Chapter 5
Handy Circuit Analysis Techniques

- Linear Elements and Linear Circuits

linear element: a passive element that has a linear voltage-current relationship

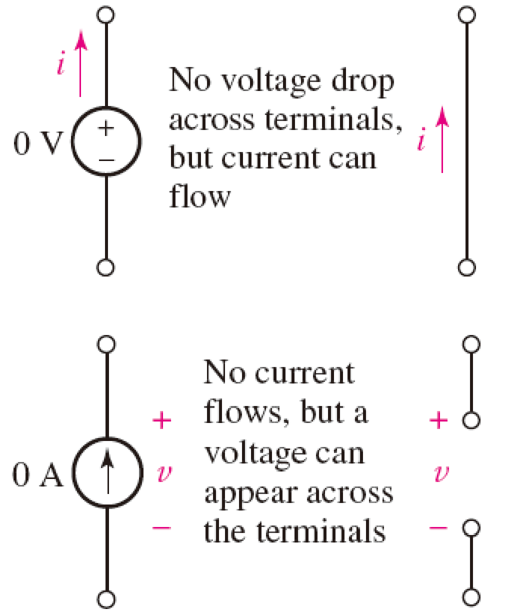
linear dependent source: a dependent source whose output is proportional only to the first power of a specified variable in the circuit (or to the sum of such quantities)

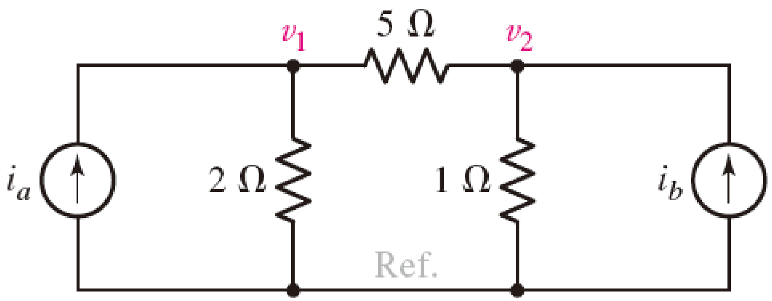
$v_s = 0.5 i_1 + 3 v_2$ vs. $v_s = 0.5 i^2{}_1$

linear circuit: a circuit composed entirely of independent sources, linear dependent sources, and linear elements.

- Superposition Theorem

In any linear resistive network, the voltage across or the current through any resistor or source may be calculated by **adding algebraically all the individual voltages or currents** caused by **the separate independent sources acting alone**, with all other independent sources "turned off" or "zeroed out".

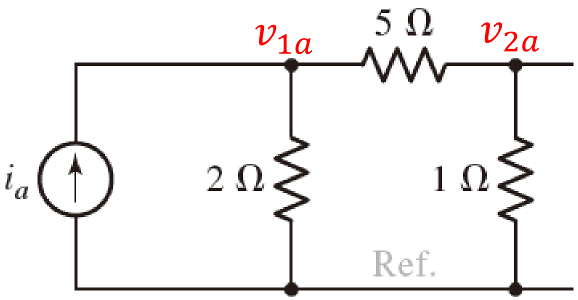




Node 1: $i_a = \frac{v_1}{2} + \frac{v_1 - v_2}{5} = 0.7v_1 - 0.2v_2$

Node 2: $i_b = \frac{v_2}{1} + \frac{v_2 - v_1}{5} = -0.2v_1 + 1.2v_2$

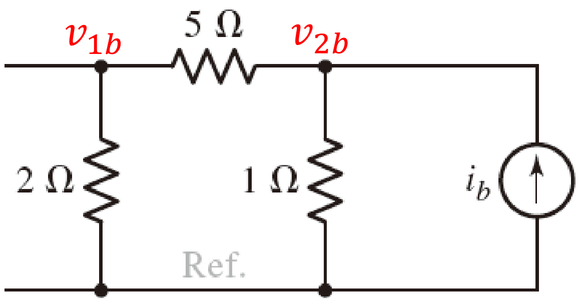
$\Longrightarrow$ $v_1 = 1.5i_a + 0.25i_b$, $v_2 = 0.25i_a + 0.875i_b$



Node 1: $i_a = \frac{v_1}{2} + \frac{v_1 - v_2}{5} = 0.7v_1 - 0.2v_2$

Node 2: $\frac{v_2}{1} = \frac{v_1 - v_2}{5} \rightarrow 0.2v_1 - 1.2v_2 = 0$

$\Longrightarrow$ $v_{1a} = 1.5i_a$, $v_{2a} = 0.25i_a$



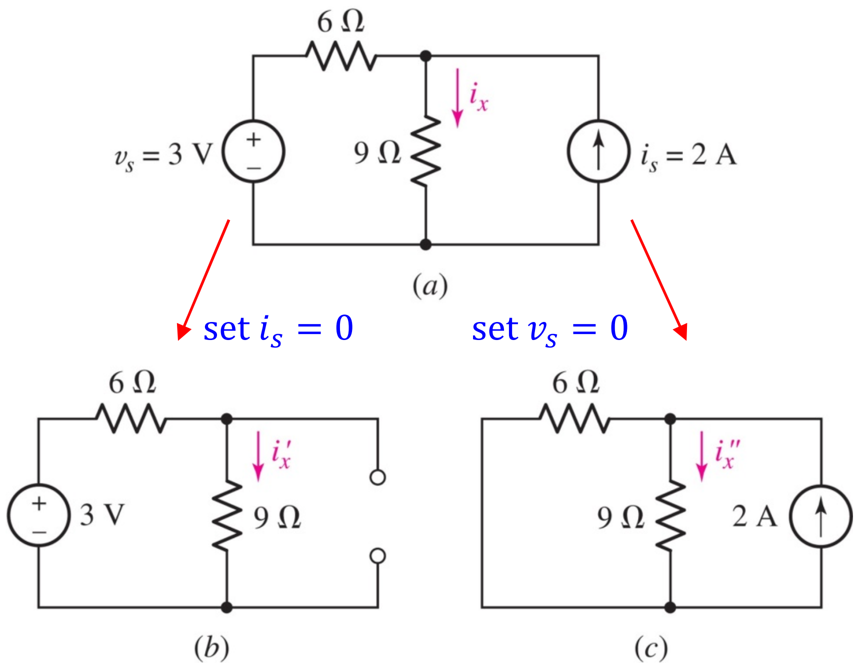
Node 1: $\frac{v_1}{2} = \frac{v_2 - v_1}{5} \rightarrow 0.7v_1 - 0.2v_2 = 0$

Node 2: $i_b = \frac{v_2}{1} + \frac{v_2 - v_1}{5} = -0.2v_1 + 1.2v_2$

$\Longrightarrow$ $v_{1b} = 0.25i_b$, $v_{2b} = 0.875i_b$

$Superposition$: $v_1 = v_{1a} + v_{1b}$, $v_2 = v_{2a} + v_{2b}$

Example 5.1 Use superposition to determine the branch current i_x



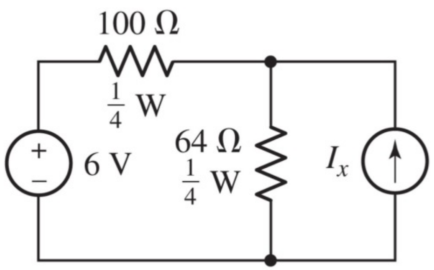
$$(b)\ i_x' = \frac{3}{6+9} = \frac{3}{15}\ A$$

$$(c)\ i_x'' = \frac{6}{6+9} i_s = \frac{6}{15} 2 = \frac{12}{15}\ A$$

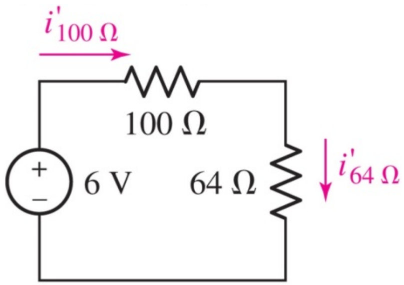
(by current division)

$$\Longrightarrow \quad i_x = i_x' + i_x'' = \frac{3}{15} + \frac{12}{15} = 1\ A$$

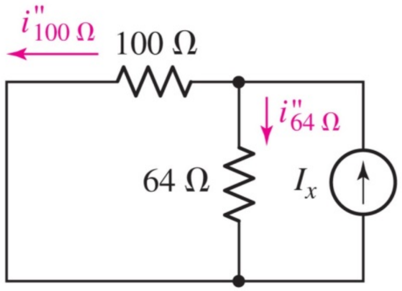
Example 5.2 Determine the maximum *positive* current to which the source I_x can be set.



(a)



(b)



(c)

$$P = i^2 R \rightarrow i^2 = \frac{P}{R}$$

$$\Rightarrow i_{\max(100\Omega)} = \sqrt{\frac{P_{max}}{R}} = \sqrt{\frac{1/4}{100}} = 50\ mA$$

$$\Rightarrow i_{\max(64\Omega)} = \sqrt{\frac{P_{max}}{R}} = \sqrt{\frac{1/4}{64}} = 62.5\ mA$$

$$(b)\ \ i'_{100\Omega} = i'_{64\Omega} = \frac{6}{100+64} = \frac{6}{164} mA$$

$$(c)\ \ i''_{100\Omega} = \frac{64}{100+64} I_x,$$

$$i''_{64\Omega} = \frac{100}{100+64} I_x$$

$$(a)\ \ i_{100\Omega} = -\frac{6}{164} + \frac{64}{164} I_x,$$

positive I_x

$$i_{64\Omega} = \frac{6}{164} + \frac{100}{164} I_x$$

$$i_{100\Omega} = -\frac{6}{164} + \frac{64}{164} I_x < 50 \times 10^{-3}$$

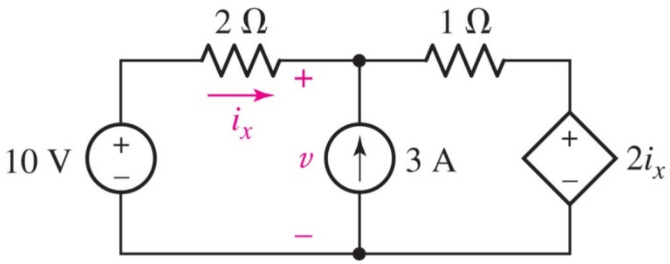
$$\Rightarrow I_x < (86.59 \times 10^{-3}) \left(\frac{164}{64}\right) = 222 mA$$

$$i_{64\Omega} = \frac{6}{164} + \frac{100}{164} I_x < 62.5 \times 10^{-3}$$

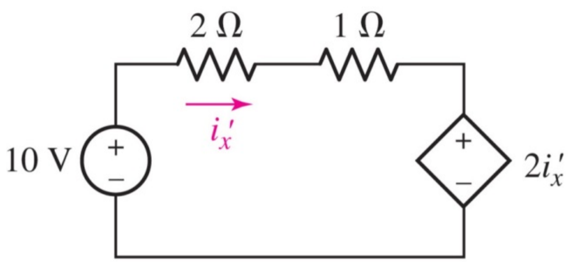
$$\Rightarrow I_x < (25.91 \times 10^{-3}) \left(\frac{164}{100}\right) = 42.5 mA$$

$$I_{x\_max} = 42.5\ mA$$

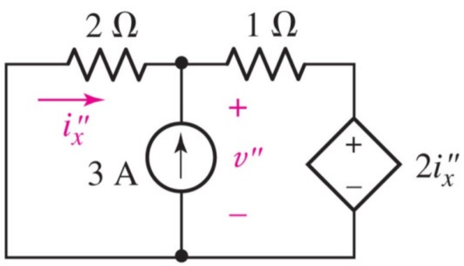
Example 5.3 Use the superposition principle to determine the current i_x



(a)



(b)



(c)

$$(b)\quad -10 + 2i_x' + i_x' + 2i_x' = 0$$

$$\rightarrow i_x' = 2\ A$$

$$(c)\quad \frac{0 - v''}{2} + 3 + \frac{2i_x'' - v''}{1} = 0$$

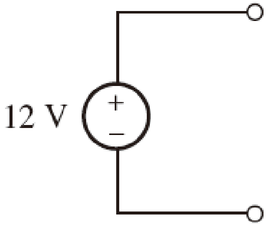
$$\rightarrow i_x'' + 3 + 2i_x'' + 2i_x'' = 0$$

$$\rightarrow i_x'' = -0.6\ A \qquad \boxed{v'' = -2i_x''}$$

$$\therefore i_x = i_x' + i_x'' = 2 - 0.6 = 1.4\ A$$

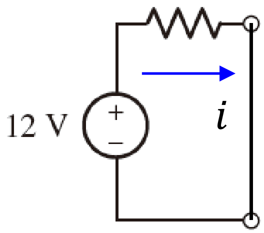
- Practical Voltage Sources

ideal 12 *V* voltage source



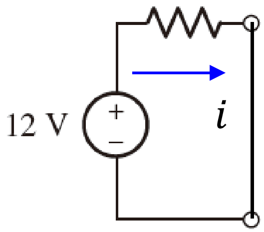
independent of the current flowing through them

$1\ \Omega$



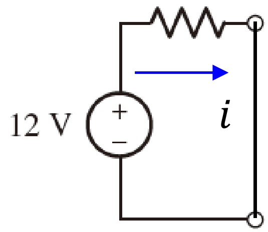
$$i = \frac{12}{1} = 12\ A$$

$1\ m\Omega$



$$i = \frac{12}{10^{-3}} = 12{,}000\ A$$

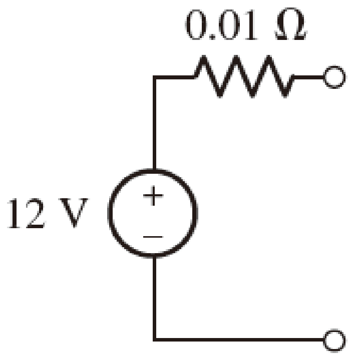
$0\ \Omega$

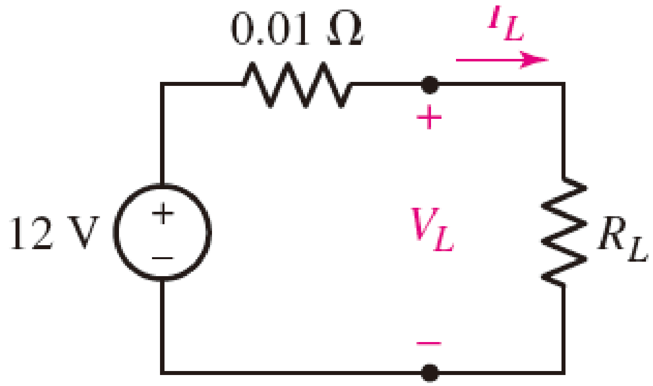


What happens?

$i = \infty$?

practical 12 *V* voltage source

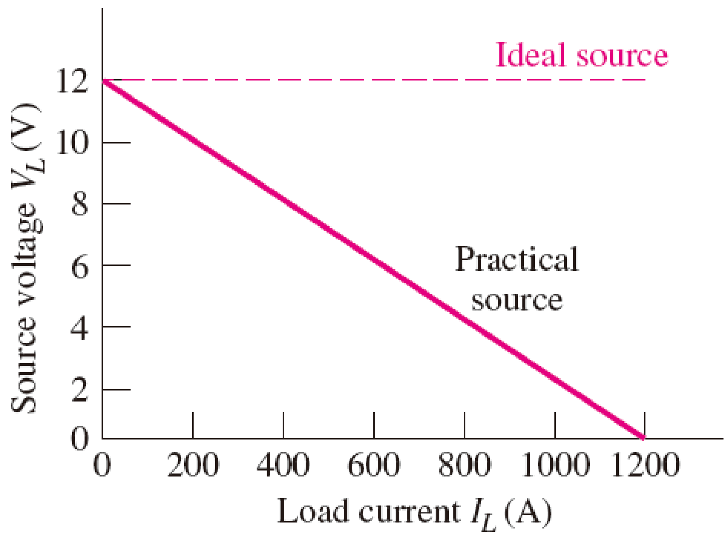




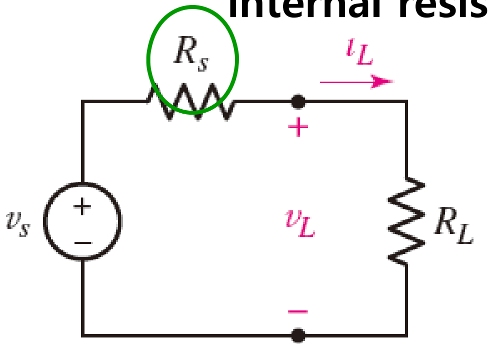
$$-12 + 0.01I_L + V_L = 0$$

$$\rightarrow V_L = -0.01I_L + 12$$

$$\rightarrow V_L = -0.01\frac{12}{R_L + 0.01} + 12$$



internal resistance



$$-v_s + i_L R_S + v_L(= i_L\ R_L) = 0$$

$$\rightarrow v_L = v_s - i_L R_S$$

$$\rightarrow i_L = \frac{v_S - v_L}{R_S}$$

$R_L = \infty$ (open circuit)

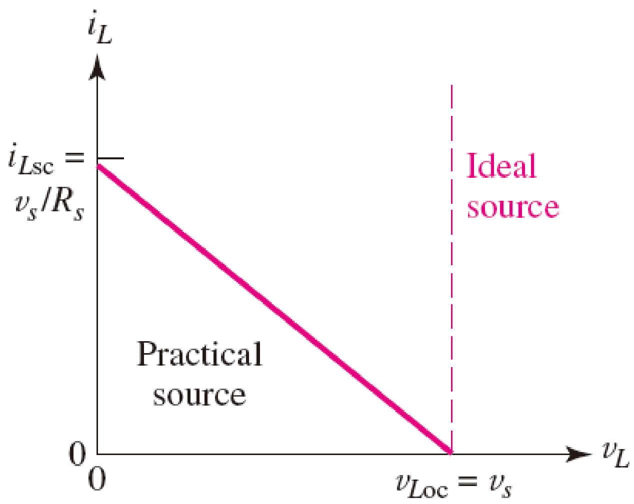
⇨ $i_L = 0$

$v_{Loc} = v_s$ **open circuit voltage**

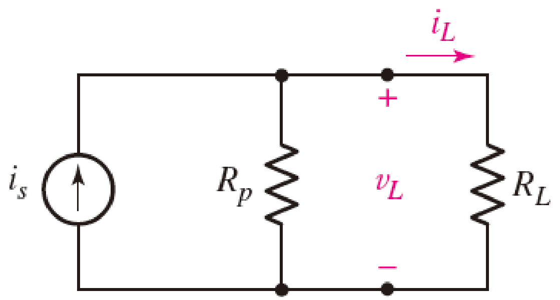
$R_L = 0$ (short circuit)

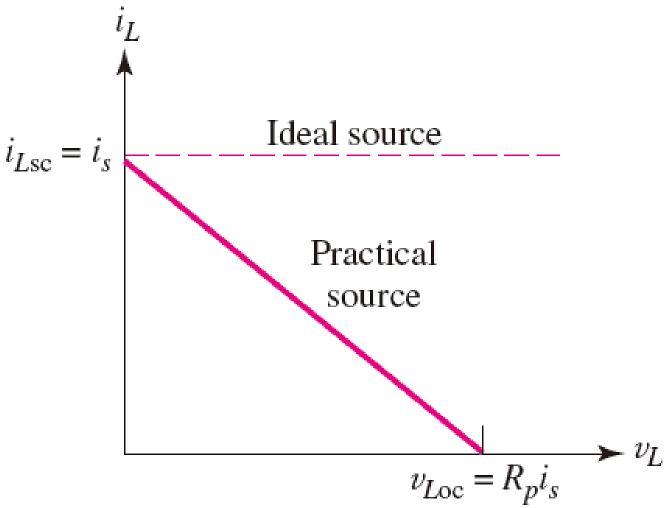
⇨ $v_L = 0$

$i_{Lsc} = \frac{v_s}{R_s}$ **short circuit current**



- Practical Current Sources





$\Rightarrow$ $i_L = i_s - \frac{v_L}{R_p}$

$R_L = \infty$ (open circuit)

$\Rightarrow$ $i_L = 0$

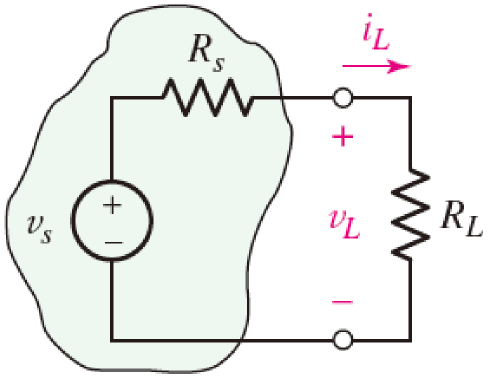
$v_{Loc} = i_s R_P$ **open circuit voltage**

$R_L = 0$ (short circuit)

$\Rightarrow$ $v_L = 0$

$i_{Lsc} = i_s$ **short circuit current**

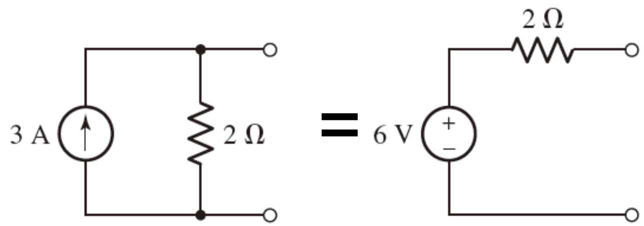
- Equivalent Practical Sources



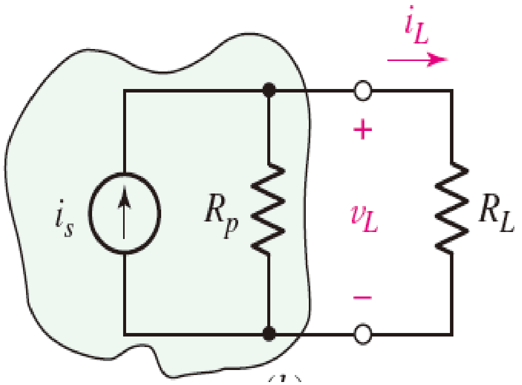
$$v_L = \frac{R_L}{R_s + R_L} v_s$$

equivalent if

$$R_s = R_p$$
$$v_s = i_s R_p = i_s R_s$$

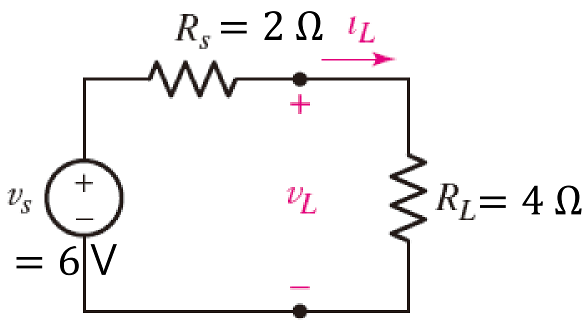


$$6 = 3A \times 2\Omega$$



$$\begin{aligned} v_L &= i_L R_L \\ &= \left(\frac{R_p}{R_p + R_L}\right) i_s R_L \\ &= \frac{R_L}{R_p + R_L}(i_s R_p) \end{aligned}$$

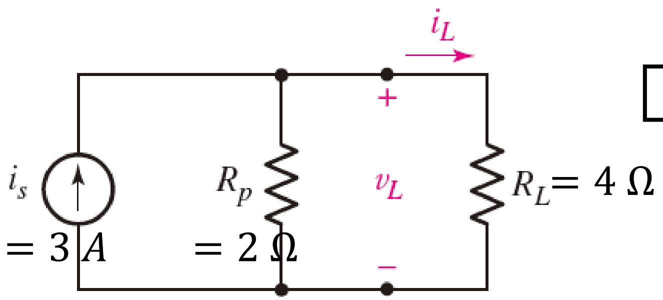
- Equivalent Practical Sources



$$v_L = \frac{4}{2+4}6 = 4\,V$$
$$i_L = \frac{6}{2+4} = 1\,A$$
$$p_{R_L} = 4\,W$$

$$p_{v_S} = v_s i_L = 6 \times 1 = 6\,W$$
$$p_{R_S} = 2 \times 1^2 = 2\,W$$

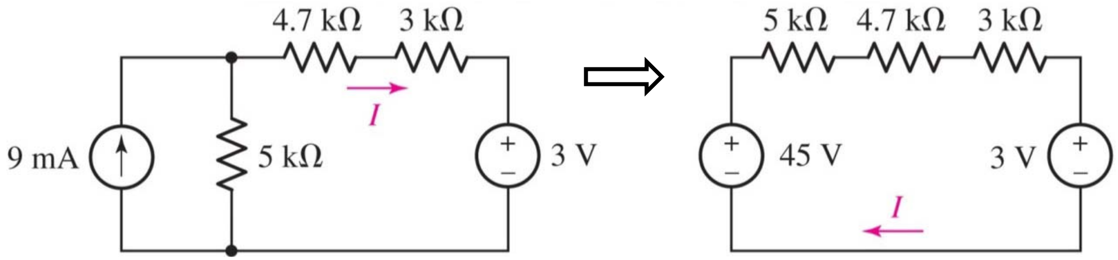
Equivalent at load



$$v_L = \frac{2}{2+4}(3 \times 4) = 4\,V$$
$$i_L = \frac{2}{2+4}3 = 1\,A$$
$$p_{R_L} = 4\,W$$

$$p_{i_S} = v_L i_s = 4 \times 3 = 12\,W$$
$$p_{R_p} = (3-1) \times 2^2 = 8\,W$$

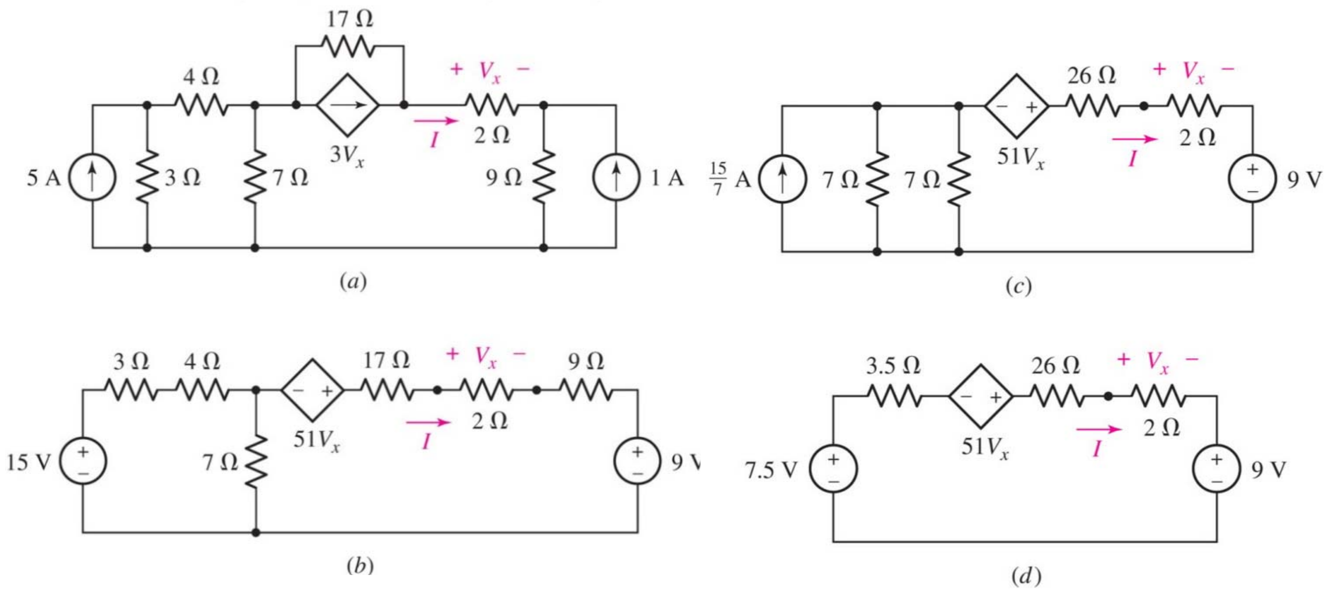
Example 5.4 Compute the current through the 4.7 kΩ resistor



$$-45 + (5k + 4.7k + 3k)I + 3 = 0$$

$$\rightarrow I = \frac{42}{5k + 4.7k + 3k} = 3.307\ mA$$

Example 5.5 Calculate the current through the 2 Ω resistor



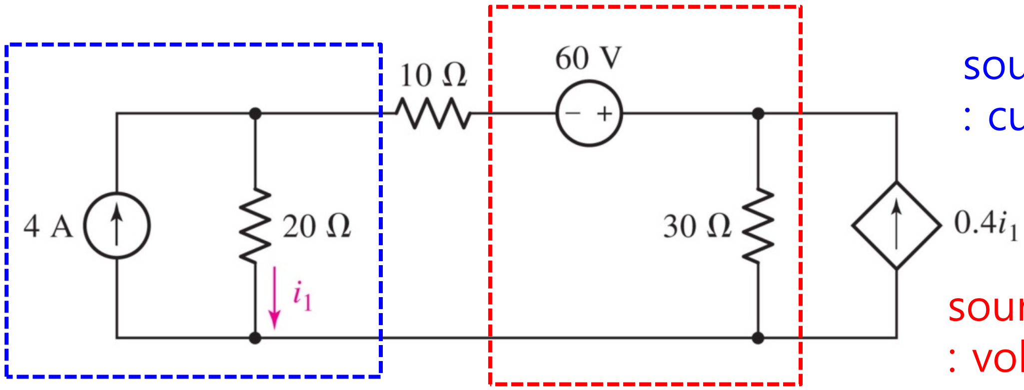
$$-7.5 + 3.5I - 51V_x + 26I + 2I + 9 = 0$$

$$(V_x = 2I)$$

$$\Rightarrow 31.5I - 51(2I) = -1.5$$

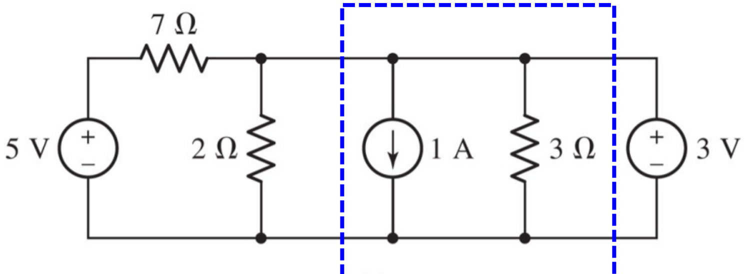
$$\rightarrow I = \frac{1.5}{102 - 31.5}$$

$$= 21.28\ mA$$

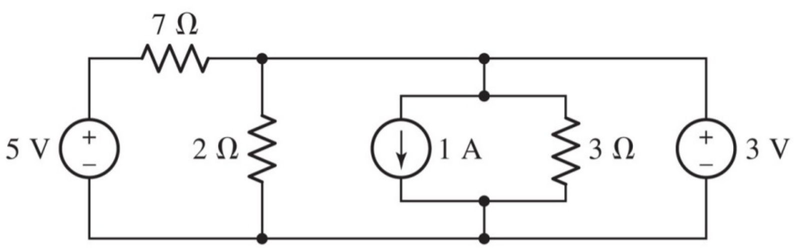


source transformation: OK!
: current source and resistor are in parallel

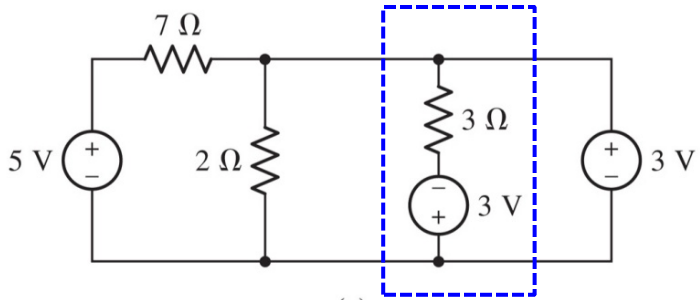
source transformation: invalid!
: voltage source and resistor are NOT in series



(a)

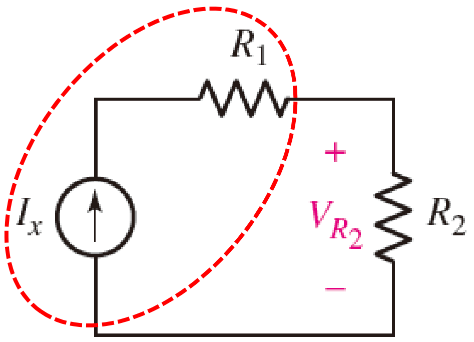


(b)



(c)

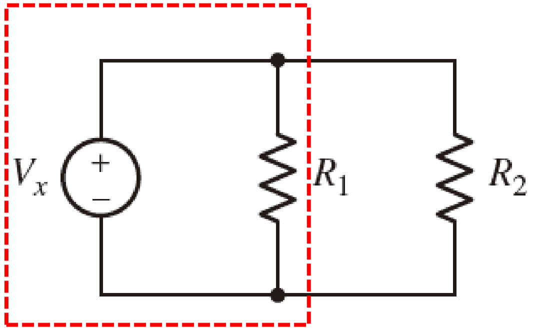
- Unusual Cases



Current source and resistor are in series

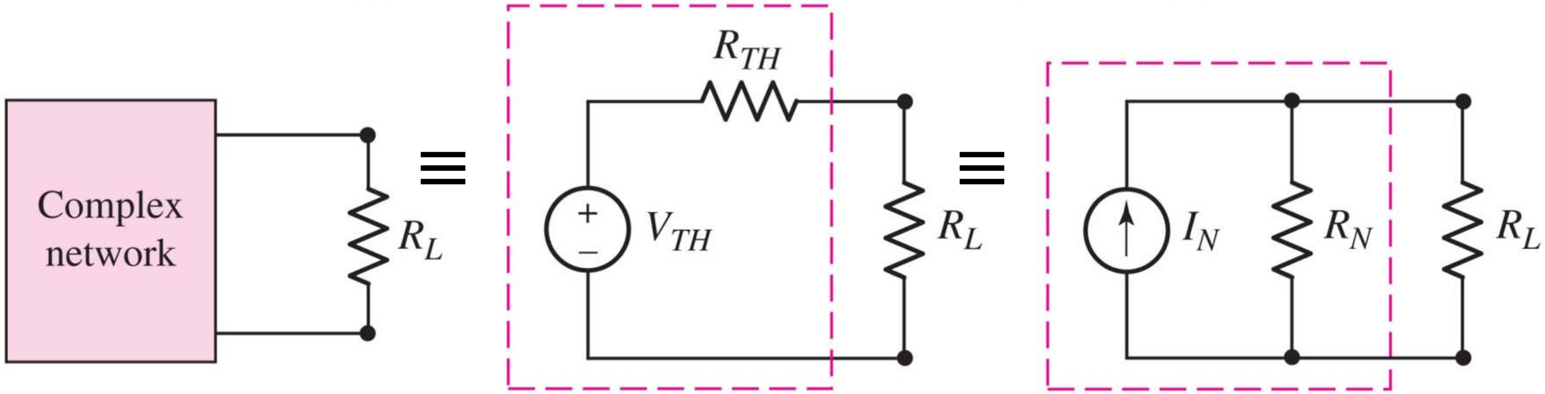
Regardless of R_1 value, may omit R_1

$V_{R_2} = I_x R_2$



Voltage source and resistor are in parallel

R_1 does not alter the voltage across, the current through, or the power dissipated by R_2

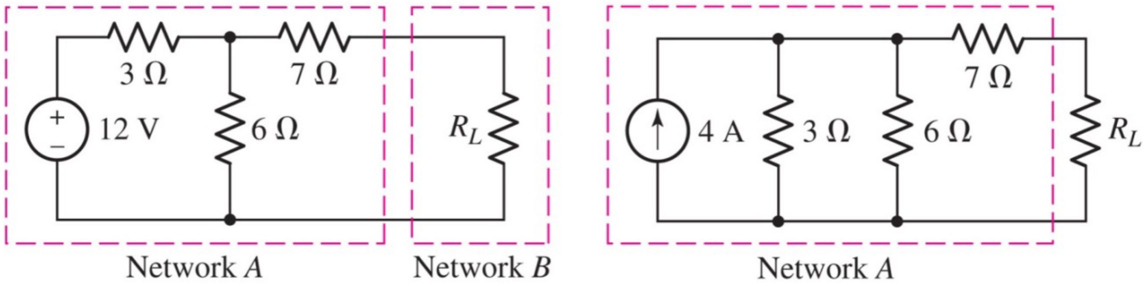


Thévenin equivalent network

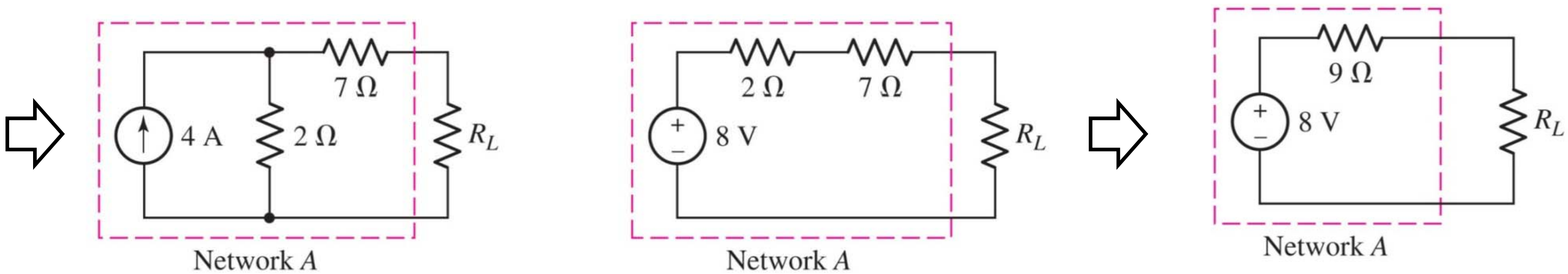
Norton equivalent network

$$R_{TH} = R_N$$

$$V_{TH} = I_N R_N$$



Voltage, current, and power at the load is maintained !!!



- Thévenin's Theorem

1. Given any linear circuit, rearrange it in the form of two networks, A and B, connected by two wires. A is the network to be simplified; B will be left untouched.
2. Disconnect network B. Define a voltage V_{oc} as the voltage now appearing across the terminals of network A.
3. Turn off or "zero out" every independent source in the network A to form an inactive network. Leave dependent sources unchanged.
4. Connect an independent voltage source with value V_{oc} in series with the inactive network. Do now complete the circuit; leave the two terminals disconnected (open).
5. Connect network B to the terminals of the new network A. All currents and voltages in B will remain unchanged.
6. The only restriction that we must impose on A or B is that all dependent sources in A have their control variables in A, and similarly for B
7. The inactive network A can be represented by a single equivalent resistance R_{TH}, which we will call the *Thévenin equivalent resistance*.

- Norton's Theorem

1. Given any linear circuit, rearrange it in the form of two networks, A and B, connected by two wires. Network A is the network to be simplified; B will be left untouched. As before, if either network contains a dependent source, *its controlling variable must be in the same network*

2. Disconnect network B and short the terminals of A. Define a current i_{sc} as the current now flowing through the shorted terminals of network A.

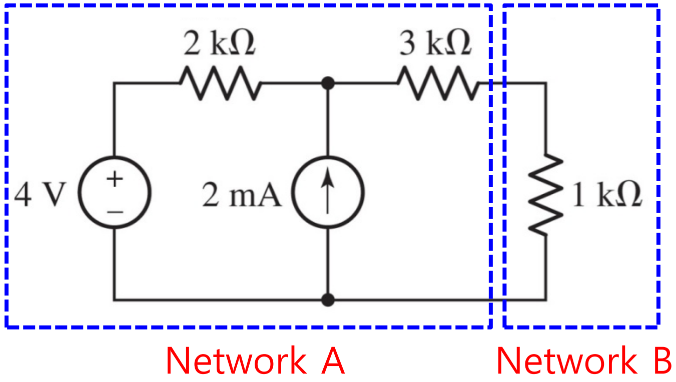
3. Turn off or "zero out" every independent source in the network A to form an inactive network. Leave dependent sources unchanged.

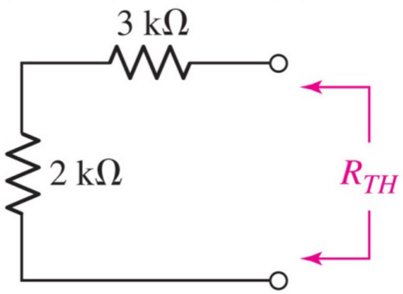
4. Connect an independent voltage source with value i_{sc} in parallel with the inactive network. Do now complete the circuit; leave the two terminals disconnected.

5. Connect network B to the terminals of the new network A. All currents and voltages in B will remain unchanged.

$$v_{oc} = R_{TH} i_{sc}$$

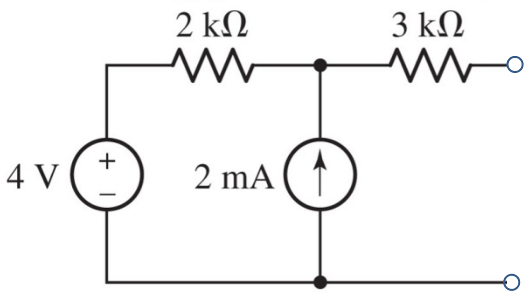
Example 5.8 Find the Thévenin and Norton equivalent circuits, faced by the 1 kΩ resistor.





Zero-out the independent sources in network A.

$$R_{TH} = 3 + 2 = 5\ \Omega$$

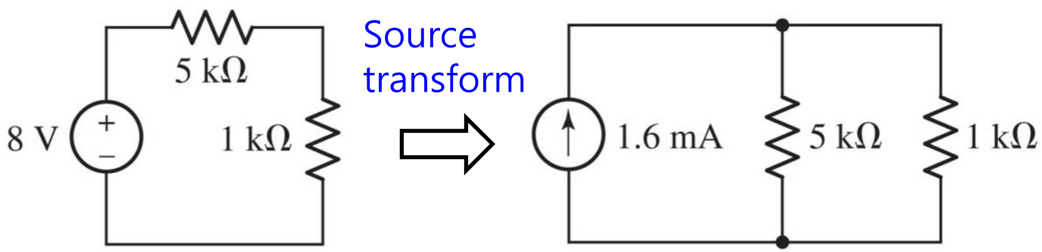


$$V_{OC} = V_{OC|4V} + V_{OC|2mA}$$

$$V_{OC|4V} = 4\ V$$

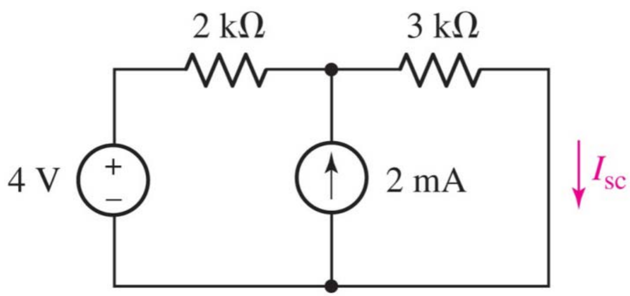
$$V_{OC|2mA} = (2 \times 10^{-3})(2 \times 10^{3}) = 4\ V$$

$$\rightarrow V_{OC} = V_{OC|4V} + V_{OC|2mA} = 4 + 4 = 8\ V$$



Thévenin equivalent network

Norton equivalent network



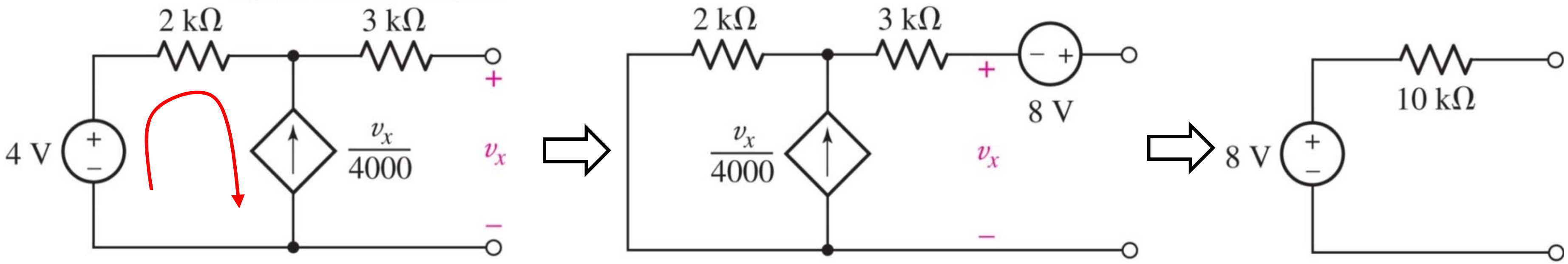
$$I_{SC} = I_{SC|4V} + I_{SC|2mA}$$

$$= \frac{4}{2+3} + \frac{2}{2+3} 2 = \frac{8}{5}\ A$$

- When Dependent sources are present

  - Controlling variable and its associated element(s) should be in the same network.

Example 5.9 Determine the Thévenin equivalent circuits



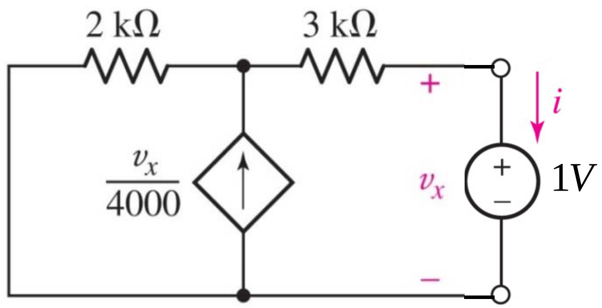
$$-4 + 2 \times 10^3 \left(-\frac{v_x}{4000}\right) + v_x = 0$$

$$\rightarrow v_x = v_{OC} = V_{TH} = 8\ V$$

Useless form of Thévenin circuit

$$v_{oc} = R_{TH} i_{sc}$$

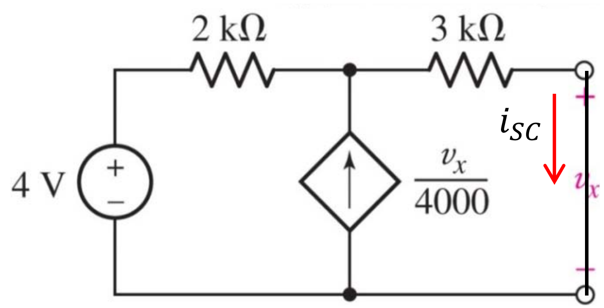
- Finding R_{TH}



$$v_x = 1\ V$$

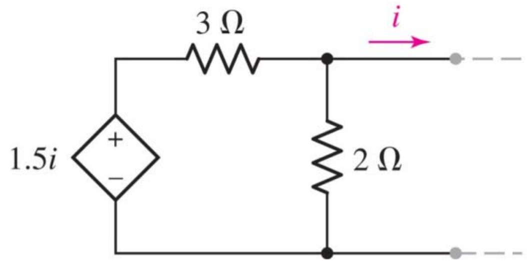
$$i = \frac{1}{4000} \times \frac{2}{3+2} = 0.1\ mA$$

$$R_{TH} = \frac{1\ V}{i} = 10\ k\Omega$$

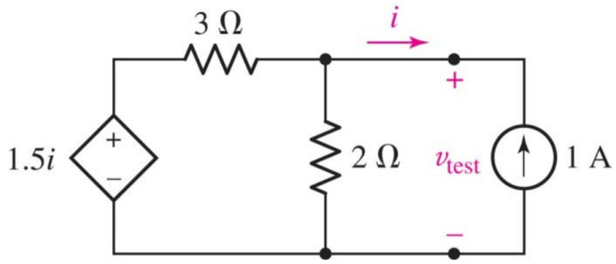


$$i_{SC} = \frac{4}{5k} = 0.8\ mA$$

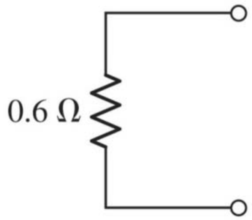
Example 5.10 Determine the Thévenin equivalent circuits (No independent source)



$$v_{oc} = 0$$
$$i_{sc} = 0$$
$$\Longrightarrow R_{TH} = \frac{0}{0} : Too\ much\ answer$$

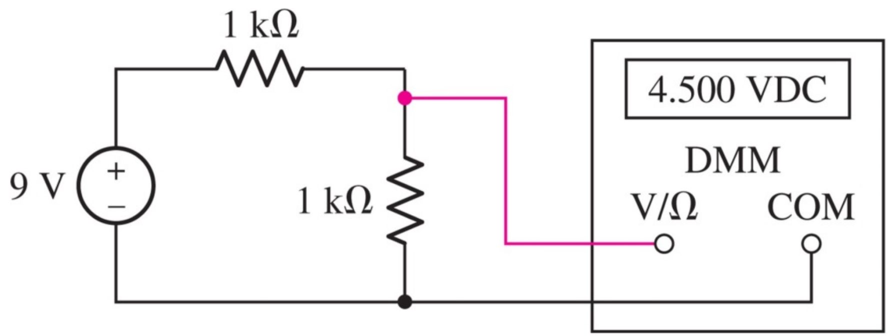


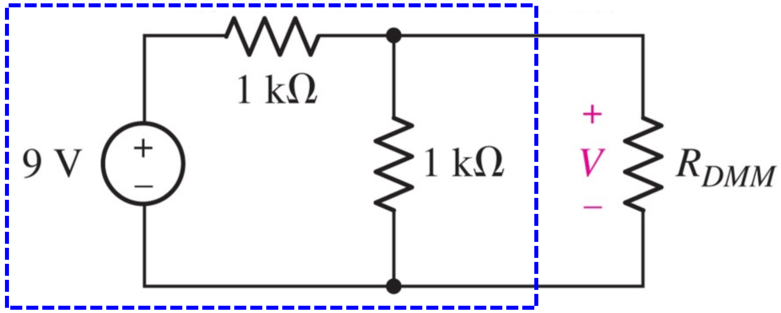
$$1 = \frac{v_{test}}{2} + \frac{v_{test} - 1.5i}{3}$$
$$\rightarrow 6 = 3v_{test} + 2v_{test} - 1.5 \times (-1) \times 2$$
$$\rightarrow v_{test} = \frac{3}{5}\ V$$



$$R_{TH} = \frac{v_{test}}{1\ A} = 0.6\Omega$$

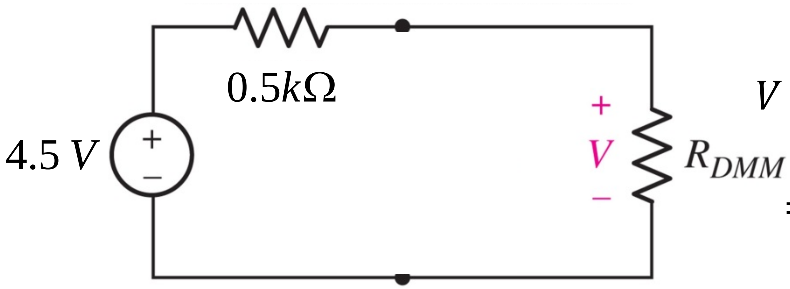
- Digital Multimeter: Voltage





$$R_{TH} = 1||1 = 500\ \Omega$$

$$V_{OC} = \frac{1}{1+1}9 = 4.5\ V$$



$$V = \frac{R_{DMM}}{R_{TH} + R_{DMM}} V_{OC}$$

$$= \frac{10 \times 10^6}{500 + 10 \times 10^6} 4.5 = 4.4998\ V$$

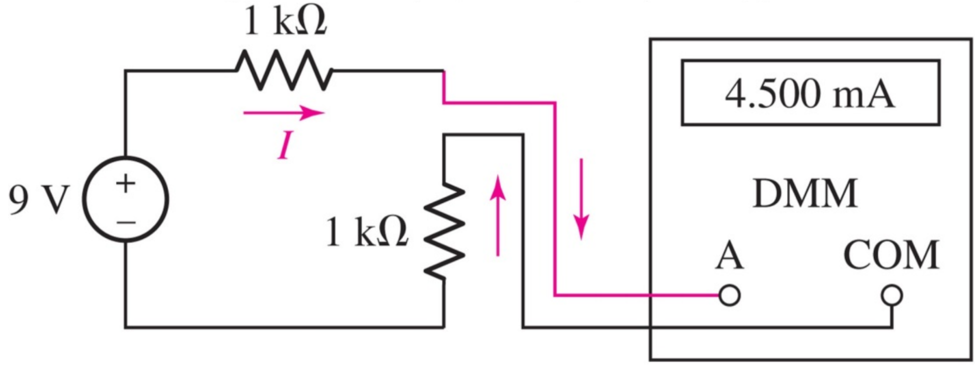
- Digital Multimeter: Current

$$-9 + 1000I + R_{DMM}I + 1000I = 0$$

$$\rightarrow I = \frac{9}{2000 + R_{DMM}}$$

$$R_{DMM} = 0.1\,\Omega \quad \rightarrow \quad I = 4.4998\,mA$$

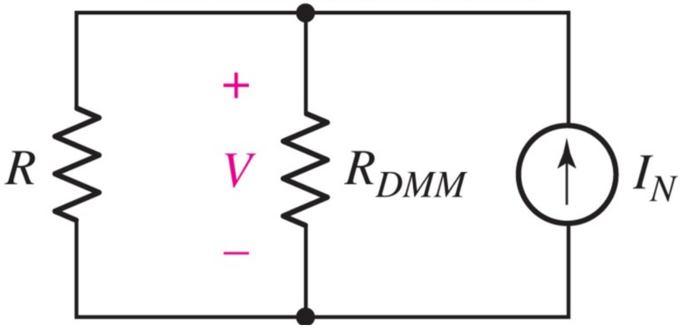
Less than 4.5 mA (ideal value)



Ideal DMM
$R_{DMM} = 0$ for current measurment
$R_{DMM} = \infty$ for voltage measurmetn

- Digital Multimeter: Resistance

DMM actually measure $R||R_{DMM}$

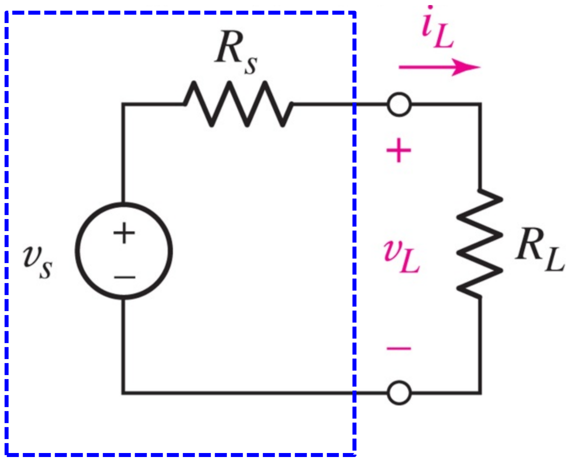


$$V = (R||R_{DMM})I_N$$

$R_{DMM} = 10\ M\Omega$
$R = 10\Omega$
$\rightarrow R_{DMM}||R = 9.9999\Omega$

$R_{DMM} = 10\ M\Omega$
$R = 10\ M\Omega$
$\rightarrow R_{DMM}||R = 5\ M\Omega$

- An independent voltage source in series with a resistance R_s, or an independent current source in parallel with a resistance R_s, delivers maximum power to a load resistance R_L such that $R_L = R_s$.



$$i_L = \frac{v_s}{R_s + R_L}$$

$$p_L = i_L^2 R_L = \frac{R_L v_s^2}{(R_s + R_L)^2}$$

⇨ Minimum power transfer

$R_L = 0 \rightarrow p_L = 0$,
$R_L = \infty \rightarrow p_L = 0$

$$\frac{dp_L}{dR_L} = \frac{(R_s + R_L)^2 v_s^2 - 2v_s^2 R_L (R_s + R_L)}{(R_s + R_L)^4} = 0$$

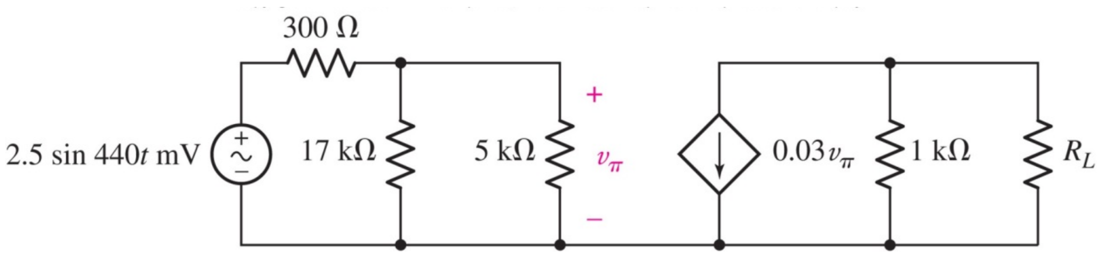
$$\Rightarrow 2R_L(R_s + R_L) = (R_s + R_L)^2$$

$R_{TH} = R_s$
$V_{TH} = v_s$

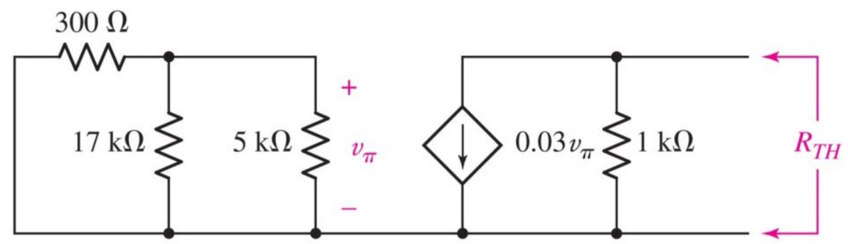
$\Rightarrow R_L = R_s$ Condition for maximum power transfer

$$p_L = \frac{R_s v_s^2}{(R_s + R_s)^2} = \frac{v_s^2}{4R_s} = \frac{v_{TH}^2}{4R_{TH}}$$

Example 5.11 Choose a load resistance so that maximum power is transferred.



$R_{TH} = 1\ k\Omega$

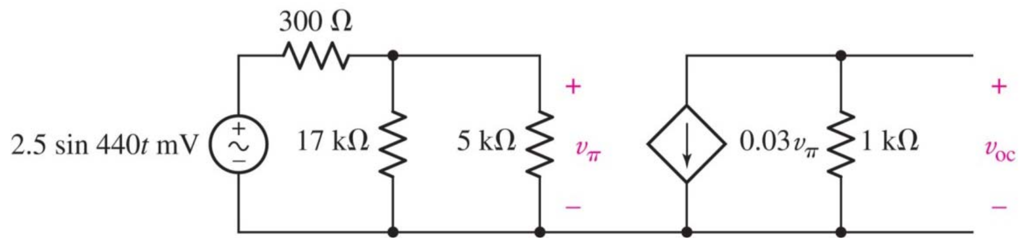


(a)

$v_{oc} = 1k \times (-0.03v_\pi) = -30v_\pi$

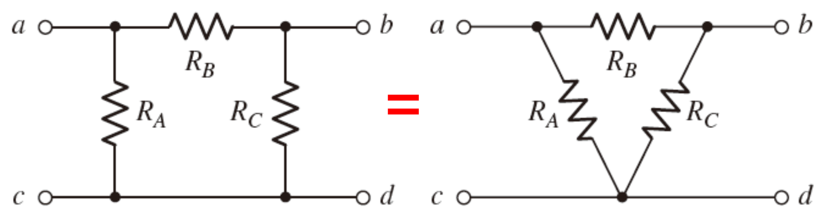
$v_\pi = \frac{17||5}{0.3+17||5}(2.5sin440t)\ mV$

$\Rightarrow v_{oc} = -69.6\sin 440t\ \ mV$

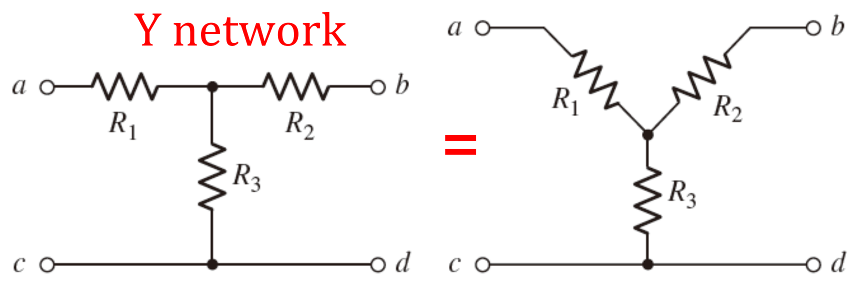


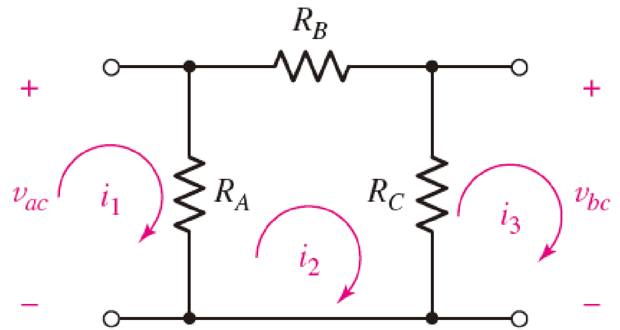
$$p_{max} = \frac{v_{oc}^2}{4R_{TH}}$$
$$= 1.211\sin^2 440t\ \mu W$$

Δ network

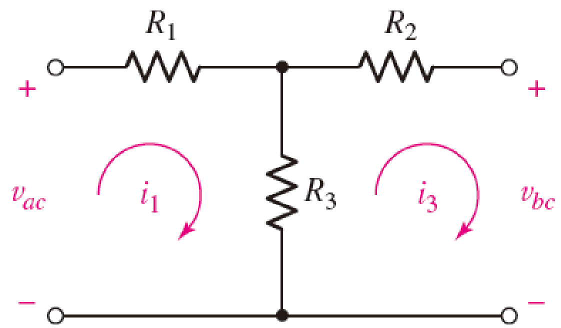


Y network





v_{ac}, v_{bc}, i_1, and i_3 are same in both networks



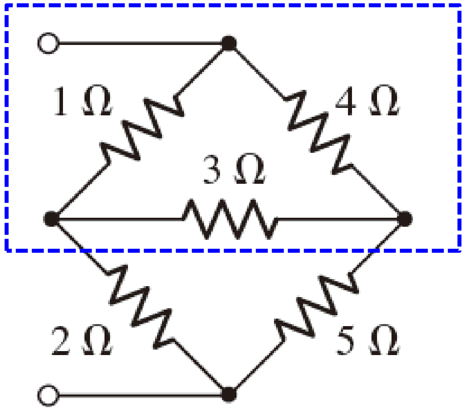
$$v_{ac} = R_A(i_1 - i_2)$$
$$R_A(i_2 - i_1) + R_B i_2 + R_C(i_2 - i_3) = 0$$
$$-v_{bc} = R_C(i_3 - i_2)$$

$$v_{ac} = R_1 i_1 + R_3(i_1 - i_3)$$
$$-v_{bc} = R_2 i_3 - R_3(i_3 - i_1)$$

$$R_A = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$
$$R_B = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$
$$R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_1 = \frac{R_AR_B}{R_A + R_B + R_C}$$
$$R_2 = \frac{R_BR_C}{R_A + R_B + R_C}$$
$$R_3 = \frac{R_CR_A}{R_A + R_B + R_C}$$

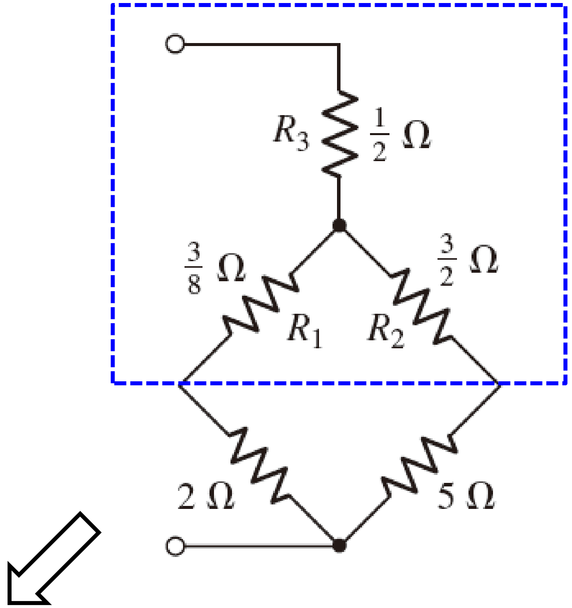
Example 5.12 Find the Thévenin equivalent circuits (using Δ-Y conversion)

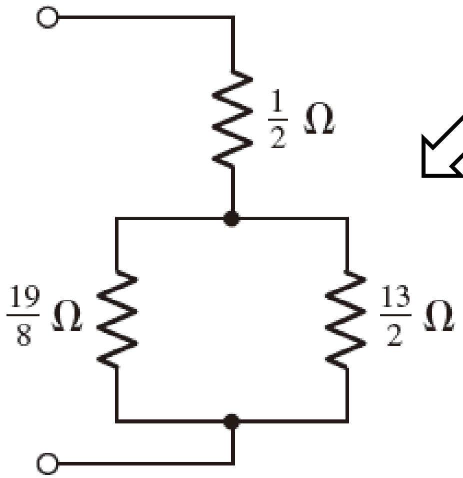


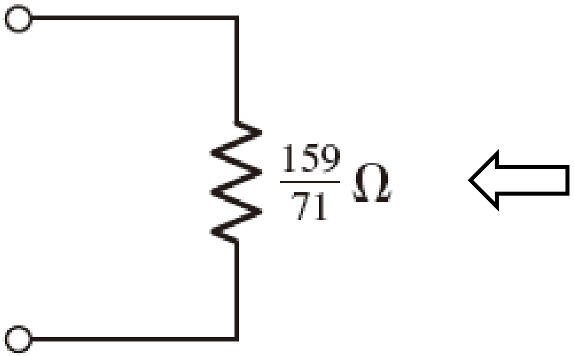
$$R_1 = \frac{1 \times 3}{1+3+4} = \frac{3}{8}$$

$$R_2 = \frac{3 \times 4}{1+3+4} = \frac{12}{8}$$

$$R_3 = \frac{4 \times 1}{1+3+4} = \frac{4}{8}$$







Homework : 5장 Exercises 5의 배수 문제 (70번 문제까지)

- Due day : 5장 수업 끝나고 일주일 후 수업시작 전까지 제출.

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