경제수학 제 9 장

제약하의 최적화

제약하의 최적화 (Constrained Optimization)

- Constrained optimization: choosing the values of the variables that maximize or minimize the objective function from the values that are permitted by the constraint
- We work simultaneously with the objective function and the constraint
- We see two different ways of identifying the constrained optimum position

Check for a Maximum or a Minimum

- Once we have identified where an optimum occurs, we should check to see whether the point is a maximum or a minimum
- The second order conditions are complicated
- We assume that the turning point we identify is either a maximum or a minimum as the economic theory requires
- Or we evaluate the function at and near the turning point and compare the values it takes

The Substitution Method

1. Rewrite the constraint and substitute into the objective function, thus eliminating one variable from it
2. Find partial derivatives
3. Set equal to zero and solve
4. Substitute for values of the variable eliminated earlier and the objective function

Lagrange Multiplier

- Lagrange multiplier: a variable that is introduced to allow the constraint to be combined with the objective function giving a Lagrangian expression
- Lagrangian expression: combines the objective function and the constraint
- The maximum or minimum of the Lagrangian expression is the maximum or minimum of the objective function subject to the constraint

The Lagrange Multiplier Method 1

1. Rewrite the constraint as an expression which equals zero and multiply by a Lagrange multiplier
2. Add this to the objective function forming a Lagrangian expression which incorporates the constraint

$$L = (\text{목적함수}) + \lambda\ (\text{제약식})$$

3. Differentiate partially with respect to each variable, including the Lagrange multiplier, and set all partial derivatives equal to zero

The Lagrange Multiplier Method 2

4. Solve the resulting equations
   - A useful method is to move terms containing the Lagrange multiplier to the right-hand side and then divide one equation by another
   - We find the optimal values of the variables subject to the constraint

Interpreting the Lagrange Multiplier

- The Lagrange multiplier which multiplies the constraint in the Lagrangian expression is in fact a multiplier in the economic sense
- If the constraint limit changes a little, multiplying by λ gives the approximate change that ensues in the value of the objective function
- We can solve for λ as we solve for the other variables