경제수학 제 4 장

변화, 변화율에 대하여

비율과 백분율 (Rates and Percentages)

- Rates are often quoted as percentages, but they are used as decimals in calculations
- The term percentage means divided by 100, so $6\% = 6/100 = 0.06$
- Rates are applied by multiplication
- If you put \$1000 in a deposit account at 6% rate of interest, the interest that is added at the end of the time period is $0.06 \times 1000 = \$60$

시간변수에 따른 변화

- V_t = value at time t
- the subscript t indicates the time period
- At the start of the process t is 0, after one time period it is 1, when two time periods have elapsed t is 2, and so on
- If the initial amount V_0 earns interest at rate r, where r is a decimal, after 1 time period it becomes

$$V_1 = V_0(1 + \mathrm{r})$$

퍼센트 변화 (Percentage Changes)

- Percentage change $= \dfrac{(V_1 - V_0)}{V_0} \times 100$
- When a price index rises from 140 to 160

percentage change $= \dfrac{(160 - 140)}{140} \times 100$

$= 14.29\ \%$

Percentage Points Change

- When we compare two percentages, the difference between them is measured in percentage points (%p)
- When a price index rises from 140 to 160, percentage points change = 160 – 140

  = 20 %p

호탄력성 (Arc Elasticity)

$$\text{Arc Elasticity} = \frac{\dfrac{(Q_1 - Q_0)}{[(Q_1 + Q_0)/2]}}{\dfrac{(P_1 - P_0)}{[(P_1 + P_0)/2]}}$$

단리와 복리 (Simple and Compound Interest)

- If r is a decimal, V_0 is the principal and V_n is the future value
- With simple interest at rate r

$$V_n = V_0(1 + nr)$$

- With interest compounded for each time period at rate r

$$V_n = V_0(1 + \mathrm{r})^n$$

Compounding more frequently

- Compounding can take place every quarter, every month or every week
- Use the same formula, choosing the frequency with which compounding occurs as unit time and ensuring *n* and r refer to this time period
- With continuous compounding the future value is given by

$$V_n = V_0 \mathrm{e}^{nr}$$

Annual Equivalent Rate

- Annual Equivalent Rate =

$$100 \times \frac{\text{total interest payable in a year}}{V_0}\ \%$$

- When compounding takes place m times a year

$$\text{AER} = 100 \times [(1 + \text{r})^{m} - 1]$$

- With continuous compounding

$$\text{AER} = 100 \times (\text{e}^{r} - 1)$$

Nominal and Real Interest Rates

- The nominal percentage rate of interest is the number of $ earned if $100 is lent for one year
- The real rate is the extra goods or services that can be bought if $100 is lent for one year

Calculating Real Interest Rates

- Nominal rate of interest = i
- Rate of inflation = p
- Real rate of interest = r
- Approximately (valid for rates < 10%)

$$r = i - p$$

- The exact formula is $r = \dfrac{1 + i}{1 + p} - 1$

Growth Rates and Logarithms

- For a variable growing at a steady rate the graph of values against time forms a curve
- The graph of logarithms of the values of the variables against time is a straight line
- Using logarithms
  - ➢ steady growth is a straight line
  - ➢ a growth rate which is increasing over the time gives a curve bending upwards
  - ➢ growth at a decreasing rate gives a curve that bends downwards

Depreciation

- Straight line depreciation

$$V_t = V_0(1 - tr)$$

- Reducing balance depreciation

$$V_t = V_0(1 - \mathrm{r})^t$$

순 현재가치 (Net Present Value)

- Present value (현재가치): the value of some future amount in the current time period, obtained by discounting
- Discount rate (할인률): used in the discount factor formula, it is a rate that represents the cost of capital
- Discount factor (할인요소): the amount by which a future value is multiplied to obtain its present value

Finding Discount Factors

- Choose an appropriate discount rate, r, writing it as a decimal
- Find the discount factors

$$\frac{1}{(1+r)^t}$$

Time period, t	Discount Factor
0	1
1	$\frac{1}{(1+r)}$
2	$\frac{1}{(1+r)^2}$

순현재가치 구하기 (NPV)

- For each year, list the net return

$$V_t = (\text{revenue} - \text{cost})$$

- For each year, multiply V_t by the discount factor to find the present value

$$V_0 = V_t \times \frac{1}{(1+r)^t}$$

- Sum the present values to obtain the NPV of the project

내부수익률 (Internal Rate of Return)

- If the discount rate increases, the NPV of the project falls
- Internal rate of return: the discount rate at which the net present value of a project is 0
- Decision rule: undertake the project if the IRR is greater than the discount rate

순열과 급수 (Series)

- Geometric progression: a sequence of terms each of which is formed by multiplying the previous term by the same amount
- Common ratio: the amount by which each term in a geometric progression is multiplied to form the next term in the sequence
- Series: a sum of a sequence of terms

Sum of a Geometric Progression

- The sum of a GP to *n* terms is given by the formula

$$S_n = \frac{a\left(1 - c^n\right)}{\left(1 - c\right)}$$

- The formula for the sum of a large number of terms, n, of a GP with c < 1 is given by

$$S_n = \frac{a}{(1-c)}$$

Savings with Regular Payments

- Regular payments: adding amount W over each of n time periods to the initial amount V_0

$$V_n = V_0(1+r)^n + \frac{W\left[(1+r)^n - 1\right]}{r}$$

Savings with Regular Payments

- Sinking fund: saving amount W each period until time period n when the money is withdrawn

$$V_n = \frac{W(1+r)\left[(1+r)^n - 1\right]}{r}$$

Annuities

- Annuity value

$$V_0 = \frac{A\left[1-(1+r)^{-n}\right]}{r}$$

- Annuity factor: the amount by which the annuity payment A is multiplied,

$$\frac{\left[1-(1+r)^{-n}\right]}{r}$$

Perpetuities

- Perpetuity: an annuity with no time limit on the length of time for which it is paid
- If n is very large $(1 + r)^{-n}$ » 0 so the annuity value formula simplifies to give
- Perpetuity value

$$V_0 = \frac{A}{r}$$

Mortgage Repayment 1

- If you borrow $M = -V_0$
- With interest payments calculated annually on the money owing at the start of the year
- Annual Mortgage Repayment

$$W = \frac{rM}{\left[1-(1+r)^{-n}\right]}$$

Mortgage Repayment 2

- Capital recovery factor: multiplies the amount you borrow M to show the size of the repayments required

- Capital recovery factor $= \dfrac{r}{\left[1-(1+r)^{-n}\right]}$

Prices of Bonds and the Rate of Interest 1

- For a perpetual bond
- Coupon payment =
  coupon value of bond × coupon rate
- Bond price = $$\frac{\text{coupon payment}}{r}$$
- Bond price falls as r rises

Prices of Bonds and the Rate of Interest 2

- For a fixed term bond
- Bond price = NPV of returns to bond holder
- Bond price falls as r rises