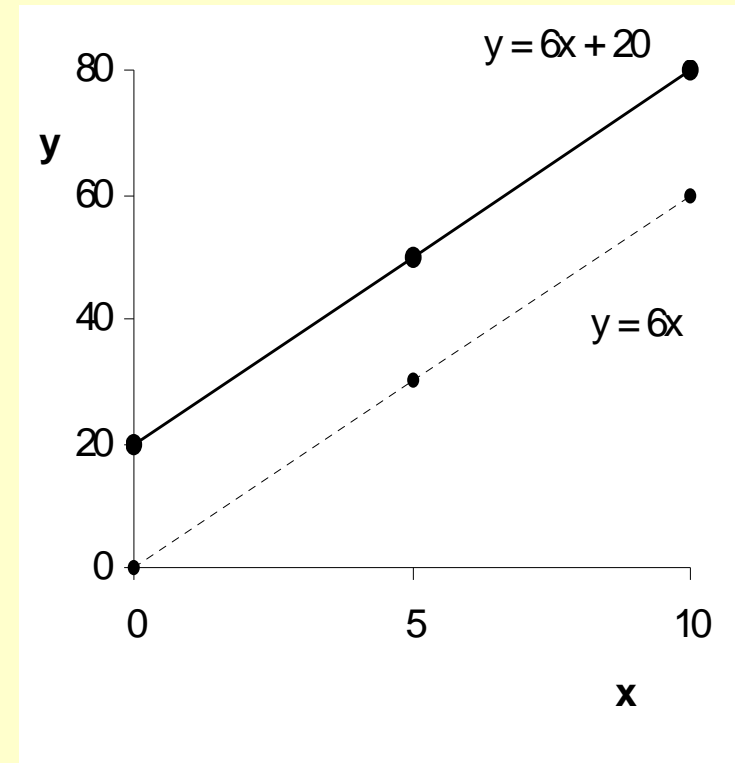
선형함수의 일반적인 형태

- A function with just a term in x and (perhaps) a constant is a linear function
- It has the general form

$$y = a + bx$$

- b is the slope of the line
- a is the y-intercept

1) Finding the Slope

2) Point-slope formula of a straight line

If a straight line passes through the point (x_1, y_1) with a slope value of a,

$$y - y_1 = a(x - x_1)$$

Ex) A line passes through a point (-2, 3) with a slope of (- 4). Then Draw a diagram.

3) Point-point formula of a straight line

A straight line passes through (x_1, y_1) and (x_2, y_2), $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

Ex) Find the equation of the line passing through (- 1, 3) and (5, - 2).

4) The General Equation for a Straight Line: $Ax + By + C = 0$, $y = -\dfrac{A}{B}x - \dfrac{C}{B}$

5) Graphic Solutions of a Straight Line

Ex) (a) $\begin{aligned} x + y &= 5 \\ x - y &= -1 \end{aligned}$ (b) $\begin{aligned} 3x + y &= -7 \\ x - 4y &= 2 \end{aligned}$ (c) $\begin{aligned} 3x + 4y &= 2 \\ 6x + 8y &= 24 \end{aligned}$

6) Linear Inequalities

Sketch in the xy-plane the set of all pairs of numbers (x, y) that satisfy the inequality $2x + y \leq 4$

It can be written as a set of $\{(x, y) : 2x + y \leq 4\}$.

<Practice 1>

1. Draw graphs for the following straight lines:

  (a) $3x + 4y = 12$ (b) $x = 3$ (c) $y = -3$

<Application 1>

Consumption Function: $C = f(Y)$, $C = a + bY$ (by John Maynard Keynes)

T. Haavelmo found the following consumption function for the period of 1929-1941. $C = 95.05 + 0.712Y$. How can we interpret 0.712?

Power Functions

- Power: an index indicating the number of times that the item to which it is applied is multiplied by itself
- Quadratic function: a function in which the highest power of x is 2
  - ➤ The only other terms may be a term in x and a constant
- Cubic function: a function in which the highest power of x is 3
  - ➤ The only other terms may be in x^2, x and a constant

$$f(x) = ax^2 + bx + c \, (a \neq 0)$$

The shape of this *parabola* roughly resembles $\cap$ when (   ) and $\cup$ when (   ). The graphs are symmetric about the axis of symmetry. Here are the questions:

(a) For which value of x (if any) is $ax^2 + bx + c = 0$?

(b) What are the coordinates of the maximum/minimum point P, also called the vertex of the parabola?

By completing the square, we can solve both of the problems above:

$$f(x) = ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a}$$

$$ax^2 + bx + c = 0 \Leftrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Ex) Complete the square for the following functions, and then find the maximum/minimum point of each:

(a) $f(x) = -2x^2 + 40x - 600$

(b) $f(x) = \frac{1}{3}x^2 + \frac{2}{3}x - \frac{8}{3}$

Ex) Market price and total cost in producing units are $P = 102 - 2Q$ and $TC = 2Q + 0.5Q^2$, respectively. Find the value of Q which maximizes profits, and corresponding maximal profit.

인수분해 (Factorizing)

- Look for a common factor, or for expressions that multiply together to give the original expression
- Example:

  $45x^2 - 60x = 15x\,(3x - 4)$
- Factorizing a quadratic expression may involve some intelligent guesswork
- Example:

  $45x^2 - 53x - 14 = (9x + 2)\,(5x - 7)$

소거 (Cancelling)

- Cancelling is dividing both numerator and denominator by the same amount

- Examples:

$$\frac{70}{105} = \frac{7 \times 2 \times 5}{5 \times 7 \times 3} = \frac{2}{3}$$

$$\frac{8x^3z^2}{28x^2z^5} = \frac{4 \times 2(x.x^2z^2)}{4 \times 7(x^2z^2.z^3)} = \frac{2x}{7z^3}$$

지수에 관하여 (Working with Indices)

- To multiply, add the indices
- To divide, subtract the indices
- $x^{-n} = 1/\ x^n$
- $x^0 = 1$
- $x^{1/2} = \sqrt{\ x}$
- $(ax)^n = a^n\ x^n$
- $(x^m)^n = x^{mn}$

다변수 함수

- Multivariate function: the dependent variable, y, is a function of more than one independent variable

- If $y = f(x, z)$
  y is a function of the two variables x and z

- We substitute values for x and z to find the value of the function

- If we hold one variable constant and investigate the effect on y of changing the other, this is a form of comparative statics analysis

엑셀 계산기호 (Arithmetic Operators)

( )   brackets

+   add

–   subtract

*   multiply

/   divide

^   raise to the power of