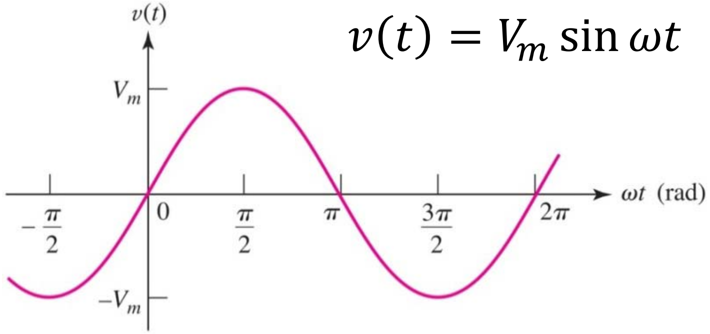
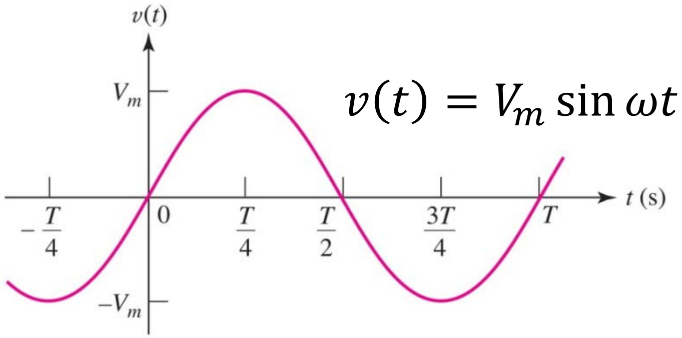
Chapter 10
Sinusoidal Steady-State Analysis

Sinusoids



$$v(t) = V_m \sin \omega t$$

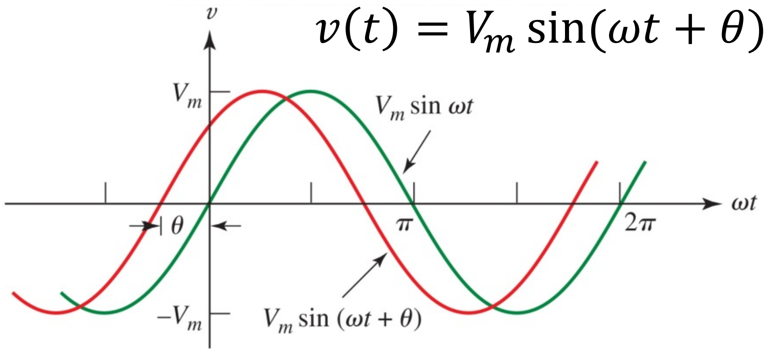
- the *amplitude* of the wave is V_m
- the *argument* is ωt
- the *radian or angular frequency* is ω
- note that sin() is *periodic*



$$v(t) = V_m \sin \omega t$$

- the period of the wave is T
- the frequency f is 1/T: units Hertz (Hz)

$$f = \frac{1}{T}, \qquad \omega = \frac{2\pi}{T} = 2\pi f$$



$$v(t) = V_m \sin(\omega t + \theta)$$

- The new wave (in red) is said to *lead* the original (in green) by θ.
- The original *sin(ωt)* is said to *lag* the new wave by θ.
- θ can be in degrees or radians, but the argument of sin() is always *radians*.

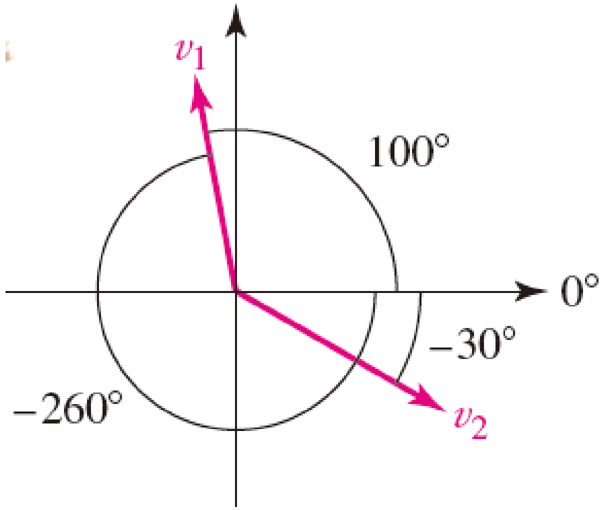
- Converting Sines to Cosines

$$-\sin\omega t = \sin(\omega t \pm 180^\circ)$$

$$-\cos\omega t = \cos(\omega t \pm 180^\circ)$$

$$\mp\sin\omega t = \cos(\omega t \pm 90^\circ)$$

$$\pm\cos\omega t = \sin(\omega t \pm 90^\circ)$$



$$\begin{aligned} v_1 &= V_{m_1}\cos(5t + 10^\circ) \\ &= V_{m_1}\sin(5t + 10^\circ + 90^\circ) \\ &= V_{m_1}\sin(5t + 100^\circ) \end{aligned} \longrightarrow v_1 = V_{m_1}\sin(5t - 260^\circ)$$

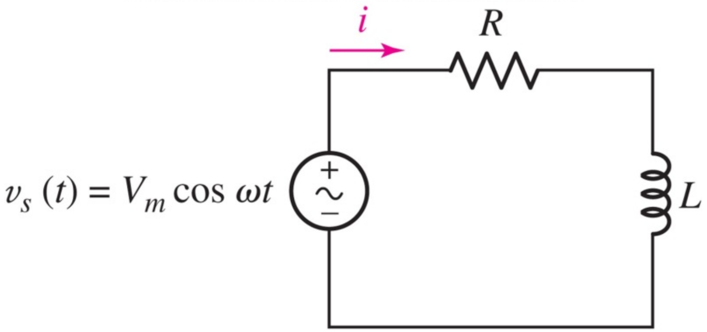
$$v_2 = V_{m_2}\sin(5t - 30^\circ)$$

v_1 ***leads*** v_2 by 130°

v_1 ***lags*** v_2 by 230°

The Steady-State Response : the condition that is reached after the transient or natural response has died out

When the source is sinusoidal, we often ignore the transient/natural response and consider only the forced or "steady-state" response.
The source is assumed to exist forever: $-\infty<t<\infty$



$$L\frac{di}{dt} + Ri = V_m \cos \omega t$$

Let $i(t) = I_1 \cos \omega t + I_2 \sin \omega t$

$$-LI_1\omega \sin \omega t + LI_2\omega \cos \omega t + R(I_1 \cos \omega t + I_2 \sin \omega t) = V_m \cos \omega t$$

$$\Rightarrow (-LI_1\omega + RI_2) \sin \omega t + (LI_2\omega + RI_1 - V_m) \cos \omega t = 0$$

$$\Rightarrow \quad -LI_1\omega + RI_2 = 0, \qquad LI_2\omega + RI_1 - V_m = 0 \qquad \Rightarrow \quad I_1 = \frac{RV_m}{R^2 + \omega^2 L^2}, \quad I_2 = \frac{\omega L V_m}{R^2 + \omega^2 L^2}$$

$$i(t) = \frac{RV_m}{R^2 + \omega^2 L^2} \cos \omega t + \frac{\omega L V_m}{R^2 + \omega^2 L^2} \sin \omega t$$

- More Compact and User-Friendly Form

Let $i(t) = A\cos(\omega t - \theta)$ instead of $i(t) = I_1 \cos\omega t + I_2 \sin\omega t$

$$i(t) = A\cos\omega t \cos\theta + A\sin\omega t \sin\theta = \frac{RV_m}{R^2 + \omega^2 L^2}\cos\omega t + \frac{\omega L V_m}{R^2 + \omega^2 L^2}\sin\omega t$$

$$A\cos\theta = \frac{RV_m}{R^2 + \omega^2 L^2}, \qquad A\sin\theta = \frac{\omega L V_m}{R^2 + \omega^2 L^2}$$

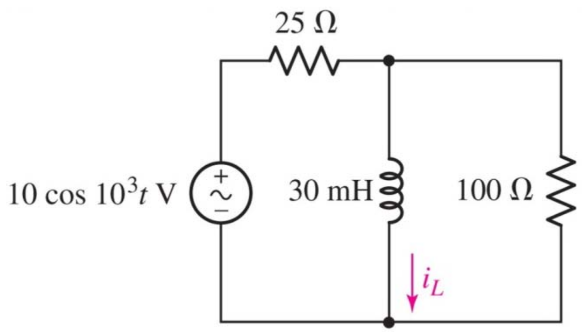
$$A^2(\cos^2\theta + \sin^2\theta) = A^2 = \frac{R^2 V_m^2 + \omega^2 L^2 V_m^2}{(R^2 + \omega^2 L^2)^2} = \frac{V_m^2 (R^2 + \omega^2 L^2)}{(R^2 + \omega^2 L^2)^2} = \frac{V_m^2}{R^2 + \omega^2 L^2}$$

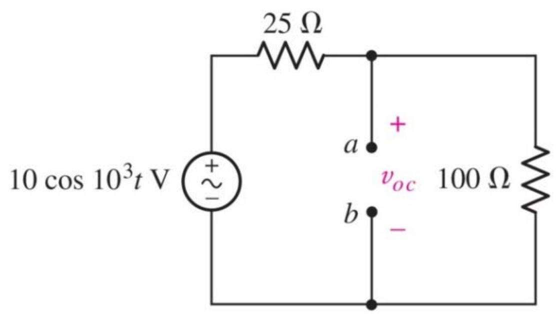
$$\Rightarrow A = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \qquad \frac{A\sin\theta}{A\cos\theta} = \tan\theta = \frac{\dfrac{\omega L V_m}{R^2 + \omega^2 L^2}}{\dfrac{RV_m}{R^2 + \omega^2 L^2}} = \frac{\omega L}{R} \qquad \Rightarrow \theta = \tan^{-1}\frac{\omega L}{R}$$

$$i(t) = A\cos(\omega t - \theta) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}\cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)$$

$i(t)$ ***lags*** $v(t)$ by $\tan^{-1}\dfrac{\omega L}{R}$

Example 10.1 Find the current i_L in the circuit.

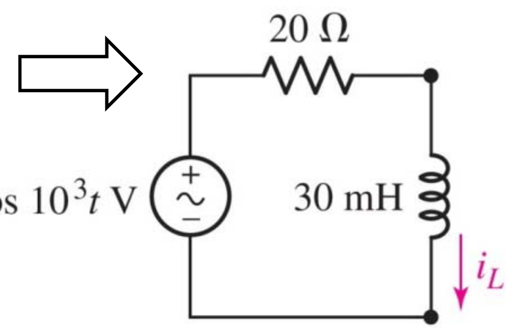


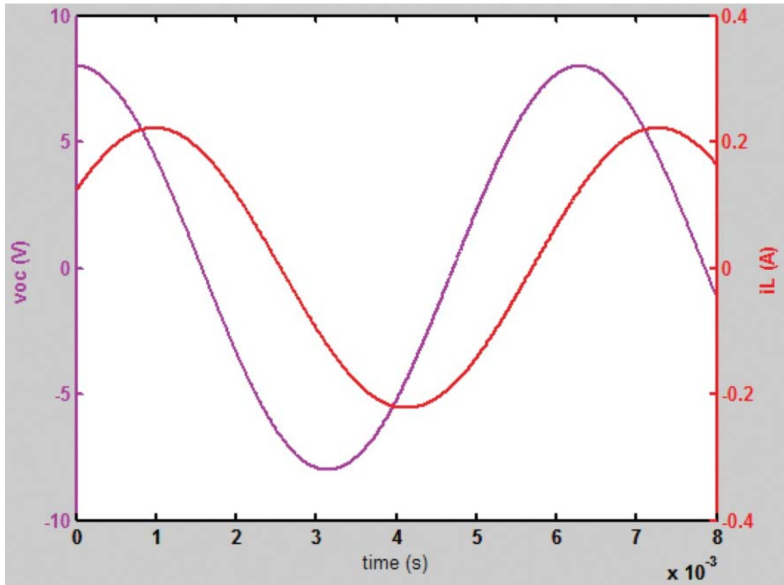


$$R_{th} = 25 \parallel 100 = 20$$

$$v_{oc} = \frac{100}{100+25}(10\cos 10^3 t)$$

$$= 8\cos 10^3 t$$





$$i_L = \frac{V_m}{\sqrt{R^2+\omega^2 L^2}}\cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)$$

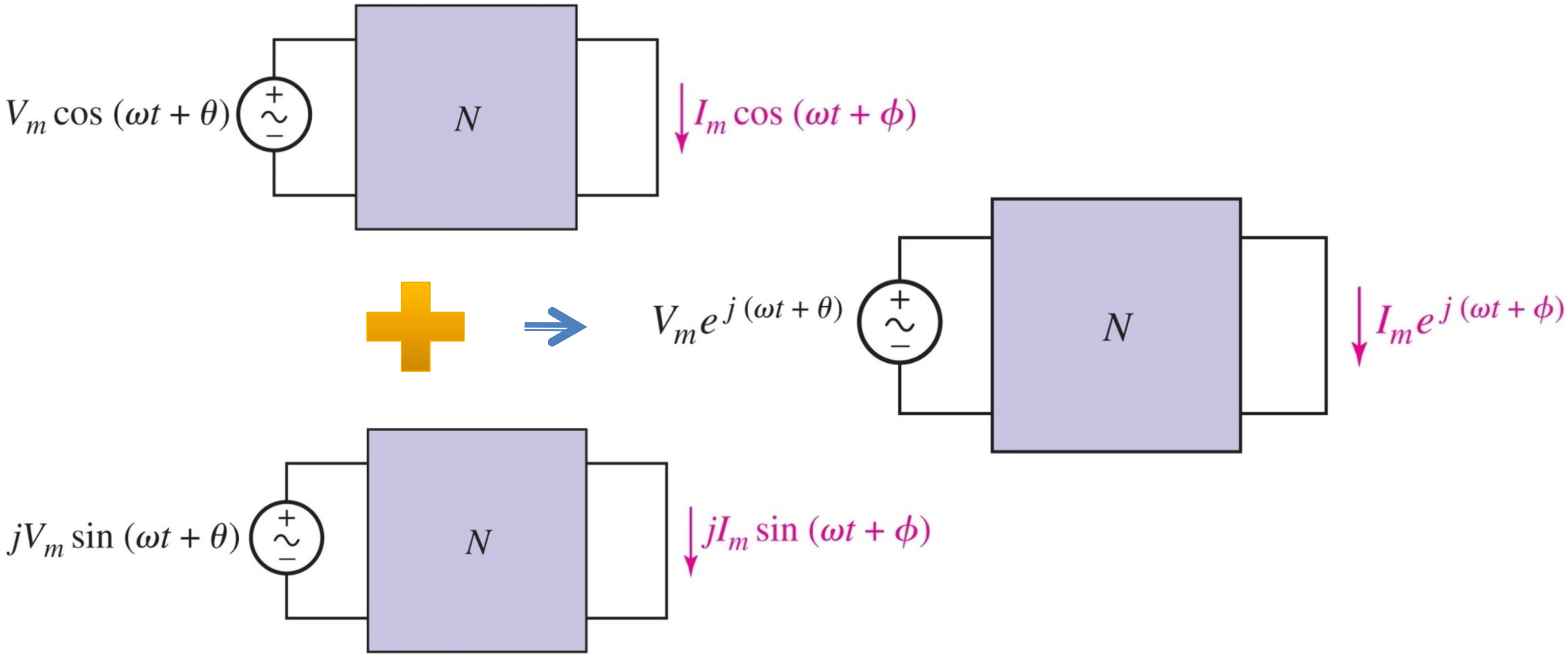
$$= \frac{8}{\sqrt{20^2+(10^3\times 30\times 10^{-3})^2}}\cos\left(10^3 t - \tan^{-1}\frac{30}{20}\right)$$

$$= 222\cos(10^3 t - 56.3^{\circ})\ mA$$

With purely resistive circuit, it is no more difficult to analyze with sinusoidal sources than with dc sources.

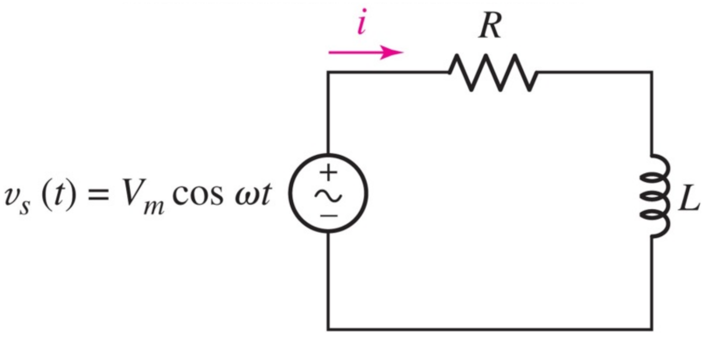
It turns out that if the transient response is of no interest to us, there is an alternative approach for obtaining the sinusoidal steady-state response of any linear circuit.

Euler's identity $e^{j\theta} = \cos\theta + j\sin\theta$



$$V_m \cos(\omega t + \theta) + jV_m \sin(\omega t + \theta) \Rightarrow I_m \cos(\omega t + \phi) + jI_m \sin(\omega t + \phi)$$

- An Algebraic Alternative to Differential Equations



$$L\frac{di}{dt} + Ri = v_s \qquad v_s = Re\{V_m e^{j\omega t}\}$$

Let $i(t) = I_m \cos(\omega t + \theta) = Re\{I_m e^{j(\omega t + \phi)}\}$

$$L\frac{d}{dt}\left(I_m e^{j(\omega t+\phi)}\right) + RI_m e^{j(\omega t+\phi)} = V_m e^{j\omega t}$$

$$\to j\omega L I_m e^{j(\omega t+\phi)} + RI_m e^{j(\omega t+\phi)} = V_m e^{j\omega t}$$

$$\to j\omega L I_m e^{j\phi} + RI_m e^{j\phi} = V_m$$

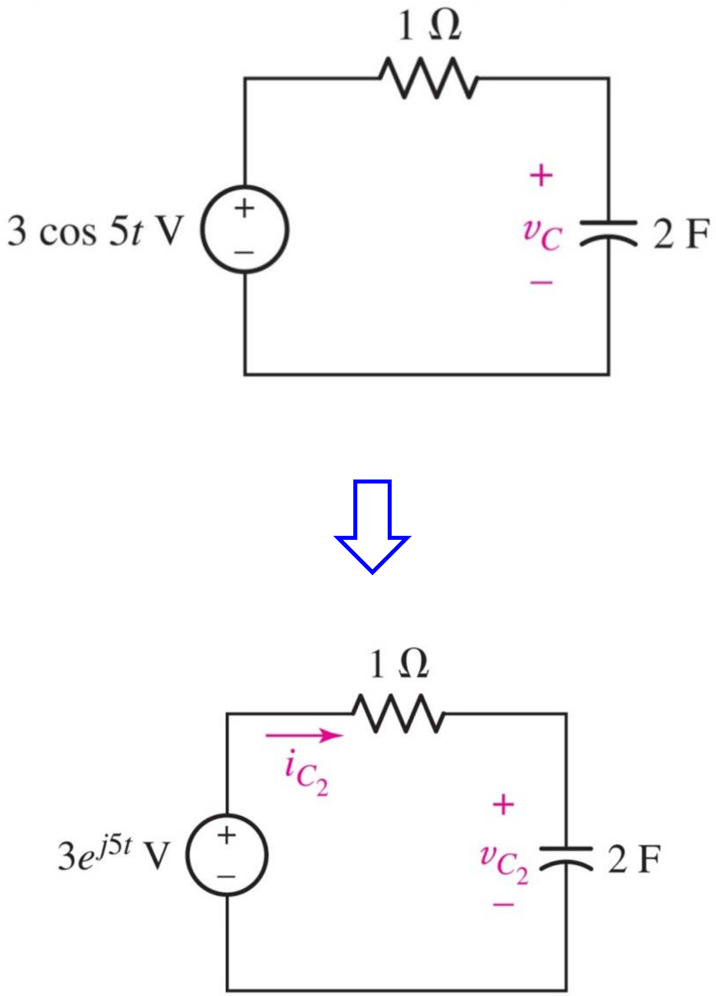
$$\to I_m e^{j\phi}(R + j\omega L) = V_m$$

$$\Rightarrow I_m e^{j\phi} = \frac{V_m}{R + j\omega L} = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} e^{j\left(-\tan^{-1}\frac{\omega L}{R}\right)}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \qquad \phi = -\tan^{-1}\frac{\omega L}{R}$$

$$i(t) = Re\{I_m e^{j(\omega t+\phi)}\} = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)$$

Example 10.2 Find the voltage on the capacitor.



let $v_{c2} = V_m e^{j5t}$

$$-3e^{j5t} + i_{c2} + v_{c2} = 0$$
$$\rightarrow -3e^{j5t} + 2\frac{dv_{c2}}{dt} + v_{c2} = 0$$

$$-3e^{j5t} + 2(j5)V_m e^{j5t} + V_m e^{j5t} = 0$$
$$\rightarrow V_m(1 + j10) = 3$$
$$\Rightarrow V_m = \frac{3}{1 + j10}$$

$$V_m = \frac{3}{\sqrt{1^2+10^2}}\angle\left(-\tan^{-1}\left(\frac{10}{1}\right)\right) \text{ V}$$

$$v_{C2} = \frac{3}{\sqrt{1^2+10^2}} e^{-j\tan^{-1}\left(\frac{10}{1}\right)} e^{j5t}$$

$$v_C = Re\{v_{C2}\} = \frac{3}{\sqrt{1^2+10^2}} \cos(5t - \tan^{-1}(10))$$
$$= 29.85\cos(5t - 84.3)$$

The term $e^{j\omega t}$ is common to all voltages and currents and can be ignored in all intermediate steps, leading to the phasor:

The phasor representation of a current (or voltage) is in the *frequency domain*

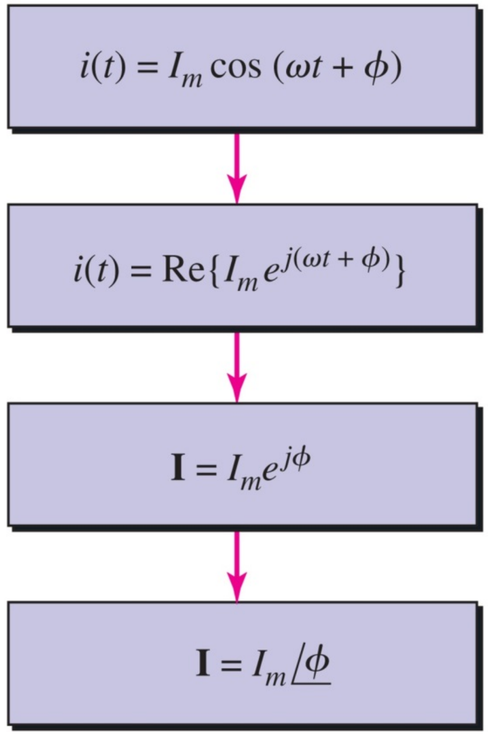
$$v(t) = V_m \cos \omega t = V_m \cos(\omega t + 0°) \rightarrow V_m \angle 0°$$

$$i(t) = I_m \cos(\omega t + \phi) \rightarrow I_m \angle \phi$$

complex value notation

$$\mathbf{I} = I_m e^{j\phi},$$

$$\mathbf{I} = I_m \angle \phi$$ Phasor

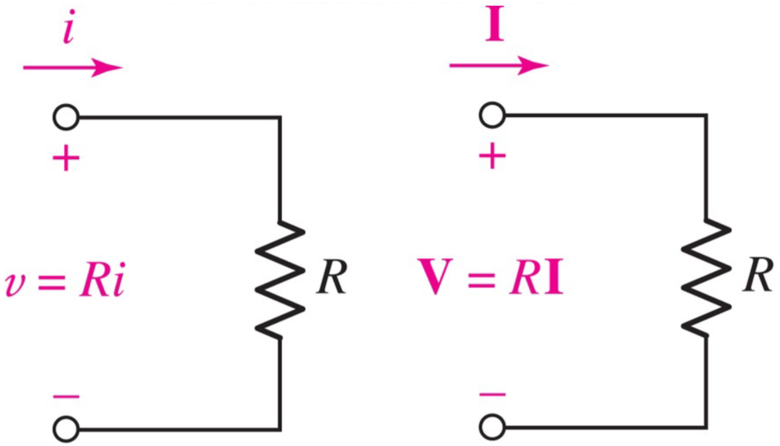


Example 10.3 Transform the time-domain voltage v(t)=100cos(400t-30°) into the frequency domain.

$$\mathbf{V} = 100\angle - 30°$$

The Resistor

In the frequency domain, Ohm's Law takes the same form:



$$v(t) = 8\cos(100t - 50°), \qquad R = 4\,\Omega$$

$$i(t) = \frac{v(t)}{R} = 2\cos(100t - 50°)$$

$$\mathbf{I} = \frac{\mathbf{V}}{R} = \frac{8\angle -50°}{4} = 2\angle -50°$$

complex voltage and current

$$v(t) = V_m e^{j(\omega t+\theta)} = V_m\cos(\omega t + \theta) + j\,V_m \sin(\omega t + \theta)$$

$$i(t) = I_m e^{j(\omega t+\phi)} = I_m\cos(\omega t + \phi) + j\,I_m \sin(\omega t + \phi)$$

$$v(t) = V_m e^{j(\omega t+\theta)} = Ri(t) = RI_m e^{j(\omega t+\phi)}$$

$$V_m\angle\theta = RI_m\angle\phi \quad \Rightarrow \quad \mathbf{V} = R\mathbf{I}$$

in phase

The Inductor

Differentiation in time becomes multiplication in phasor form: (calculus becomes algebra!)

$$v(t) = L\frac{di(t)}{dt}$$

$$v(t) = V_m e^{j(\omega t+\theta)} = L\frac{d}{dt}I_m e^{j(\omega t+\phi)} = j\omega L I_m e^{j(\omega t+\phi)}$$

$$j = e^{j\frac{\pi}{2}} = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2}$$

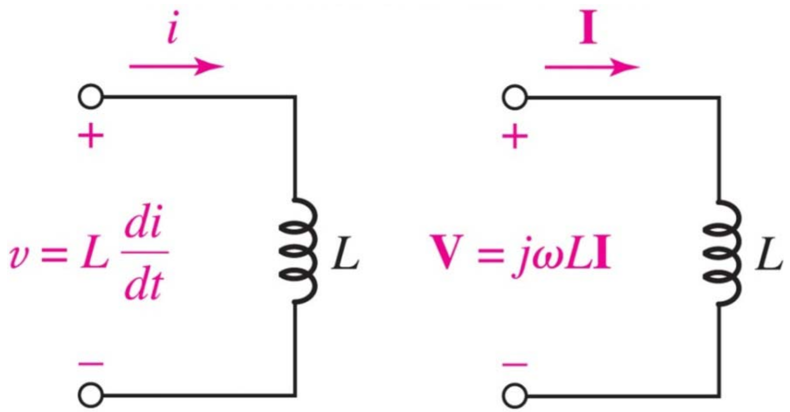
$$\text{-}j = e^{-j\frac{\pi}{2}} = \cos\frac{\pi}{2} - j\sin\frac{\pi}{2}$$

$$v(t)\ :\quad V_m e^{j\theta} = j\omega L I_m e^{j\phi}$$

$$\mathbf{V} = j\omega L\mathbf{I}$$

I *lag* V by 90

Example 10.4



$$v(t) = 8\angle -50°, \omega = 100\ rad/s, L = 4\ H$$

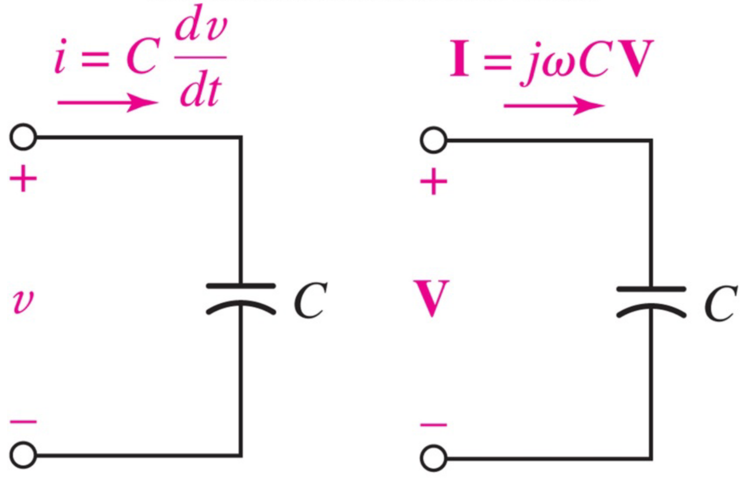
$$I = \frac{V}{j\omega L} = \frac{8\angle -50°}{j100(4)} = -j0.02\angle -50°$$

$$= (1\angle -90°)(0.02\angle -50°)$$

$$\Rightarrow \mathbf{I} = 0.02\angle -140°$$

The Capacitor

Differentiation in time becomes multiplication in phasor form: (calculus becomes algebra!)

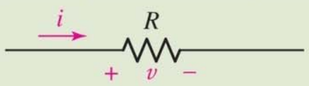
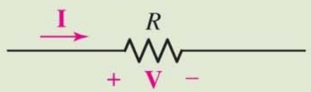
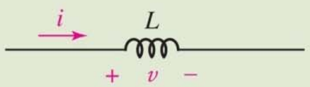
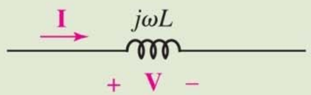
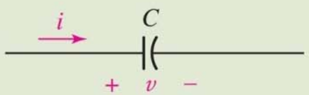
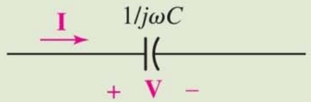


$$i(t) = C\frac{dv(t)}{dt}$$

$$i(t) = I_m e^{j(\omega t+\phi)} = C\frac{d}{dt}V_m e^{j(\omega t+\theta)} = j\omega C V_m e^{j(\omega t+\theta)}$$

$$i(t):\ I_m e^{j\theta} = j\omega C V_m e^{j\phi}$$

$\mathbf{I} = j\omega C\mathbf{V}$ **$\mathbf{I}$ *leads* V by 90**

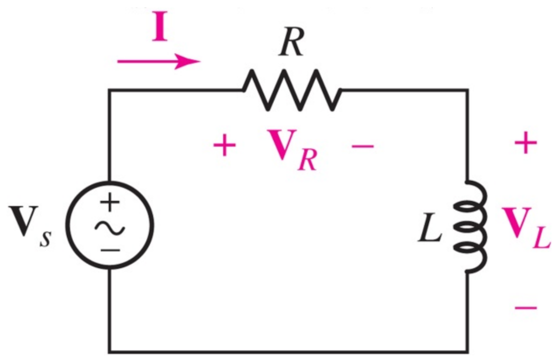
Time Domain		Frequency Domain	
	$v = Ri$	$\mathbf{V} = R\mathbf{I}$	
	$v = L\dfrac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$	
	$v = \dfrac{1}{C}\int i\,dt$	$\mathbf{V} = \dfrac{1}{j\omega C}\mathbf{I}$	

Calculus (hard but real) Algebra (easy but complex)

- Kirchhoff's Laws Using Phasors

$$KVL: \ \upsilon_1(t)+\upsilon_2(t)+\cdots+\upsilon_N(t)=0 \ \rightarrow \ \mathbf{V}_1+\mathbf{V}_2+\cdots+\mathbf{V}_N=0$$

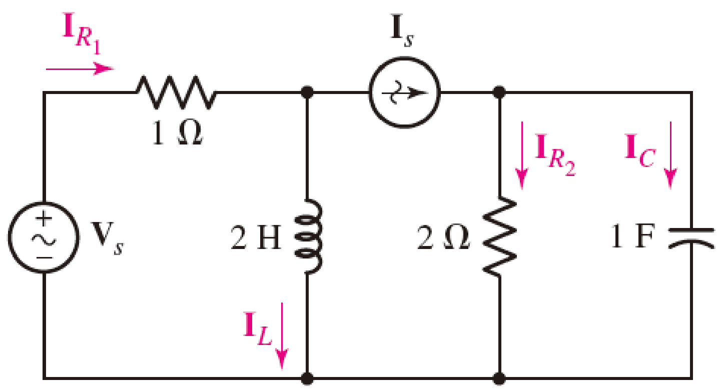
$$KCL: \ i_1(t)+i_2(t)+\cdots+i_N(t)=0 \ \rightarrow \ \mathbf{I}_1+\mathbf{I}_2+\cdots+\mathbf{I}_N=0$$



$$\mathbf{V}_s = \mathbf{V}_R + \mathbf{V}_L$$
$$= R\mathbf{I} + j\omega L\mathbf{I} \ \rightarrow \ \mathbf{I} = \frac{\mathbf{V}_s}{R + j\omega L}$$

Let $\mathbf{V}_s = V_m\angle 0°$ $\quad \mathbf{I} = \dfrac{V_m\angle 0°}{R+j\omega L} = \dfrac{V_m}{\sqrt{R^2+\omega^2 L^2}}\angle\left[-\tan^{-1}\dfrac{\omega L}{R}\right]$

Example 10.5 Determine $\mathbf{I}_s$ and $i_s(t)$ if sources are operate at ω=2 rad/s and $I_C=2\angle 28°$ A.



$$\mathbf{V}_C = \frac{1}{j\omega C}\mathbf{I}_C = -\frac{j}{2}(2\angle 28°)$$
$$= (0.5\angle -90°)(2\angle 28°) = 1\angle -62°\ V$$

$$\mathbf{I}_{R_2} = \frac{1}{2}\mathbf{V}_C = \frac{1}{2}\angle -62°\ A$$

$$\mathbf{I}_s = \mathbf{I}_{R_2} + \mathbf{I}_C = \frac{1}{2}\angle -62° + 2\angle 28° = \frac{3}{2}\angle -62°$$

$$\Rightarrow \ i_s(t) = 1.5\cos(2t - 62°)\ A$$

- Define impedance as $Z=\mathbf{V}/\mathbf{I}$, i.e. $\mathbf{V}=\mathbf{I}Z$

$Z_R=R$ $\qquad$ $Z_L=j\omega L$ $\qquad$ $Z_C=1/j\omega C$

$$\mathbf{Z} = \underbrace{R}_{\text{resistance}} + j\underbrace{X}_{\text{reactance}} \qquad \mathbf{Z} = |\mathbf{Z}|\angle\theta$$

- Impedance is the equivalent of resistance in the frequency domain.
- Impedance is a complex number (unit ohm).
- Impedances in series or parallel can be combined using "resistor rules."

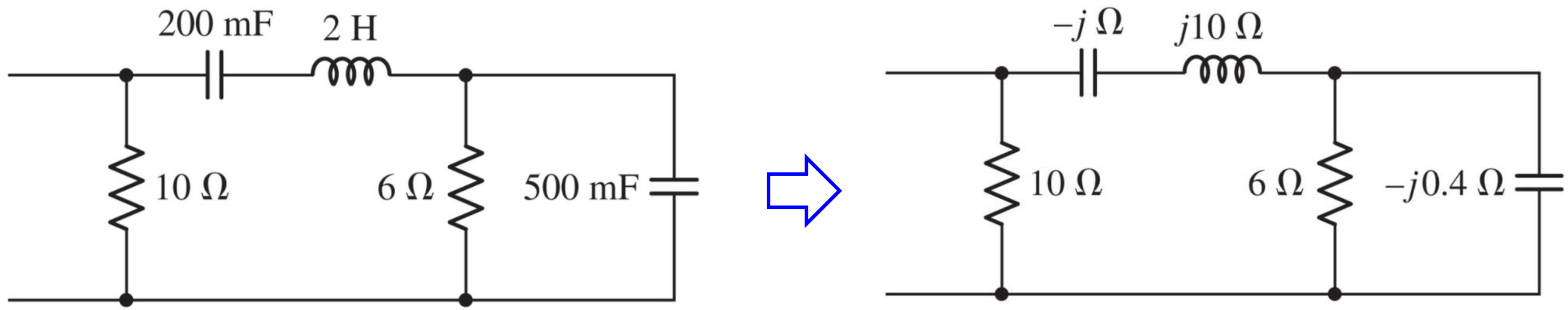
- the admittance is $Y=1/Z$

$Y_R=1/R$ $\qquad$ $Y_L=1/j\omega L$ $\qquad$ $Y_C=j\omega C$

$$\mathbf{Y} = \underbrace{G}_{\text{conductance}} + j\underbrace{B}_{\text{susceptance}} = \frac{1}{\mathbf{Z}} = \frac{1}{R+jX}$$

- if $Z=R+jX$; R is the *resistance*, X is the *reactance* (unit ohm Ω)
- if $Y=G+jB$; G is the *conductance*, B is the *susceptance*. (unit siemen S)

Example 10.6 Determine the equivalent impedance of the network working on ω=5 rad/s.



$$200mF \;\rightarrow\; \frac{1}{j\omega C} = \frac{1}{j5(0.2)} = -j1 \qquad 500mF \;\rightarrow\; \frac{1}{j\omega C} = \frac{1}{j5(0.5)} = -j0.4$$

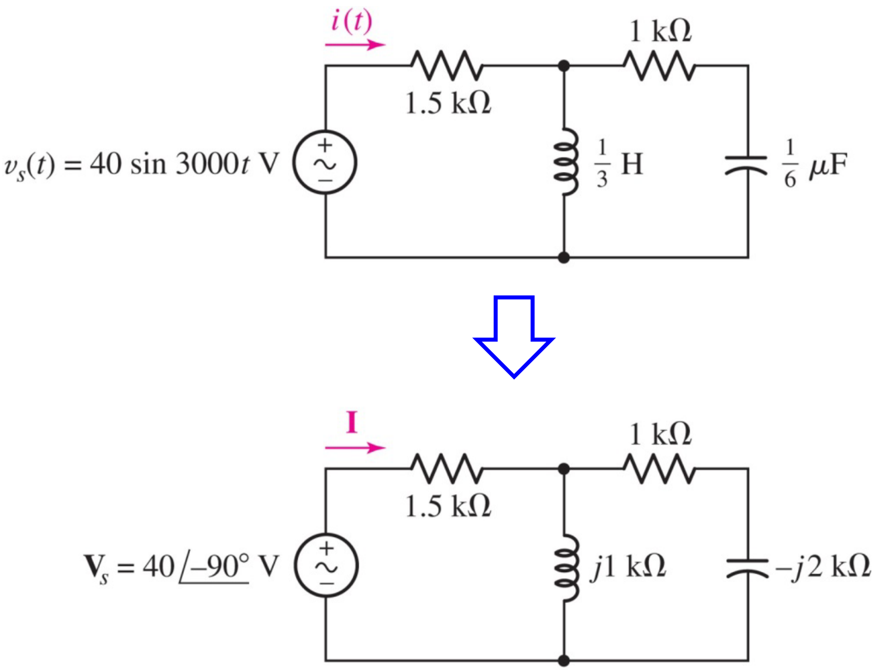
$$2\,H \;\rightarrow\; j\omega L = j10$$

$$6 \parallel j0.4 = \frac{6(-j0.4)}{6-j0.4} = 0.02655 - j0.3982$$

$$-j + j10 + 0.02655 - j0.3982$$
$$= 0.02655 + j8.602$$

$$10 \parallel (0.02655 + j8.602) = \frac{10(0.02655 + j8.602)}{10 + 0.02655 + j8.602} = 4.255 + j4.929 = 6.511\angle 49.20°$$

Example 10.7 Find the current i(t).



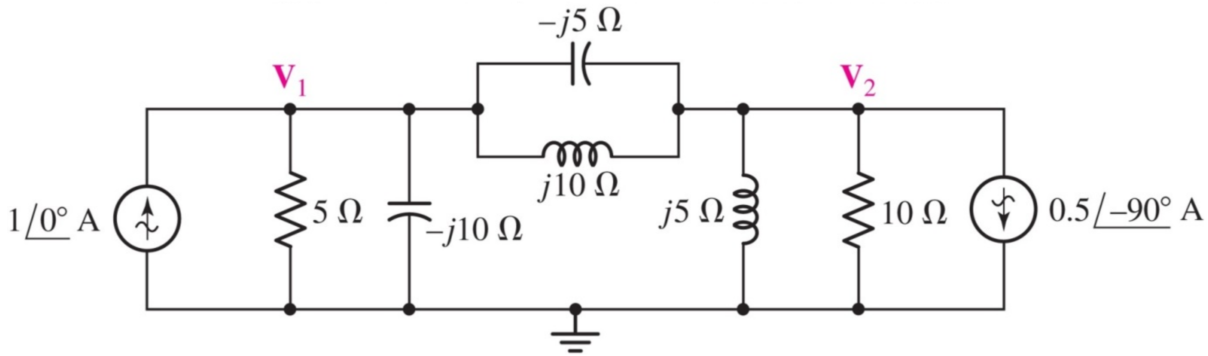
$$\mathbf{V}_s = 40\angle -90°$$

$$\begin{aligned}\mathbf{Z}_{eq} &= 1.5 + j1000 \parallel (1000 - j2000) \\ &= 1.5 + \frac{(j1)(1-j2)}{j1 + (1-j2)} = 1.5 + \frac{2+j}{1-j} \\ &= 2 + j1.5 = 2.5\angle 36.87°\ \mathrm{k\Omega}\end{aligned}$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}_{eq}} = \frac{40\angle -90°}{2.5\angle 36.87°} = 0.016\angle -126.9°\ mA$$

$$i(t) = 16\cos(3000t - 126.9°)\quad mA$$

Example 10.8 Find the node voltages $v_1(t)$ and $v_2(t)$.



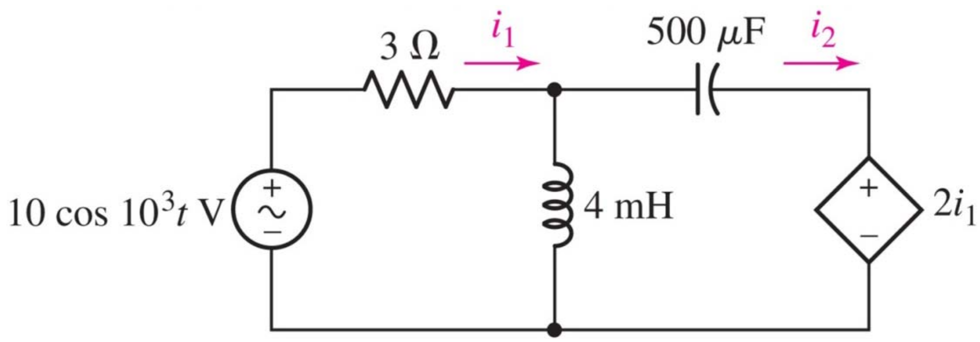
$$1\angle 0° = 1 + j0 = \frac{\mathbf{V}_1}{5} + \frac{\mathbf{V}_1}{-j10} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j10}$$

$$-0.5\angle -90° = -0.5(-j1) = \frac{\mathbf{V}_2}{10} + \frac{\mathbf{V}_2}{j5} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{-j5} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{j10}$$

$$\Rightarrow \begin{aligned}(0.2 + j0.2)\mathbf{V}_1 - j0.1\mathbf{V}_2 &= 1,\\ -j0.1\mathbf{V}_1 + (0.1 - j0.1)\mathbf{V}_2 &= j0.5\end{aligned} \Rightarrow \begin{aligned}\mathbf{V}_1 &= 1 - j2 = 2.24\angle -63.4°\\ \mathbf{V}_2 &= -2 + j4 = 4.47\angle 116.6°\end{aligned}$$

$$v_1(t) = 2.24\cos(\omega t - 63.4°),$$
$$v_2(t) = 4.47\cos(\omega t + 116.6°)$$

Example 10.9 Obtain the expression for the time-domain currents i_1 and i_2.

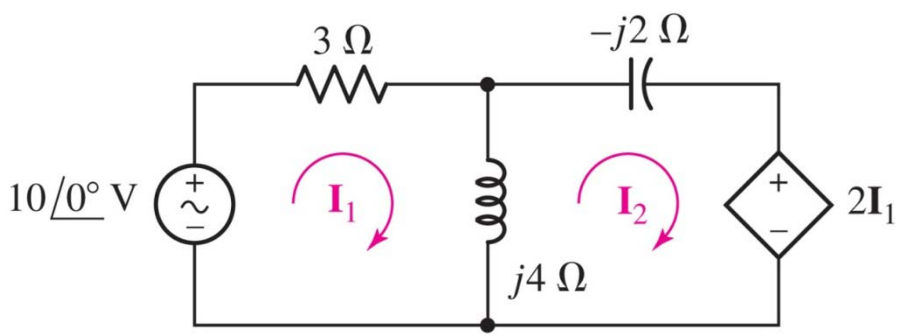


$$-10 + 3\mathbf{I}_1 + j4(\mathbf{I}_1 - \mathbf{I}_2) = 0,$$
$$-j2\mathbf{I}_2 + 2\mathbf{I}_1 + j4(\mathbf{I}_2 - \mathbf{I}_1) = 0$$

$$500\mu F = \frac{1}{j\omega C} = -j\frac{1}{(10^3)(500 \times 10^{-6})} = -j2$$

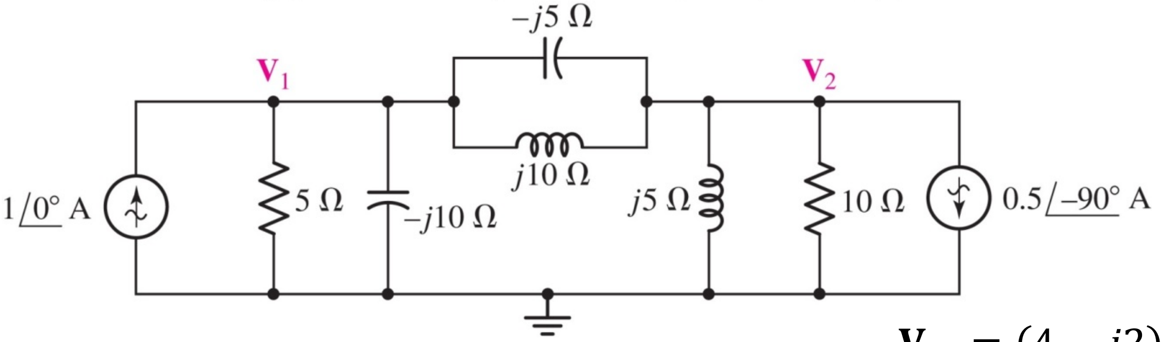
$$4mH = j\omega L = j(10^3)(4 \times 10^{-3}) = j4$$

$$\mathbf{I}_1 = \frac{14 + j8}{13} = 1.24\angle 29.7°,$$
$$\mathbf{I}_2 = \frac{20 + j30}{13} = 2.77\angle 56.3°$$



$$i_1(t) = 1.24\cos(10^3 t + 29.7°),$$
$$i_2(t) = 2.77\cos(10^3 t + 56.3°)$$

Example 10.10 Use superposition to find $\mathbf{V_1}$.



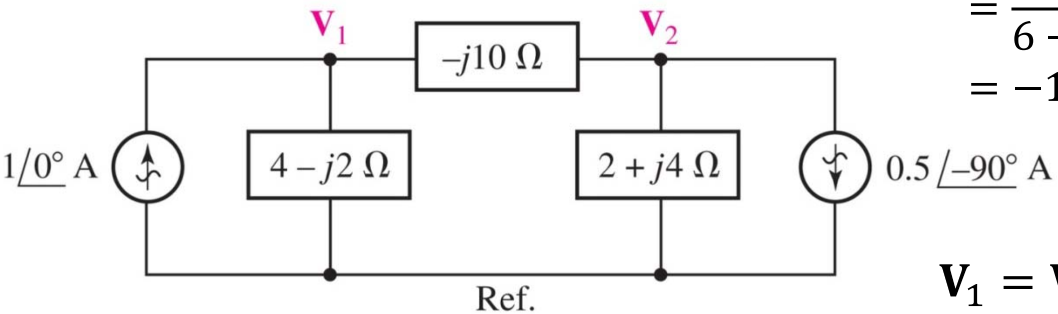
$$5 \parallel (-j10) = \frac{5(-j10)}{5-j10} = 4 - j2$$

$$-j5 \parallel (j10) = \frac{-j5(j10)}{-j5+j10} = -j10$$

$$10 \parallel (j5) = \frac{10(j5)}{10+j5} = 2 + j4$$

$$1\angle 0° = 1 + j0$$

$$0.5\angle -90° = -j0.5$$

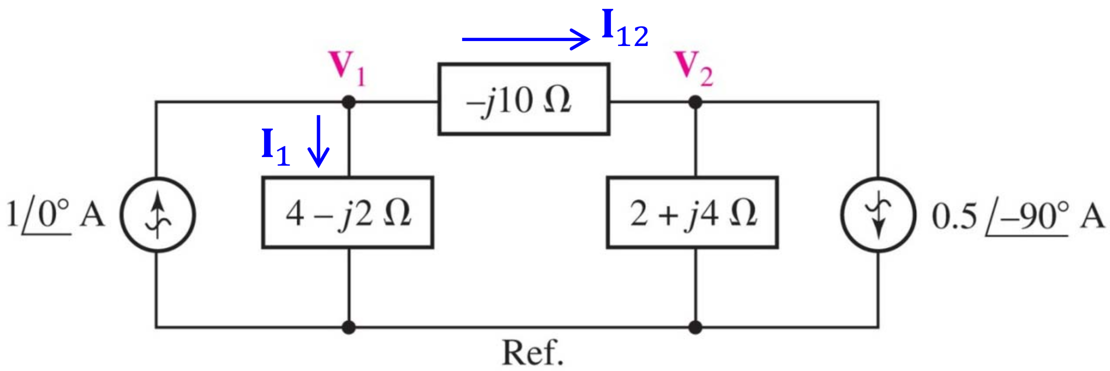


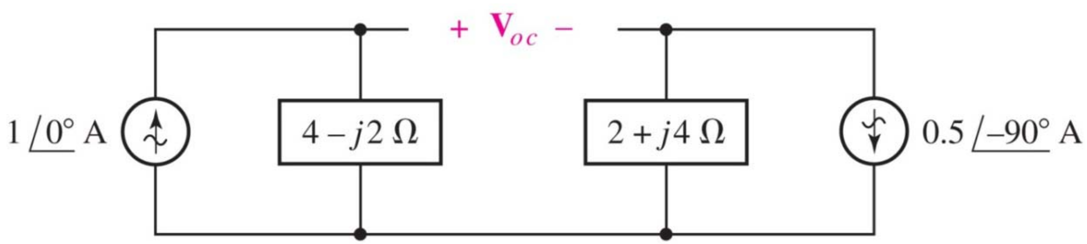
$$\mathbf{V}_{1L} = (4-j2)\boldsymbol{I}_{1L}$$
$$= (4-j2)\frac{-j10+2+j4}{(4-j2)+(-j10+2+j4)}(1+j0)$$
$$= \frac{-4-j28}{6-j8} = 2 - j2\ V$$

$$\mathbf{V}_{1R} = (4-j2)\boldsymbol{I}_{1R}$$
$$= (4-j2)\frac{2+j4}{(2+j4)+(-j10+4-j2)}(j0.5)$$
$$= \frac{6-j8}{6-j8}$$
$$= -1\ V$$

$$\mathbf{V}_1 = \mathbf{V}_{1L} + \mathbf{V}_{1R} = 2 - j2 - 1 = 1 - j2\ V$$

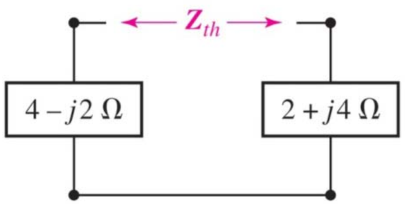
Example 10.11 Determine the Thévenin equivalent seen by the $-j10$ impedance to find $\mathbf{V_1}$.

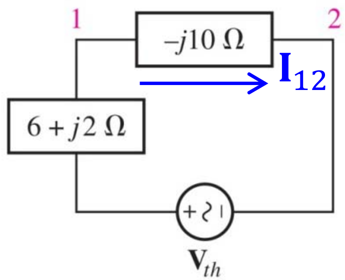




$$\mathbf{V}_{oc} = \mathbf{V}_{1o} - \mathbf{V}_{2o}$$
$$= (4 - j2)(1 + j0) - (2 + j4)(j0.5)$$
$$= 4 - j2 - j1 + 2$$
$$= 6 - j3$$

$$\mathbf{Z}_{th} = (4 - j2) + (2 + j4)$$
$$= 6 + j2$$



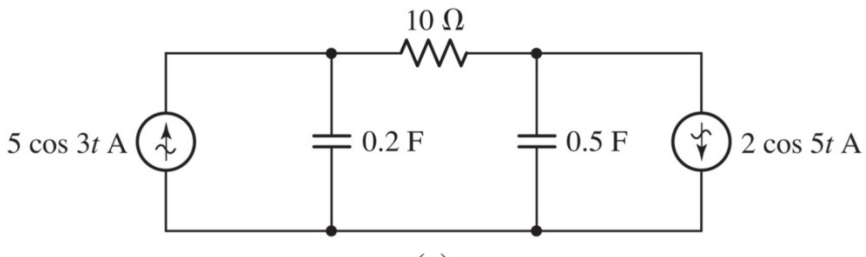


$$\mathbf{I}_{12} = \frac{6 - j3}{6 + j2 - j10}$$
$$= \frac{60 + j30}{100}$$
$$= 0.6 + j0.3$$

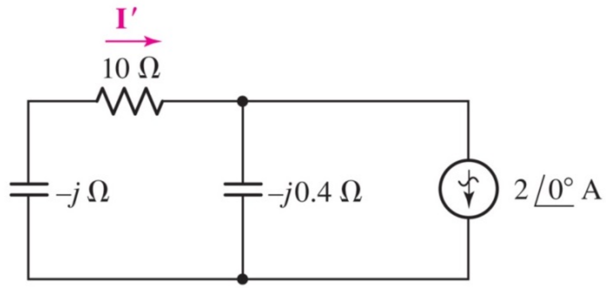
$$\mathbf{I}_1 = (1 + j0) - \mathbf{I}_{12} = 1 - 0.6 - j0.3 = 0.4 - j0.3\ A$$

$$\mathbf{V}_1 = (4 - j2)\mathbf{I}_1 = (4 - j2)(0.4 - j0.3) = 1 - j2\ V$$

Example 10.12 Determine the power dissipated by the 10 Ω resistor.



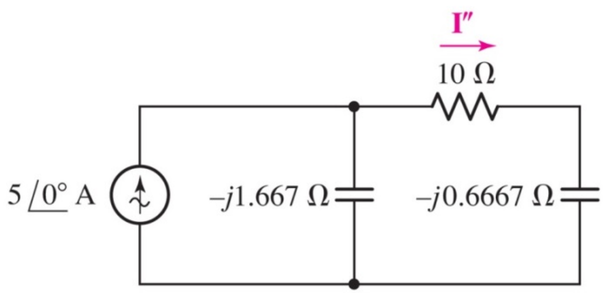
(a)



(b)

$$\mathbf{I}' = \frac{-j0.4}{(10-j)+(-j0.4)}[(2+j0)]$$
$$\Rightarrow \mathbf{I}' = 79.23\angle-82.03°\ mA$$

$$i'(t) = 79.23\cos(5t-82.03°)\ mA$$

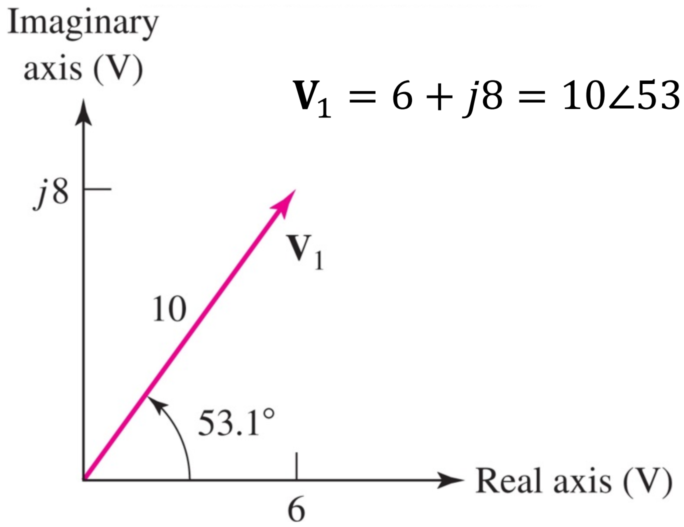


$$\mathbf{I}'' = \frac{-j1.667}{-j1.667+(10-j0.667)}[5+j0]$$
$$\Rightarrow \mathbf{I}'' = 811.7\angle-76.86°\ mA$$

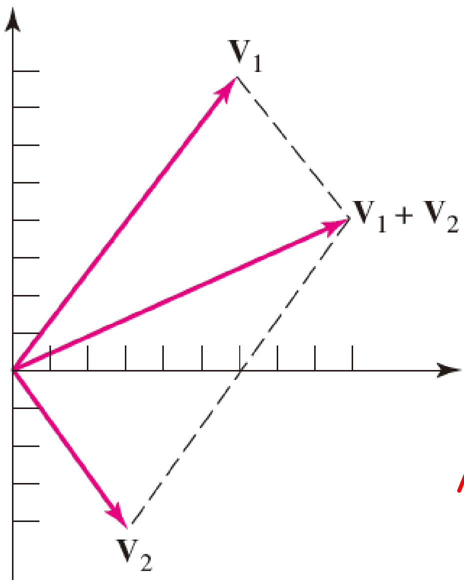
$$i''(t) = 811.7\cos(3t-76.86°)\ mA$$

$$p_{10} = i^2R = (i'+i'')^2 10$$
$$= 10[79.23\cos(5t-82.03°)+811.7\cos(3t-76.86°)]^2\ \mu W$$

The arrow for the phasor **V** on the phasor diagram is a photograph, taken at *ωt = 0*, of a rotating arrow whose projection on the real axis is the instantaneous voltage *v(t)*.



$$\mathbf{V}_1 = 6 + j8 = 10\angle 53.1°$$



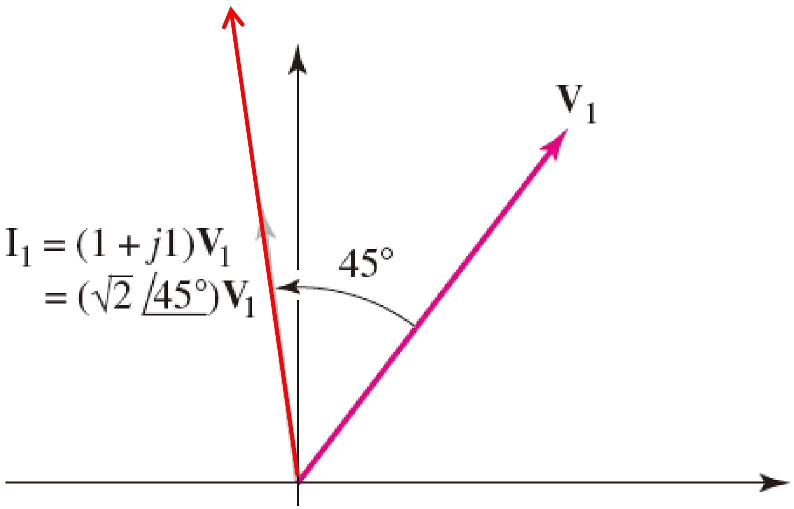
$$\mathbf{V}_2 = 3 - j4 = 5\angle -53.1°$$

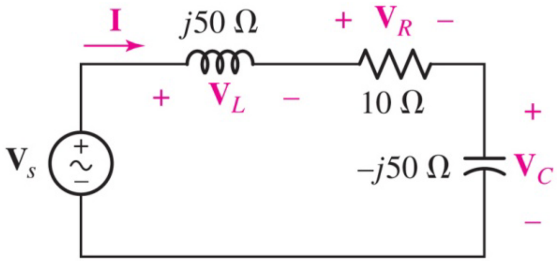
$$\downarrow$$

$$\begin{aligned}\mathbf{V}_1 + \mathbf{V}_2 &= 6 + j8 + 3 - j4 \\ &= 9 + j4 \\ &= 9.849\angle 23.96°\end{aligned}$$

$$\mathbf{Y} = 1 + j1$$

$$\Rightarrow \mathbf{I}_1 = \mathbf{V}_1\mathbf{Y} = 1.414\angle 45°(10\angle 53.1°) = 14.14\angle 98.1°$$





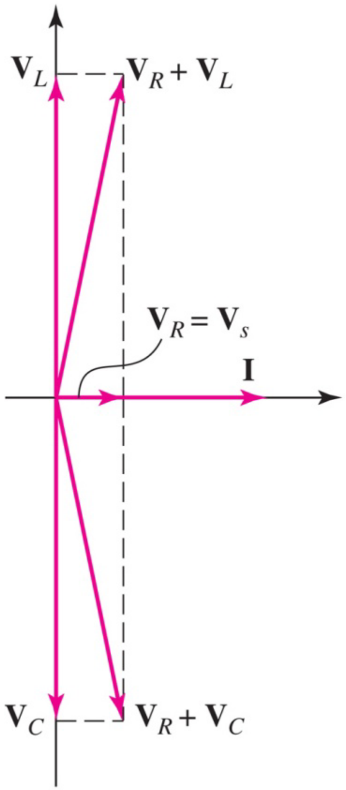
$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{\mathbf{V}_s}{j50 + 10 - j50} = \frac{\mathbf{V}_s}{10}$$

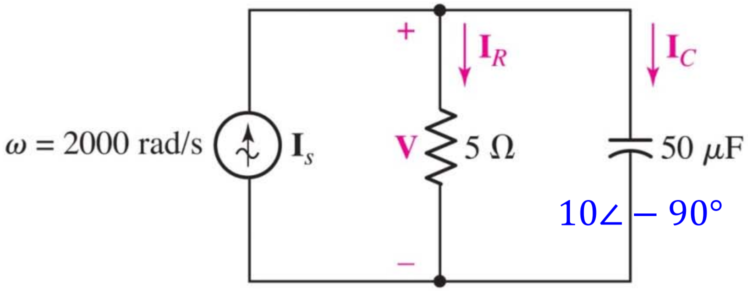
Let $\mathbf{I} = I_m \angle 0°$

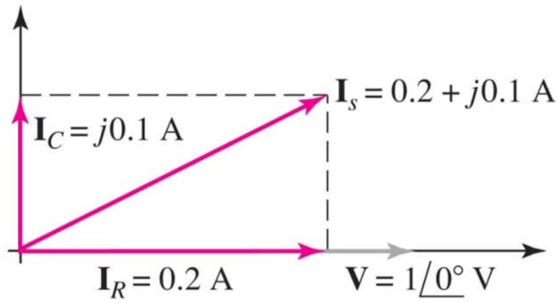
$$\mathbf{V}_L = j50\mathbf{I} = j50\frac{\mathbf{V}_s}{10} = j5\mathbf{V}_s,$$

$$\mathbf{V}_R = 10\mathbf{I} = 10\frac{\mathbf{V}_s}{10} = \mathbf{V}_s,$$

$$\mathbf{V}_C = -j50\mathbf{I} = -j50\frac{\mathbf{V}_s}{10} = -j5\mathbf{V}_s$$





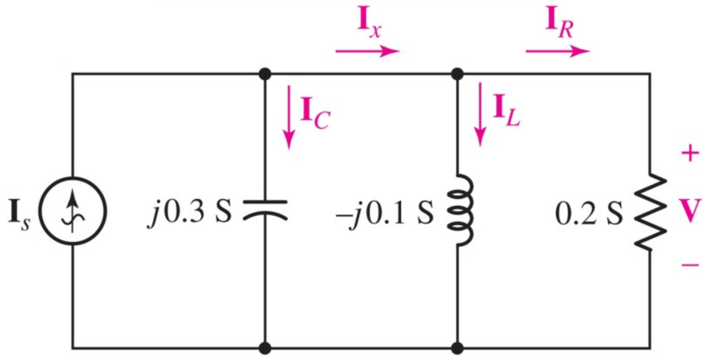


Let $\mathbf{V} = 1\angle 0° = 1 + j0$

$$\mathbf{I}_R = \frac{\mathbf{V}}{5} = 0.2\angle 0° = 0.2 + j0,$$

$$\mathbf{I}_C = \frac{\mathbf{V}}{\boldsymbol{Z}_C} = \frac{1\angle 0°}{10\angle - 90°} = 0.1\angle 90°$$

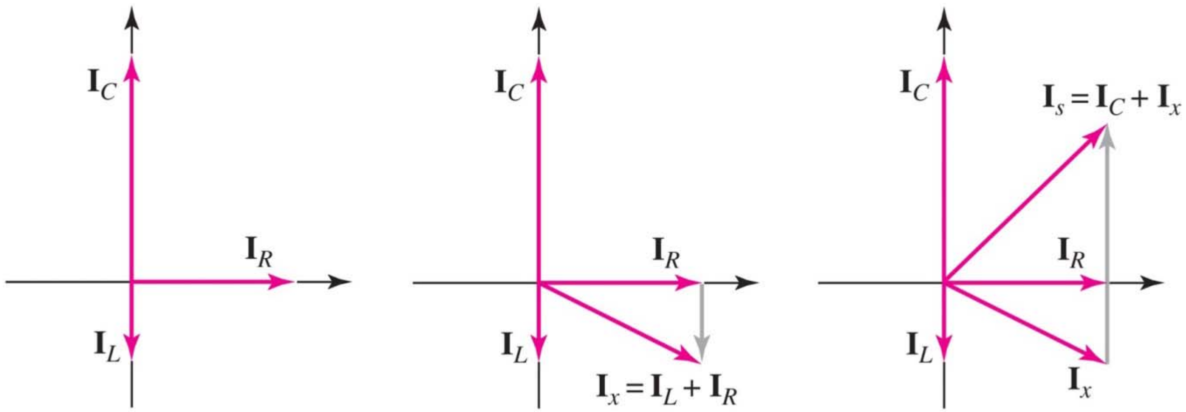
Example 10.13 Construct a phasor diagram showing I_R, I_L, and I_C.



Let $\mathbf{V} = 1\angle 0° = 1 + j0$

$$\Rightarrow \mathbf{I}_x = \mathbf{I}_R + \mathbf{I}_L$$
$$= 0.2\angle 0° + 0.1\angle -90°$$
$$= 0.2 - j0.1 = 0.224\angle -26.6°$$
$$\Rightarrow \mathbf{I}_s = \mathbf{I}_C + \mathbf{I}_x = j0.3 + 0.2 - j0.1$$
$$= 0.2 + j0.2 = 0.283\angle 45°$$

$$\mathbf{I}_R = 0.2\mathbf{V} = 0.2\angle 0° = 0.2$$
$$\mathbf{I}_L = -j0.1\mathbf{V} = 0.1\angle -90° = -j0.1$$
$$\mathbf{I}_C = j0.3\mathbf{V} = 0.3\angle 90° = j0.3$$



 Homework : 10장 Exercises 7의 배수 문제

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