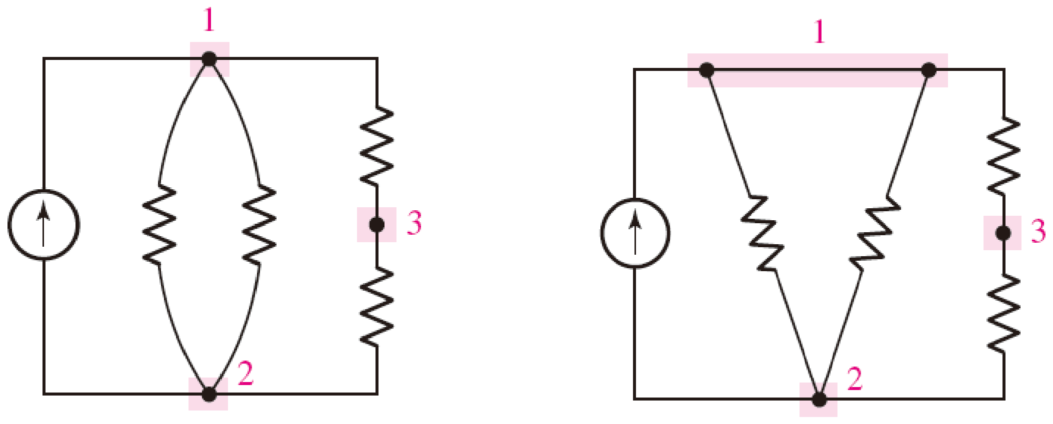
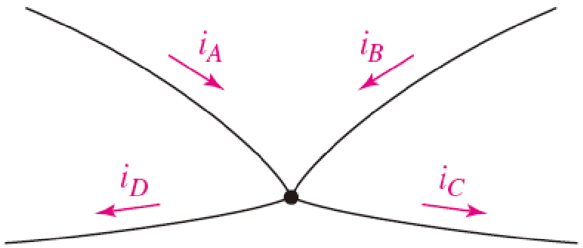
Chapter 3
Voltage and Current Laws

- Lumped-parameter network
  - network with a number of simple elements and a set of connecting leads (or wires)
- Distributed-parameter network
  - contains an essentially infinite number of vanishingly small elements

- ***Node*** : A point of connection of two or more elements
- ***Branch*** : Node + Element + Node
- ***Path*** : Node (start) ➪ Element ➪ Node ..➪ Node (end)
- Closed path (***loop***) : Path with start node = end node



- Definition
  - The algebraic sum of the currents entering any node is zero
  - Charge cannot accumulate at a node → the entered current(charge) must go out at a node !!



based on **entering** node

$$i_A + i_B + (-i_C) + (-i_D) = 0$$

based on **leaving** node

$$(-i_A) + (-i_B) + i_C + i_D = 0$$

General form

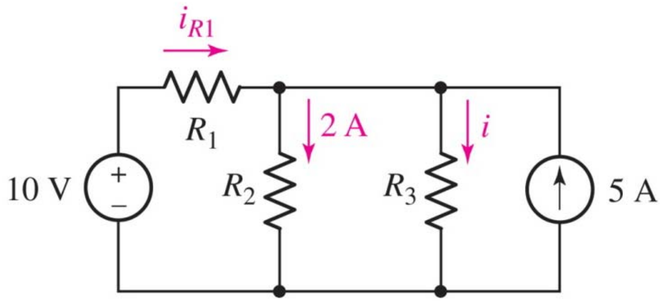
$$i_1 + i_2 + i_3 + \cdots + i_N = 0$$

$$\sum_{i=1}^{N} i_n$$

currents going in = currents going out

$$i_A + i_B = i_C + i_D$$

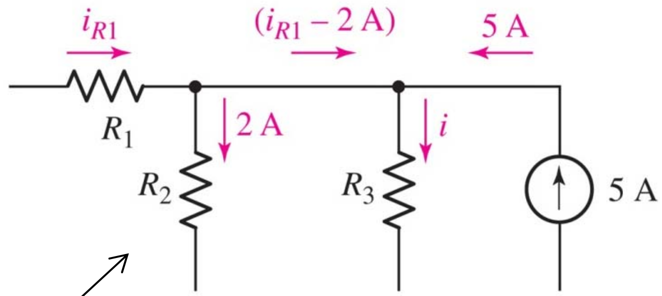
Example 3.1 *Find i*



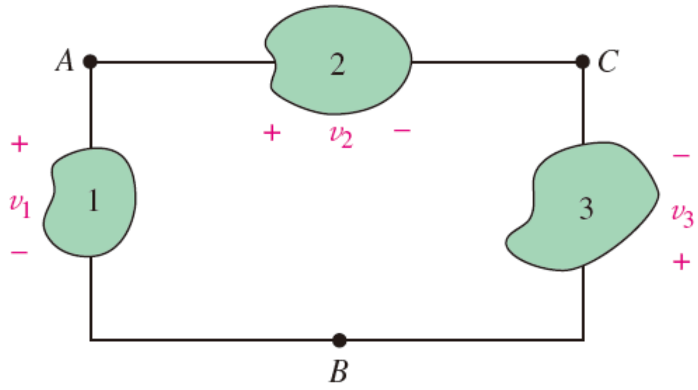
$$i_{R_1} - 2 - i + 5 = 0$$

$$i = i_{R_1} - 2 + 5$$
$$= 3 - 2 + 5 = 6A$$

$$(i_{R_1} - 2) + 5 = i$$
$$\rightarrow i = 3 - 2 + 5 = 6A$$



- Definition
  - The algebraic sum of the voltages around any closed path is zero
  - Energy required to move a unit charge from A to B in a circuit must have a value independent of the path chosen to get from A to B !!



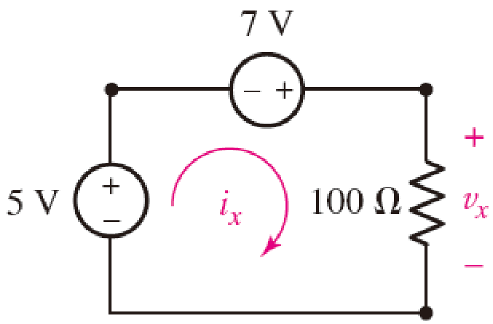
$$v_1 - v_2 + v_3 = 0$$
$$\rightarrow v_1 = v_2 - v_3$$

General form

$$v_1 + v_2 + v_3 + \cdots + v_N = 0$$

$$\sum_{i=1}^{N} v_n$$

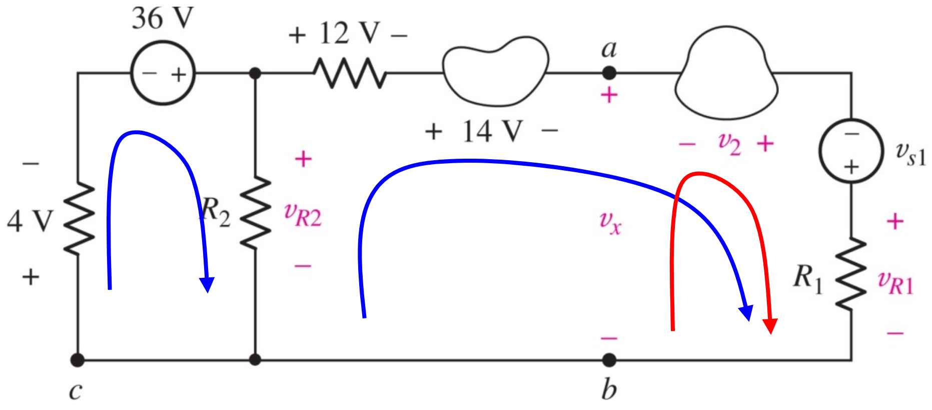
Example 3.2 *Find* i_x



$$-5 - 7 + v_x = 0$$
$$\rightarrow v_x = 12V$$

$$i_x = \frac{v_x}{100} = \frac{12}{100} = 120mA$$

Example 3.3 Find v_{R2}, v_x



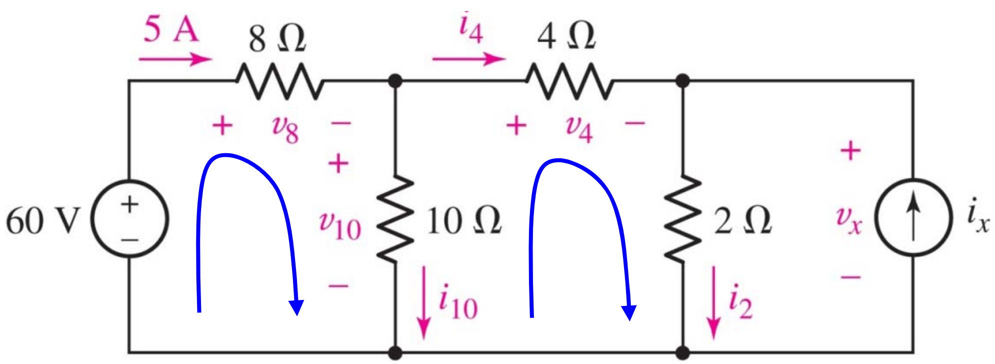
$$4 - 36 + v_{R2} = 0 \quad \rightarrow v_{R2} = 32V$$

$$-v_{R2} + 12 + 14 - v_2 - v_{s1} + v_{R1} = 0$$
$$\rightarrow v_{R2} - 12 - 14 = v_{R1} - v_2 - v_{s1}$$

$$-v_x - v_2 - v_{s1} + v_{R1} = 0$$
$$\rightarrow v_x = v_{R1} - v_2 - v_{s1}$$

$$v_x = v_{R2} - 12 - 14$$
$$\rightarrow v_x = 32 - 12 - 14 = 6V$$

Example 3.4 Find v_x



$$-60 + v_8 + v_{10} = 0$$

$$-v_{10} + v_4 + v_x = 0 \quad \rightarrow v_x = v_{10} - v_4$$

$$v_8 = 8 \times 5 = 40V$$

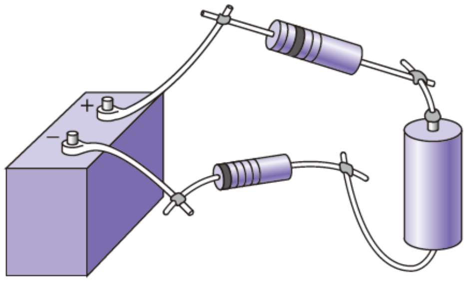
$$v_{10} = 60 - v_8 = 60 - 40 = 20V$$

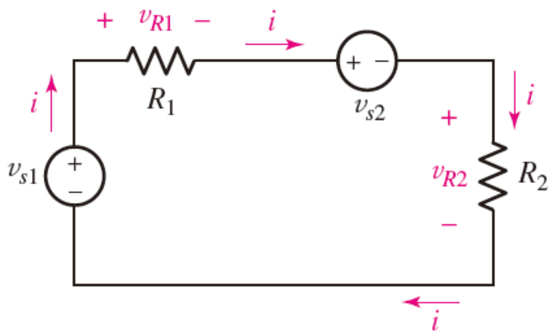
$$v_x = v_{10} - v_4 = 20 - v_4$$

$$5 = i_{10} + i_4 \quad \rightarrow i_4 = 5 - i_{10} = 5 - 2 = 3$$

$$v_x = 20 - v_4 = 20 - (i_4 \times 4) = 20 - 12 = 8V$$

- Definition
  - Connected in series
  - All of the elements face to the same current





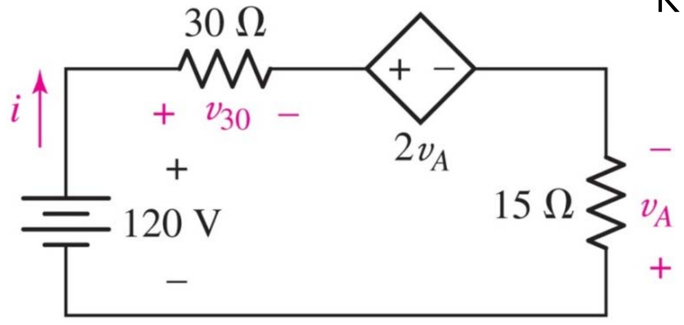
KVL: $-v_{s1} + v_{R1} + v_{S2} + v_{R_2} = 0$

Ohm's Law: $v_{R1} = R_1 i,\ \ v_{R2} = R_2 i$

$$-v_{s1} + R_1 i + v_{S2} + R_2 i = 0$$

$$\rightarrow i = \frac{v_{s1} - v_{s2}}{R_1 + R_2}$$

Example 3.5 Compute the power absorbed in each element



KVL: $-120 + v_{30} + 2v_A - v_A$

$v_{30} = 30i,\ \ v_A = -15i$

$-120 + 30i - 30i + 15i$

$\rightarrow i = \dfrac{120}{15} = 8A$

$p_{120V} = (120)(-8) = -960W$

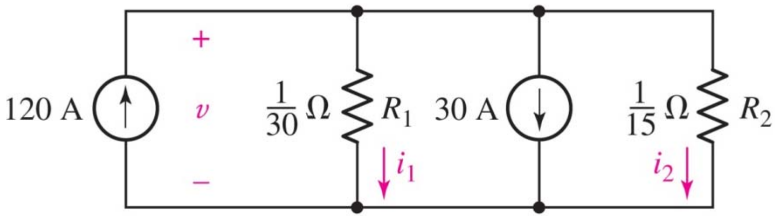
$p_{30} = (30)(8)^2 = 1920W$

$p_{2v_A} = (8)\big(2(-15)(8)\big) = -1920W$

$p_{15} = (8)\big(-(-15)\big)(8) = 960W$

- Definition
  - Connected in parallel
  - Elements in a circuit having a common voltage across them

Example 3.6 Find the voltage, current, and power associated with each element



$$-120 + i_1 + 30 + i_2 = 0$$

$$i_1 = 30v, \qquad i_2 = 15v$$

$$\rightarrow -120 + 30v + 30 + 15v = 0$$

$$\rightarrow 45v = 90$$

$$\rightarrow v = 2V$$

$$i_1 = 60A, \qquad i_2 = 30A$$

$$p_{R_1} = vi_1 = (2)(60) = 120W$$

$$p_{R_2} = vi_2 = (2)(30) = 60W$$

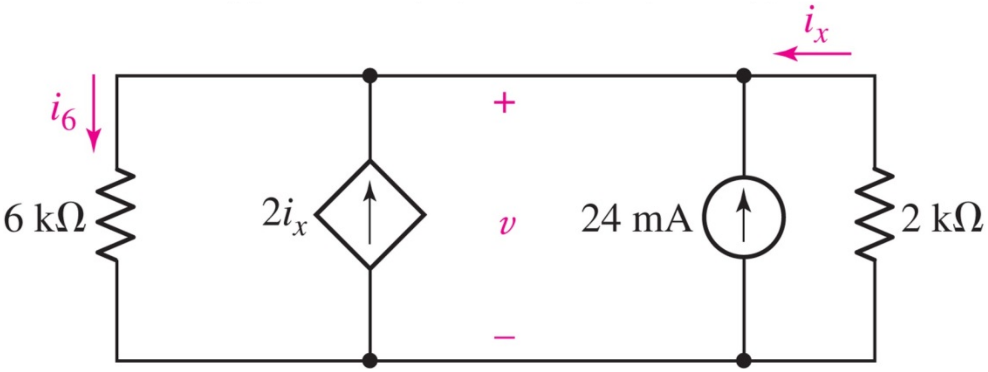
$$p_{120A} = (2)(-120) = -240W$$

$$p_{30A} = (2)(30) = 60W$$

Total power absorbed by each element

$$120 + 60 - 240 + 60 = 0$$

Example 3.7 Determine the value of υ and the power supplied by the indep. source



KCL : $i_6 - 2i_x - 24 \times 10^{-3} - i_x = 0$

$\rightarrow \quad i_6 - 3i_x - 24 \times 10^{-3} = 0$

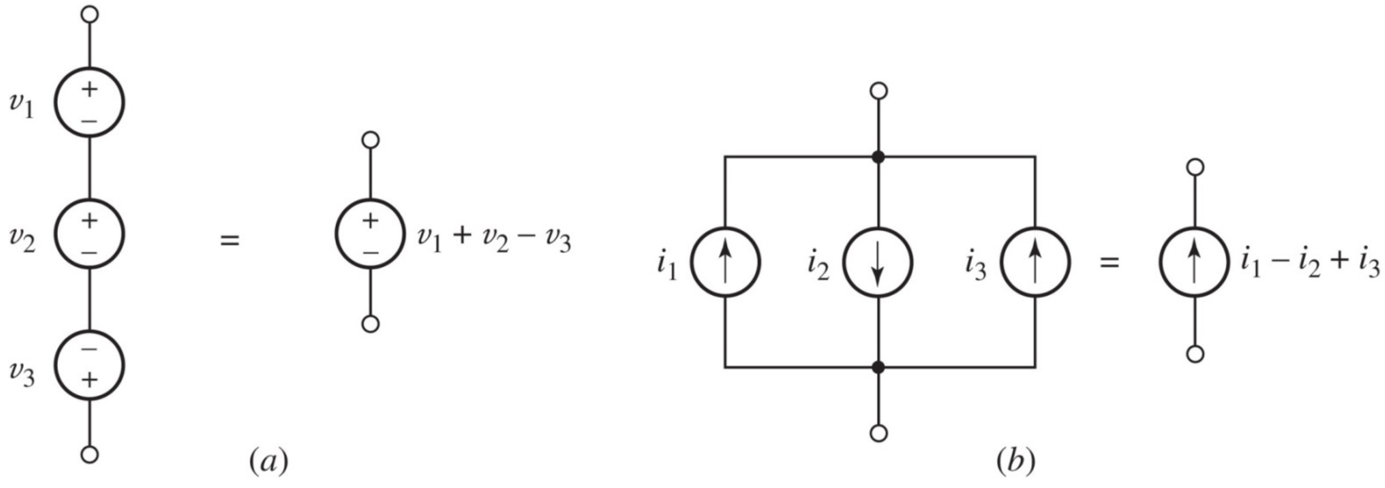
$$i_6 = \frac{v}{6000}, \quad i_x = -\frac{v}{2000}$$

$$\Rightarrow \frac{v}{6000} - 3\left(-\frac{v}{2000}\right) - 24 \times 10^{-3} = 0$$

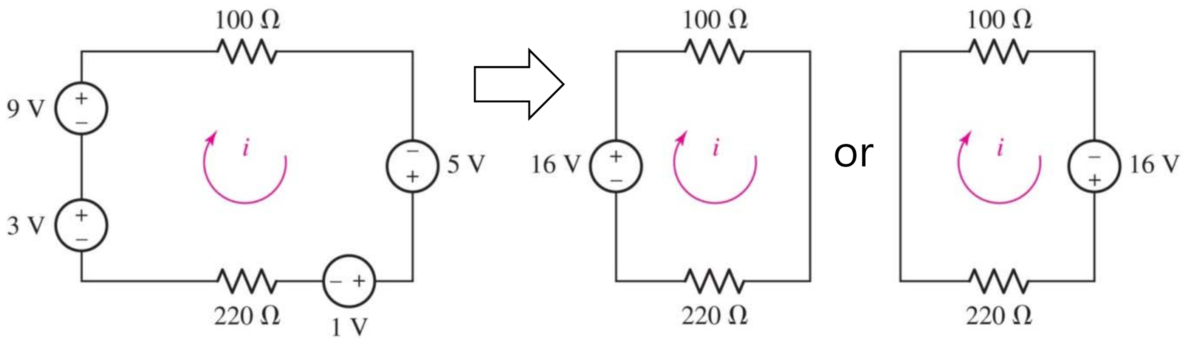
$$\rightarrow v = \left(\frac{6000}{10}\right)(24 \times 10^{-3}) = 14.4V$$

The supplying power

$$p_{24} = (14.4)(-24 \times 10^{-3})$$
$$= -345.6mW$$



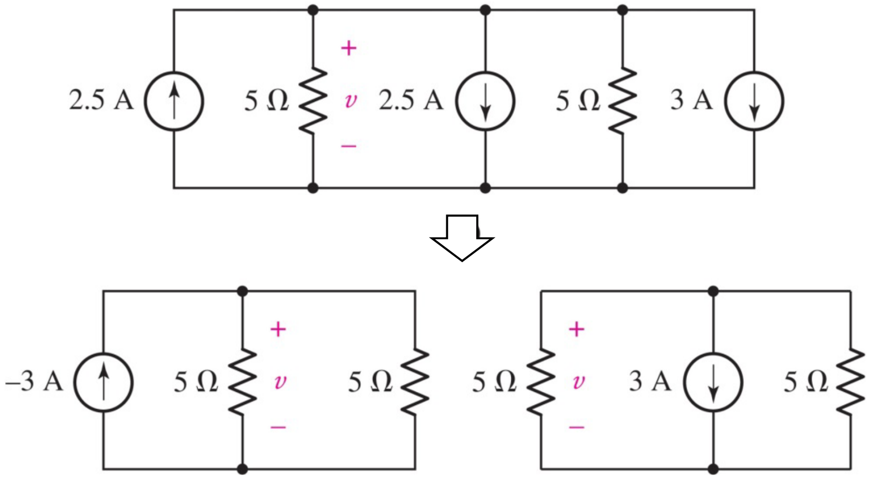
Example 3.8 Determine the current i



$$-3-9-5+1$$
$$=-16V$$

$$-16+100i+220i=0$$
$$\rightarrow i=\frac{16}{320}=50mA$$

Example 3.9 Determine the voltage v

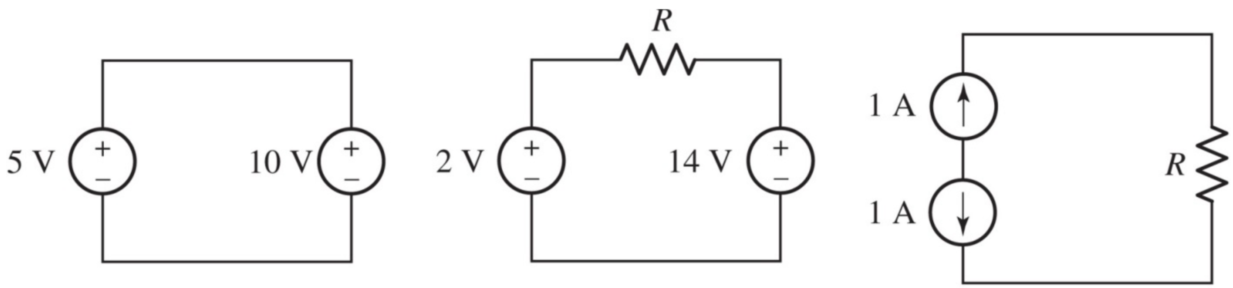


$$2.5 - 2.5 - 3 = -3A$$

$$\text{KCL}: \ -3 + \frac{v}{5} + \frac{v}{5} = 0$$

$$\rightarrow v = 7.5V$$

Example 3.10 Determine which of the circuits are valid



(a) (b) (c)

Using KVL

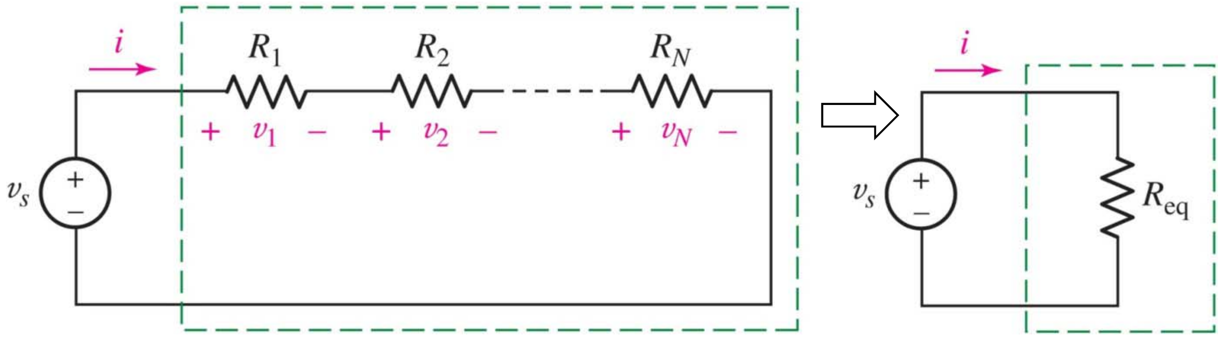
~~$-5 + 10 = 0$~~

$$-2 + iR + 14 = 0$$

KVL can hold

unclear what current actually flows through the R

Series connection

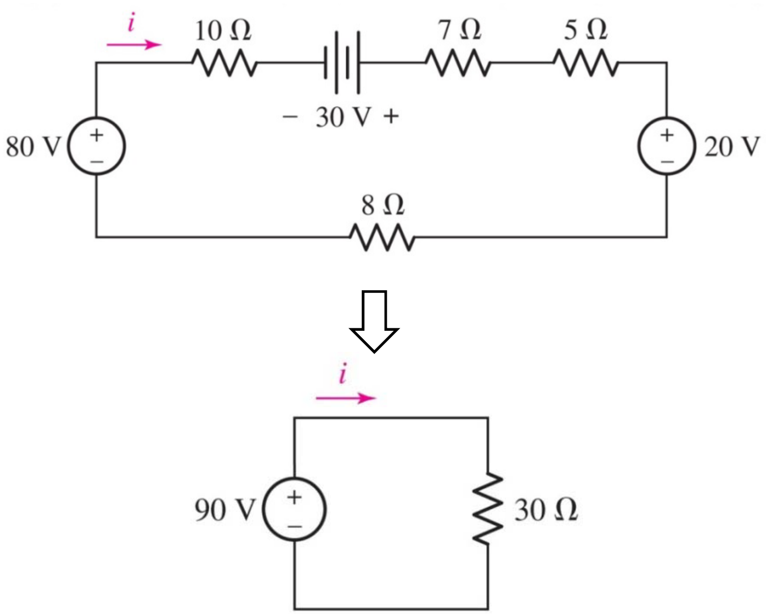


$$-v_s + v_1 + v_2 + \cdots + v_N = 0$$

$$\begin{aligned} v_s &= v_1 + v_2 + \cdots + v_N \\ &= R_1 i + R_2 i + \cdots + R_N i \\ &= (R_1 + R_2 + \cdots + R_N) i \\ &= R_{eq} i \end{aligned}$$

$$R_{eq} = R_1 + R_2 + \cdots + R_N$$

Example 3.11 Determine the current and the power delivered by the 80 V source

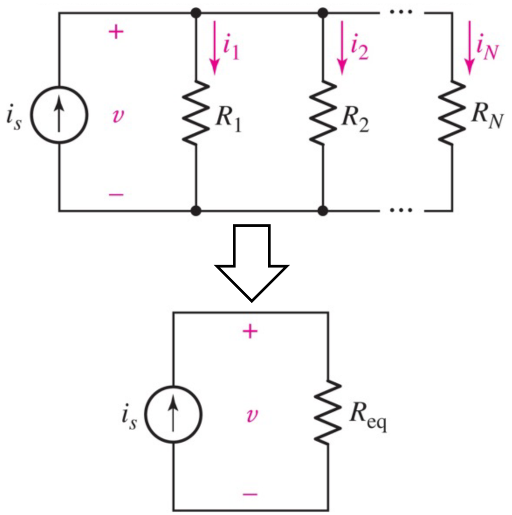


$$-80 + 10i - 30 + 7i + 5i + 20 + 8i = 0$$
$$\rightarrow (-8 - 30 + 20) + (10 + 7 + 5 + 8)i = 0$$
$$\rightarrow -90 + 30i = 0$$
$$\therefore i = 3A$$

The supplying power

$$\begin{aligned} p_{80V} &= (80)(-3) \\ &= -240\ W \end{aligned}$$

Parallel connection



$$i_s = i_1 + i_2 + \cdots + i_N$$

$$= \frac{v}{R_1} + \frac{v}{R_2} + \cdots + \frac{v}{R_N}$$

$$= \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}\right) v$$

$$\therefore i_s = \frac{v}{R_{eq}}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

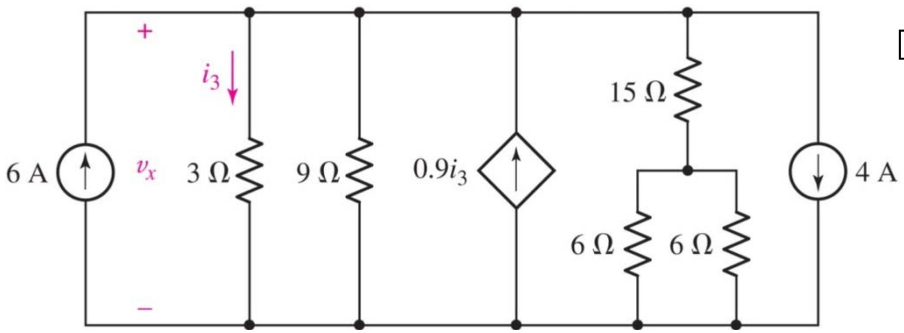
$$R_{eq}^{-1} = R_1^{-1} + R_2^{-1} + \cdots + R_N^{-1}$$

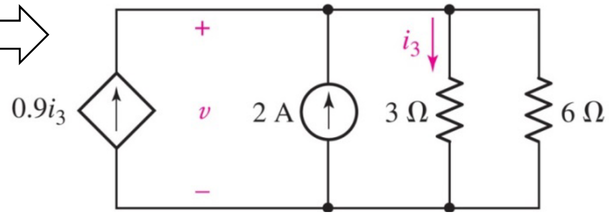
$$G_{eq} = G_1 + G_2 + \cdots + G_N$$

For only two resistors

$$R_{eq} = R_1 || R_2 = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

Example 3.12 Calculate the power and voltage of the dependent source



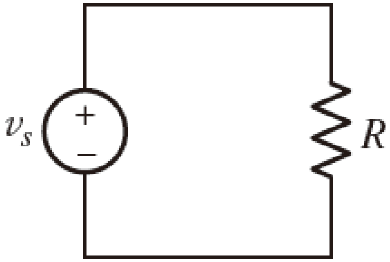


$$\left(1 + \frac{3}{6} - 0.9\right) i_3 = 2$$

KCL : $0.9i_3 + 2 = i_3 + \frac{v}{6}$ ⇨ $0.9i_3 + 2 = i_3 + \frac{3i_3}{6}$ ⇨ $\rightarrow i_3 = \frac{10}{3} A$

$$\therefore v = 3i_3 = 10V$$

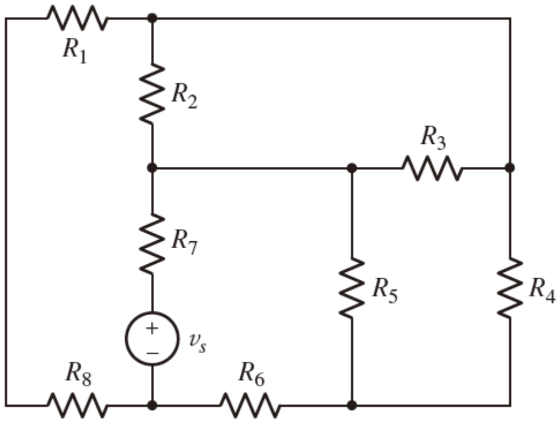
$$p = v \times -0.9i_3$$
$$= -10 \times 0.9 \times \frac{10}{3}$$
$$= -30W$$



Are v_s and R in series or in parallel?

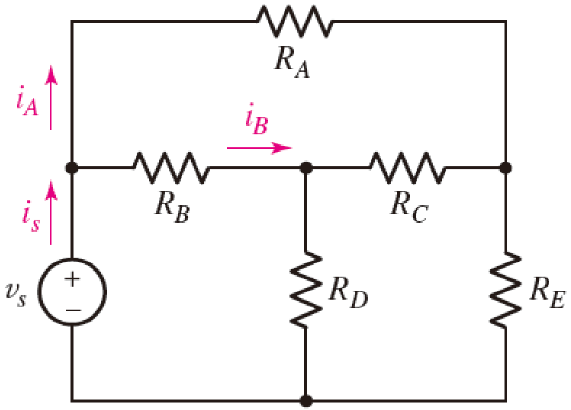
⇨ Answer: **Both**

Carry the same current or
Enjoy the same voltage



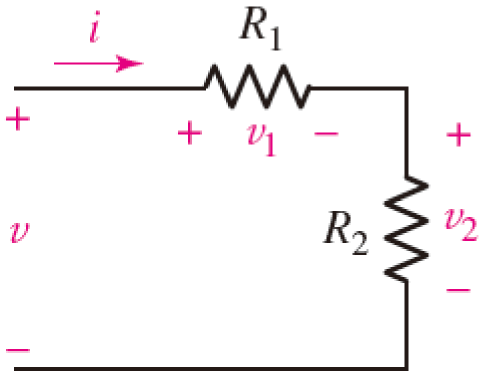
R_1 and R_8 are in series.

R_2 and R_3 are in parallel.



There are no circuit elements either in series or in parallel with one another

- Voltage division



resistors in series

KVL: $v = v_1 + v_2 = iR_1 + iR_2 = i(R_1 + R_2)$

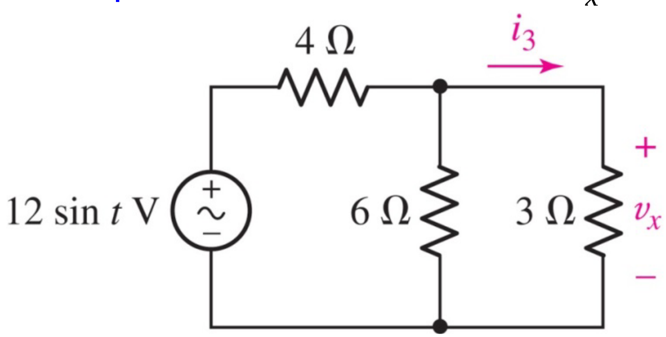
$$\Rightarrow \quad i = \frac{v}{R_1 + R_2}$$

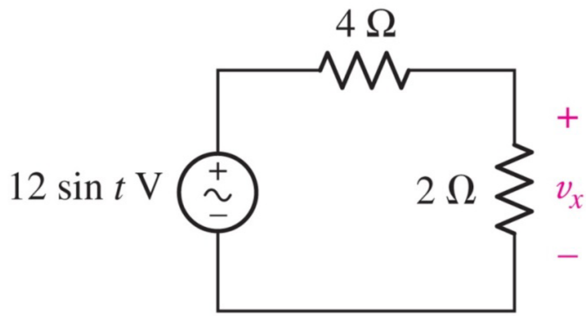
$$\Rightarrow \quad v_1 = iR_1 = \frac{v}{R_1 + R_2}R_1 = \frac{R_1}{R_1 + R_2}v$$

$$v_2 = iR_2 = \frac{v}{R_1 + R_2}R_2 = \frac{R_2}{R_1 + R_2}v$$

$$\boxed{v_k = \frac{R_k}{R_1 + R_2 + \cdots + R_N}v}$$

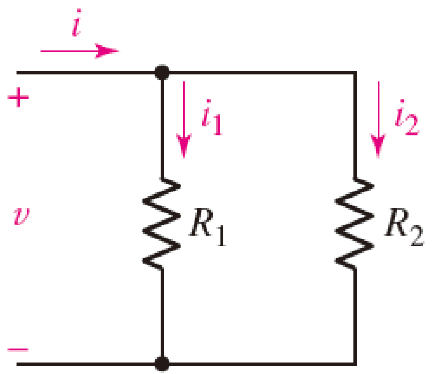
Example 3.13 Determine v_x





$$v_x = \frac{2}{4+2}12\text{sin}t = 4\text{sin}t\ V$$

- Current division



resistors in parallel

$$i = i_1 + i_2 = \frac{v}{R_1} + \frac{v}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) v \Rightarrow v = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} i$$

$$i_1 = \frac{v}{R_1} = \frac{\frac{1}{{}^1/_{R_1} + {}^1/_{R_2}}}{R_1} i = \frac{R_2}{R_1 + R_2} i$$

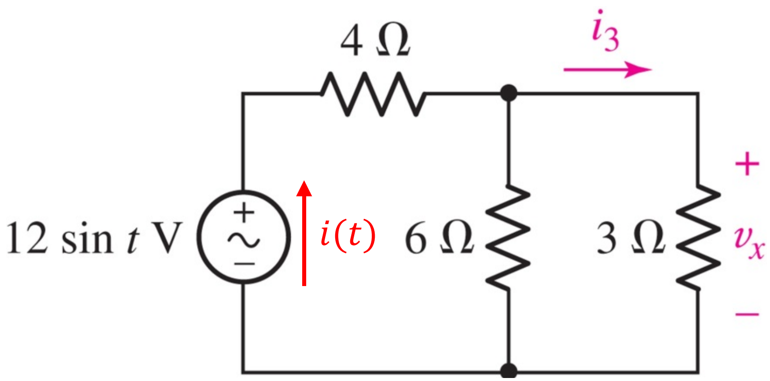
$$i_2 = \frac{v}{R_2} = \frac{\frac{1}{{}^1/_{R_1} + {}^1/_{R_2}}}{R_2} i = \frac{R_1}{R_1 + R_2} i$$

$$\Rightarrow i_k = \frac{\frac{1}{R_k}}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}} i$$

$$i_k = \frac{G_k}{G_1 + G_2 + \cdots + G_N} i$$

$$i_k = \frac{R_1 \| R_2 \| \cdots \| R_N}{R_k} i$$

Example 3.13 Determine i_3

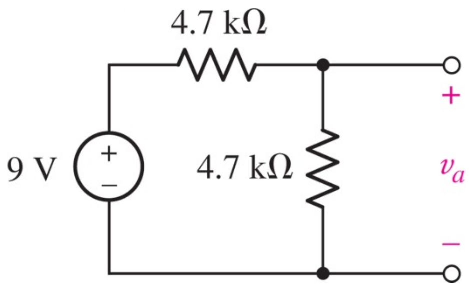


$$R_{total} = 4 + 6||3$$
$$= 4 + \frac{6 \times 3}{6 + 3} = 4 + 2 = 6$$

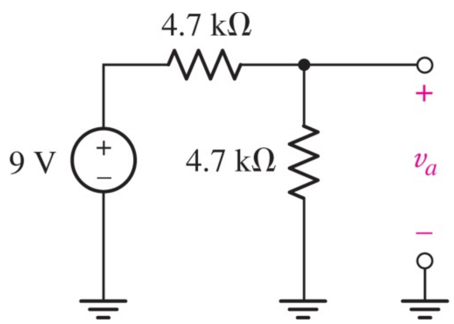
$$i(t) = \frac{12\text{sin}t}{R_{total}}$$
$$= \frac{12\text{sin}t}{6} = 2\text{sin}t$$

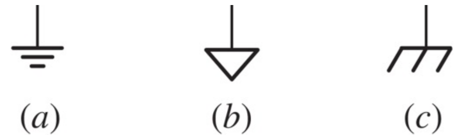
$$\Rightarrow i_3 = \frac{6}{6+3} i(t) = \frac{4}{3}\text{sin}t$$

- Earth ground

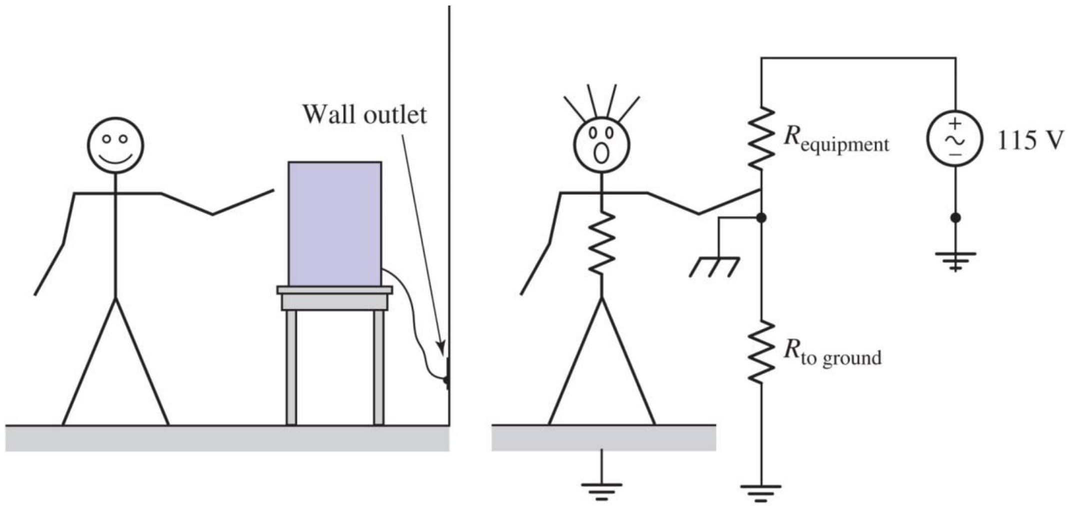


equivalent





(a) Earth ground
(b) Signal ground
(c) Chassis ground



- Big resistance between chassis ground and earth ground due to wiring fault or wear and tear in old building

Homework : 3장 Exercises 4의 배수 문제

- Due day : 3장 수업 끝나고 일주일 후 수업시작 전까지 제출.

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