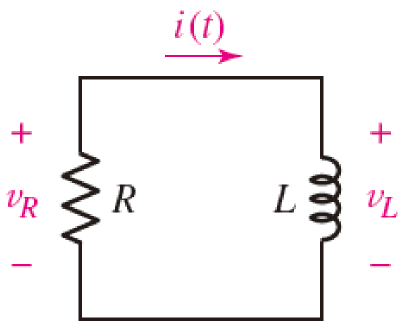
Chapter 8
Basic *RL* and *RC* Circuits

- Natural response vs. Forced response

Natural Response : depends upon the general “nature” of circuit (the type of elements, their sizes, the interconnection of the elements)
Source-free response, transient response, complementary function

Forced Response : resemble the nature of the particular source (or forcing function)
Particular solution, steady-state response,



$0 \le t' \le t$

$$Ri + L\frac{di}{dt} = 0 \;\rightarrow\; \frac{di}{dt} + \frac{R}{L}i = 0$$

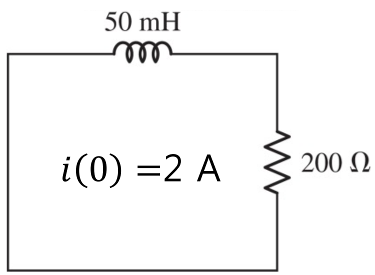
$$\Rightarrow \frac{1}{i}di = -\frac{R}{L}dt \;\Rightarrow\; \int_{I_0}^{i(t)} \frac{1}{i'}di' = -\frac{R}{L}\int_0^t dt' \longleftarrow \quad i(0) = I_0$$

$$\Rightarrow \ln i'\Big|_{I_0}^{i} = -\frac{R}{L}t\Big|_0^t \;\Rightarrow\; \ln i - \ln I_0 = -\frac{R}{L}(t-0)$$

$$\Rightarrow \ln\frac{i}{I_0} = -\frac{R}{L}t \;\Rightarrow\; i(t) = I_0 e^{-\frac{R}{L}t}$$

Energy is stored
in the inductor at $t = 0$

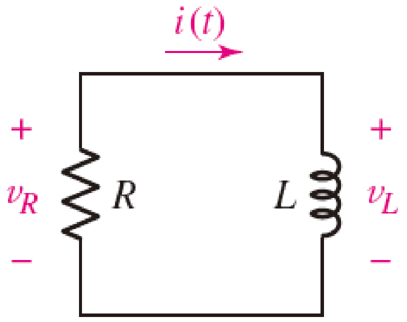
Example 8.1



$$i(t) = 2e^{-4000t}$$

$$0 \le t$$

- Alternative Approach



$$\int \frac{di}{i} = -\int \frac{R}{L} dt + K$$

$$\Rightarrow \ln i = -\frac{R}{L} t + K$$

$$i(t=0) = I_0 \quad \Longrightarrow$$

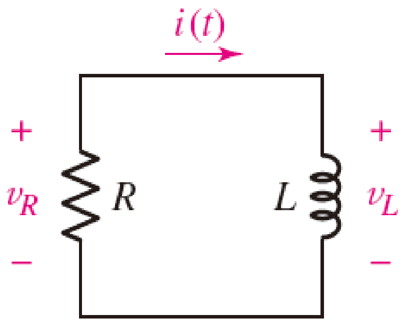
$$\ln I_0 = K$$

$$\rightarrow \ln i = -\frac{R}{L} t + \ln I_0$$

$$\Rightarrow \ln \frac{i}{I_0} = -\frac{R}{L} t$$

$$\Rightarrow i(t) = I_0 e^{-\frac{R}{L} t}$$

- A More General Solution Approach



$$\text{Let } i(t) = Ae^{s_1 t}$$

$$Ri + L\frac{di}{dt} = 0 \quad \rightarrow \quad RAe^{s_1 t} + LAs_1 e^{s_1 t} = 0$$

$$\rightarrow (R + Ls_1)Ae^{s_1 t} = 0 \quad \Rightarrow \quad s_1 = -\frac{R}{L}$$

$$\therefore i(t) = Ae^{s_1 t} = Ae^{-\frac{R}{L} t} \qquad i(0) = Ae^{-\frac{R}{L} 0} = A = I_0$$

$$\Rightarrow i(t) = I_0 e^{-\frac{R}{L} t}$$

- A Direct Route: The Characteristic Equation

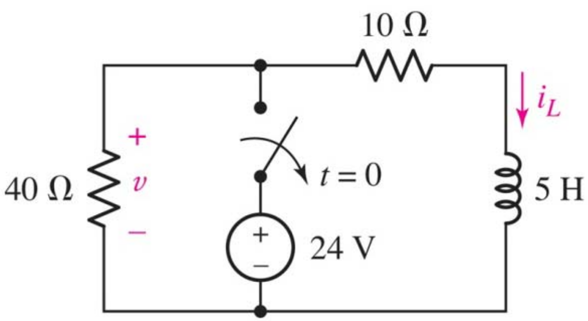
Consider $a\dfrac{df}{dt} + bf = 0$

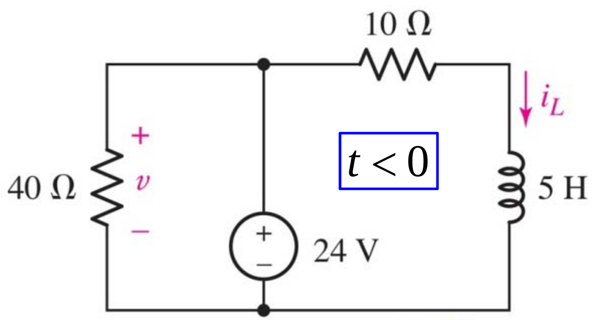
Assume $f(t) = Ae^{st}$

$$a\frac{df}{dt} + bf = as + b = 0$$
$$\rightarrow s = -\frac{b}{a}$$
$$\Rightarrow f = Ae^{-\frac{b}{a}t}$$

$$\boxed{\begin{aligned} & f \rightarrow s^0 = 1, \\ & \frac{df}{dt} \rightarrow s^1, \\ & \frac{d^2f}{dt^2} \rightarrow s^2, \\ & \quad \vdots \\ & \frac{d^nf}{dt^n} \rightarrow s^n \end{aligned}}$$

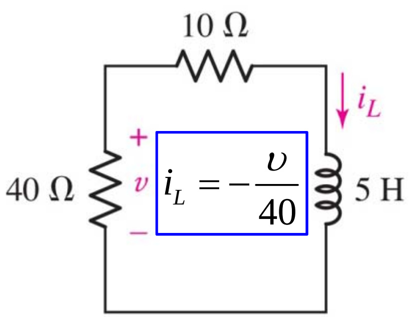
Example 8.2 Find the voltage v at t=200ms





$$i_L(0) = \frac{24}{10} = 2.4\ A$$

$$\rightarrow v(0) = -40 i_L(0) = -96\ V$$



$t \geq 0$

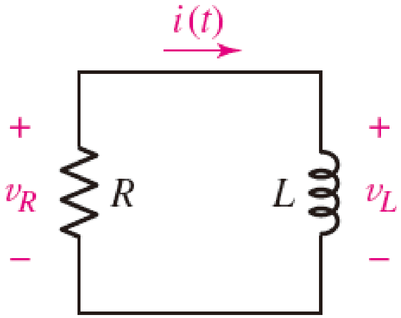
$$-v + 10 i_L + 5\frac{di_L}{dt} = 0$$
$$-v + 10\left(-\frac{v}{40}\right) + 5\left(-\frac{1}{40}\right)\frac{dv}{dt} = 0$$
$$\Rightarrow \frac{dv}{dt} + 10v = 0$$
$$\rightarrow s + 10 = 0 \Rightarrow s = -10$$

Let $v = Ae^{-10t}$

$$v(0) = -96\ V$$
$$\Rightarrow v(t) = -96e^{-10t}$$

$$\therefore v(0.2) = -96e^{-10\times 0.2}$$
$$= -12.99\ V$$

- Accounting for the Energy



$$i(t) = I_0 e^{-\frac{R}{L}t}$$

$$p_R = i^2 R = I_0^2 R e^{-\frac{2R}{L}t}$$

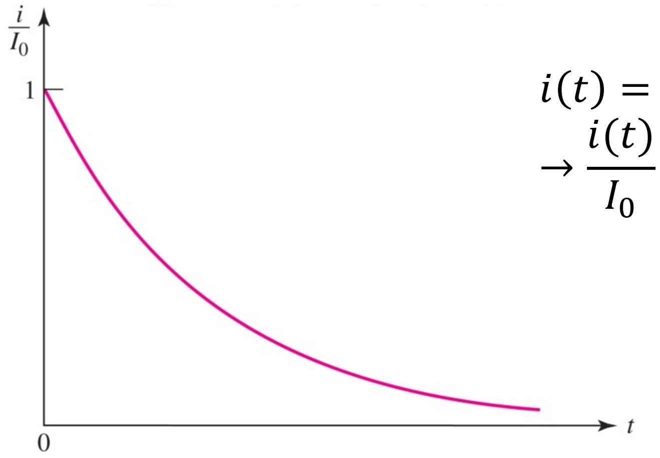
$$w_R = \int_0^\infty p_R\,dt = I_0^2 R \int_0^\infty e^{-\frac{2R}{L}t} dt$$

$$= I_0^2 R \left(-\frac{L}{2R}\right) e^{-\frac{2R}{L}t}\Big|_0^\infty = \frac{1}{2} L I_0^2$$

Energy stored initially in the inductor

- There is no longer any energy stored in the inductor for at infinite time since its current eventually drops to zero

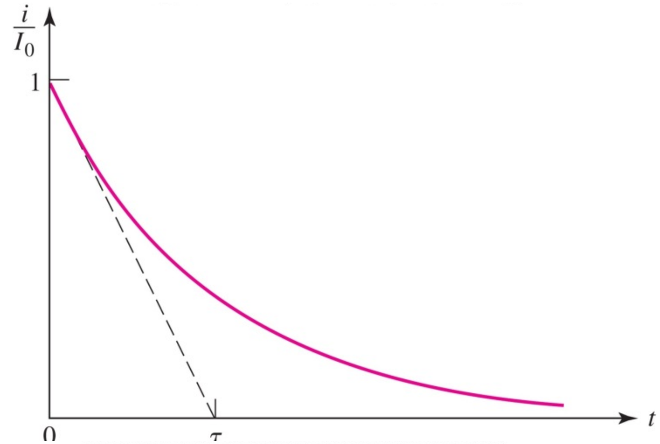
- All the initial energy is accounted for by dissipation in the resistor



$$i(t) = I_0 e^{-\frac{R}{L}t}$$
$$\rightarrow \frac{i(t)}{I_0} = e^{-\frac{R}{L}t}$$

The initial rate of decay

$$\frac{d}{dt}\frac{i(t)}{I_0}\bigg|_{t=0} = -\frac{R}{L}e^{-\frac{R}{L}t}\bigg|_{t=0} = -\frac{R}{L}$$

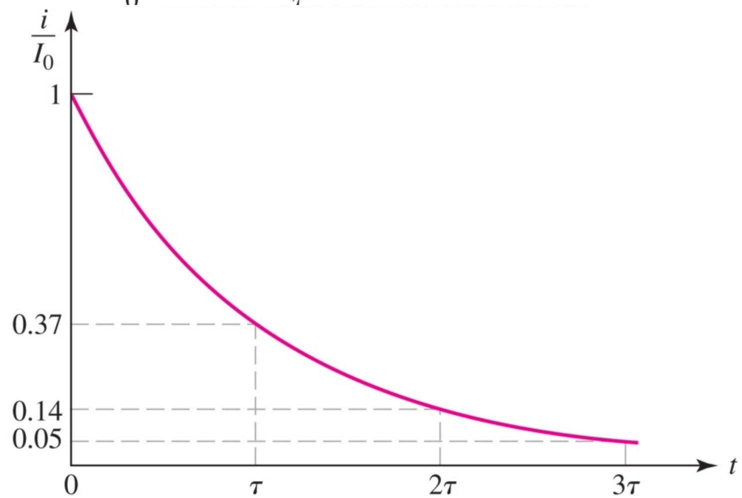


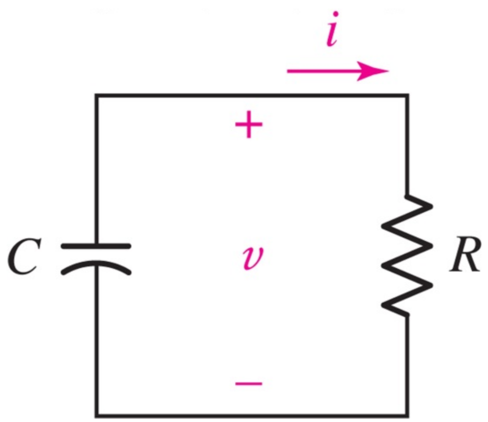
Time constant

$$\frac{i(t)}{I_0} = e^{-\frac{R}{L}t} = \frac{1}{e} \ at\ t = \tau \quad \rightarrow \tau = \frac{L}{R}$$

$$\frac{i(\tau)}{I_0} = e^{-\frac{R}{L}\tau} = e^{-\frac{RL}{LR}} = e^{-1} = 0.3679$$

$$\Rightarrow i(\tau) = e^{-1}I_0 = 0.3679I_0$$





$v(0) = V_0$

$$C\frac{dv}{dt} + \frac{v}{R} = 0$$
$$\rightarrow \frac{dv}{dt} + \frac{v}{RC} = 0$$
$$\rightarrow s + \frac{1}{RC} = 0$$
$$\Rightarrow s = -\frac{1}{RC}$$

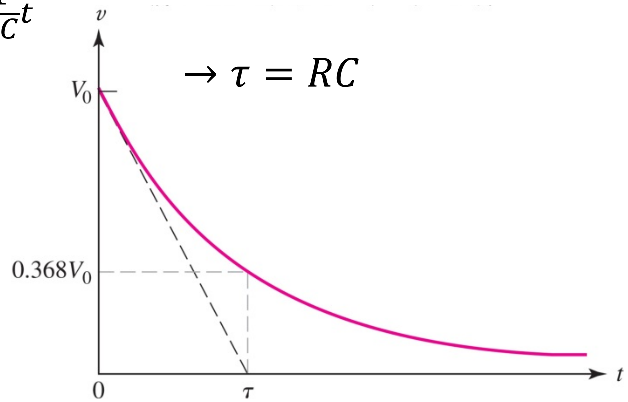
$$v(t) = Ae^{-st}$$
$$\rightarrow v(0) = A = V_0$$

$$v(t) = V_0 e^{-\frac{1}{RC}t}$$

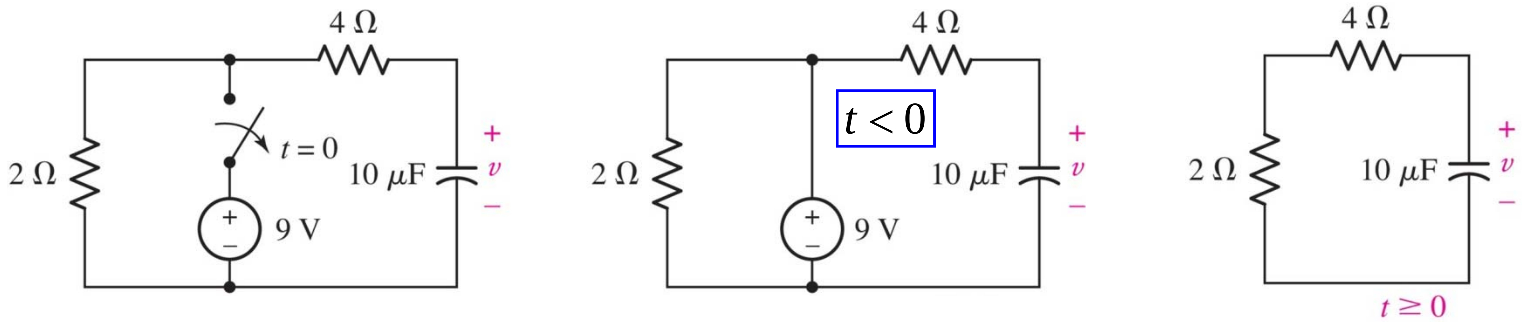
Time constant

$$\frac{v(t)}{V_0} = e^{-\frac{1}{RC}t} = \frac{1}{e} \quad at\ t = \tau$$

$$\rightarrow \tau = RC$$



Example 8.3 Find the voltage v at t=200 μs



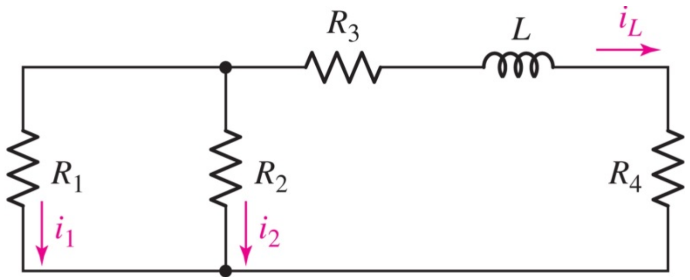
$v(0) = 9\ V$

$\tau = RC = (2+4)(10 \times 10^{-6}) = 60\ \mu s$

$$v(t) = v(0)e^{-\frac{1}{RC}t} = 9e^{-\frac{t}{60\times10^{-6}}}\ V$$

$$\Rightarrow v(200\times10^{-6}) = 321.1\ mV$$

- General *RL* Circuits

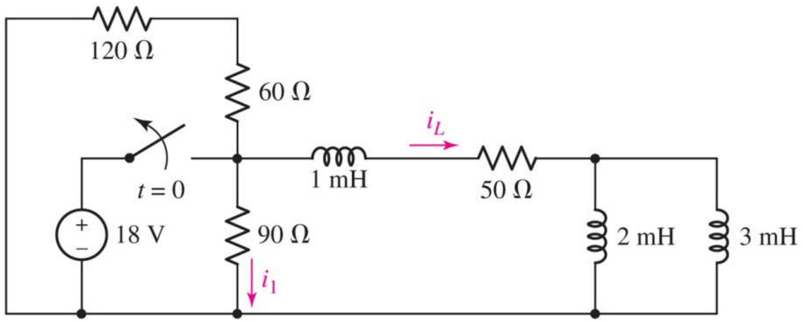


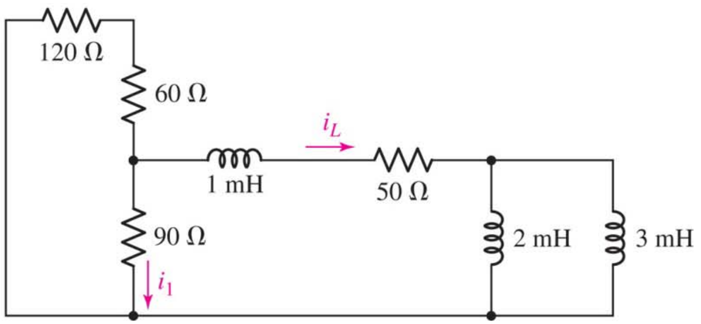
$$R_{eq} = R_1 \parallel R_2 + (R_3 + R_4) = R_3 + R_4 + \frac{R_1 R_2}{R_1 + R_2}$$

$$\tau = \frac{L}{R_{eq}},$$

$$\text{Generally } \tau = \frac{L_{eq}}{R_{eq}}$$

Example 8.4 Determine both i_1 and i_L for t >0





$$t < 0 : i_L(0^-) = \frac{18}{50} = 360\ mA, \quad L_{eq} = 1 + (2 \parallel 3) = 2.2\ mH$$

$$i_1(0^-) = \frac{18}{90} = 200\ mA \quad R_{eq} = 50 + (90 \parallel (120 + 60)) = 110\ \Omega$$

$$\tau = \frac{L_{eq}}{R_{eq}} = \frac{2.2 \times 10^{-3}}{110} = 20\ \mu s$$

$$i_L(0^+) = i_L(0^-)$$

$$i_1(0^+) = -i_L(0^+)\frac{180}{90 + 180} = -\frac{2}{3} \times 360 = -240\ mA$$

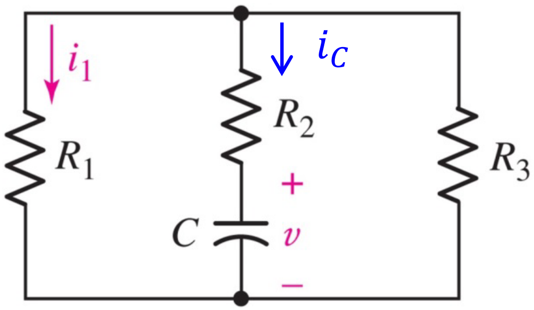
$$t \geq 0: i_L = i_L(0^+)e^{-\frac{t}{\tau}} = 360e^{-50000t}\ mA$$

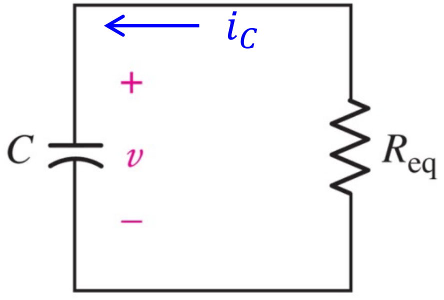
$$i_1 = i_1(0^+)e^{-\frac{t}{\tau}} = -240e^{-50000t}\ mA$$

- General *RC* Circuits

$$\tau = R_{eq}C_{eq}$$

Example 8.5 Find $v(0^+)$ and $i_1(0^+)$ if $v(0^-)=V_0$





$$R_{eq} = R_2 + (R_1 \parallel R_3) = R_2 + \frac{R_1R_3}{R_1 + R_3}$$

$$\tau = R_{eq}C = \left(R_2 + \frac{R_1R_3}{R_1 + R_3}\right)C$$

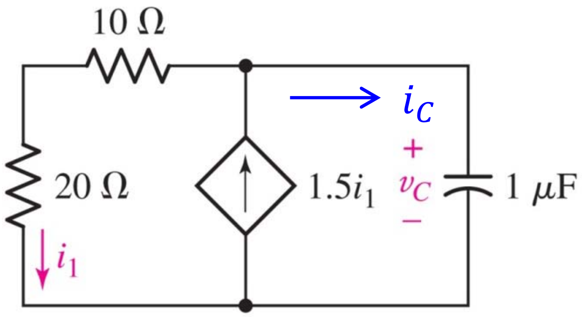
$$v(0^-) = V_0 \qquad v(0^+) = v(0^-) = V_0 \qquad v = V_0e^{-\frac{t}{\tau}} = V_0e^{-\frac{t}{R_{eq}C}}$$

$$i_1 = -\frac{R_3}{R_1 + R_3}i_C = -\frac{R_3}{R_1 + R_3}\frac{-v}{R_{eq}}$$

$$\Rightarrow i_1(0^+) = \frac{R_3}{R_1 + R_3}\frac{v(0^+)}{R_{eq}} = \frac{V_0}{R_2 + \frac{R_1R_3}{R_1 + R_3}}\frac{R_3}{R_1 + R_3}$$

$$i_1 = i_1(0^+)e^{-\frac{t}{\tau}} = \frac{V_0}{R_2 + \frac{R_1R_3}{R_1 + R_3}}\frac{R_3}{R_1 + R_3}e^{-\frac{t}{\tau}}$$

Example 8.6 Find $v_C(t)$ for t > 0 if $v_C(0^-)$=2 V



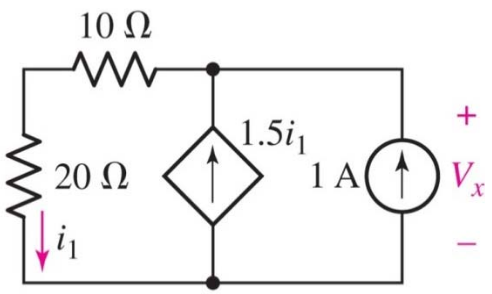
$$i_C = 1.5i_1 - i_1 = 0.5i_1$$
$$i_1 = \frac{v_C}{10+20}$$

$$i_C = 10^{-6}\frac{dv_C}{dt}$$
$$\rightarrow 0.5i_1 = 10^{-6}\frac{dv_C}{dt}$$
$$\rightarrow 0.5\frac{v_C}{30} = 10^{-6}\frac{dv_C}{dt}$$

$$\rightarrow \frac{dv_C}{dt} - \frac{1}{60\times10^{-6}}v_C = 0 \qquad \rightarrow s = \frac{1}{60\times10^{-6}} = \tau$$

$$\rightarrow v_C(t) = 2e^{t/(60\times10^{-6})}\ V \quad \longleftarrow \quad v_C(0^-) = v_C(0^+) = 2$$

Another method for Example 8.6 Finding R_{TH} on the left side of capacitor

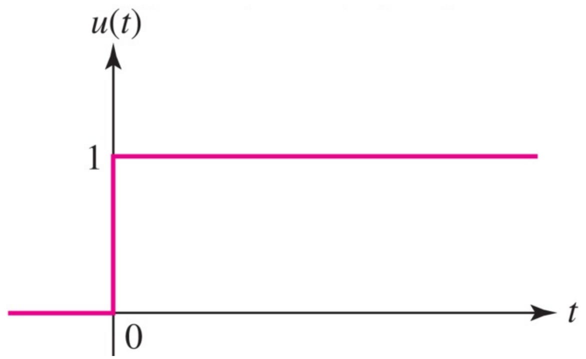


$$i_1 = \frac{V_x}{30}$$
$$V_x = (1+1.5i_1)(30)$$
$$= 30 + 45i_1 = 30 + 1.5V_x$$
$$\rightarrow V_x = -60V$$

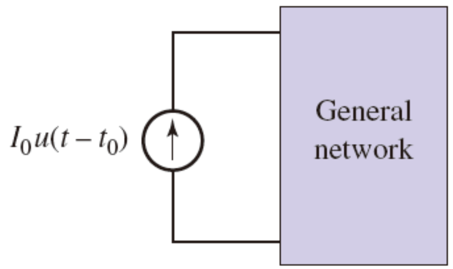
$$\therefore R_{TH} = \frac{V_x}{1} = -60\ \Omega$$

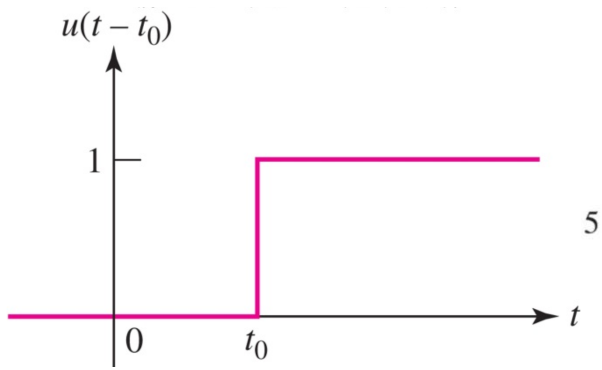
$$\tau = RC = -60\times10^{-6}\ s$$

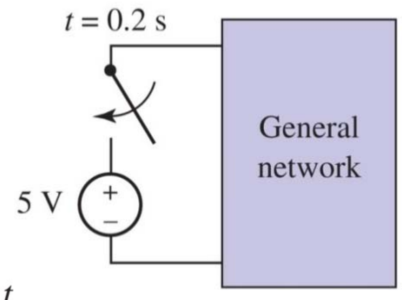
$$\upsilon_C(t) = 2e^{-\frac{t}{\tau}} = 2e^{-\frac{t}{-60\times10^{-6}}} = 2e^{\frac{t}{60\times10^{-6}}}\ V$$

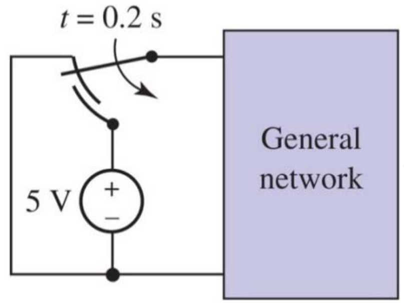


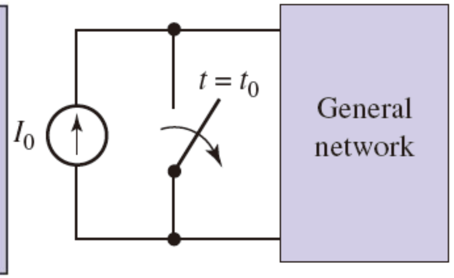
$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$

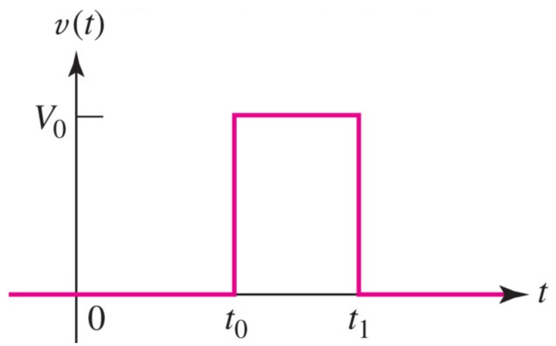




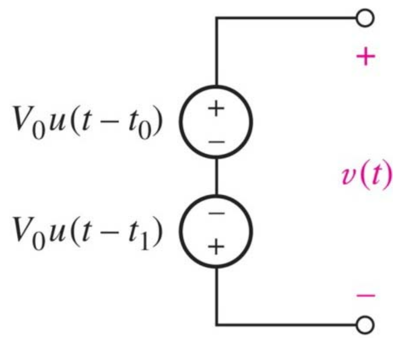




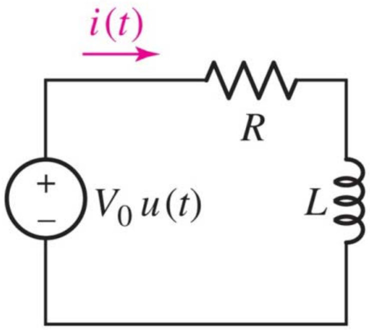




$$v(t) = \begin{cases} 0 & t < t_0 \\ V_0 & t_0 < t < t_1 \\ 0 & t > t_1 \end{cases}$$



$$v(t) = V_0[u(t - t_0) - u(t - t_1)]$$



$$Ri + L\frac{di}{dt} = V_0 u(t)$$

$$t < 0:\ i(t) = 0$$

$$t > 0:\ Ri + L\frac{di}{dt} = V_0$$

$$i(0^-) = i(0^+) = 0$$

$$\rightarrow \frac{Ldi}{V_0 - Ri} = dt$$

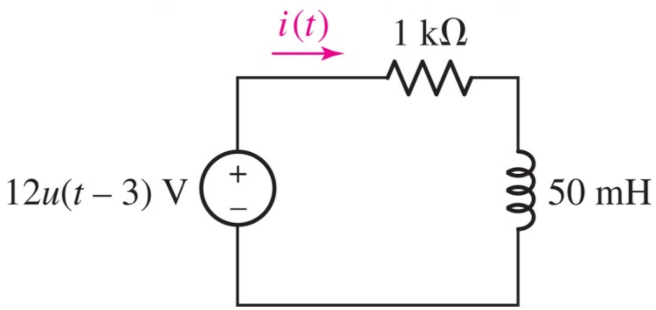
$$\rightarrow -\frac{L}{R}\ln(V_0 - Ri) = t + K$$

$$\text{at } t = 0^+ :\ -\frac{L}{R}\ln V_0 = K$$

$$-\frac{L}{R}[\ln(V_0 - Ri) - \ln V_0] = t \ \rightarrow\ \frac{V_0 - Ri}{V_0} = e^{-\frac{R}{L}t} \ \Rightarrow\ i(t) = \frac{V_0}{R} - \frac{V_0}{R}e^{-\frac{R}{L}t}, t > 0$$

$$\boxed{\therefore\ i(t) = \frac{V_0}{R}\left(1 - e^{-\frac{R}{L}t}\right)u(t)}$$

Example 8.7 Find i(t) for t =∞, 3⁻, 3⁺, and 3.0001 s.



$$i(3^+) = i(3^-) = 0$$
$$i(\infty) = 12\,mA$$

$$\frac{V_0}{R} = \frac{12}{1000} = 12\,mA,\qquad \frac{R}{L} = \frac{1\times 10^3}{50\times 10^{-3}} = 20000$$

For $12u(t)$ *input*,

$$i(t) = \left(\frac{V_0}{R} - \frac{V_0}{R}e^{-\frac{R}{L}t}\right)u(t) = (12 - 12e^{-20000t})u(t)$$

$$\Rightarrow i(t) = \left(12 - 12e^{-20000(t-3)}\right)u(t-3)$$

$$i(3.0001) = 10.38\,mA$$

Consider general equation

$$\frac{di}{dt} + Pi = Q(t) \longrightarrow \textbf{forcing function}$$

$$\rightarrow di + Pidt = Q(t)dt$$

multiply e^{Pt} (P: positive constant)

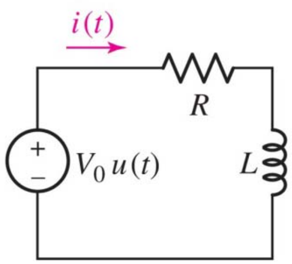
$$e^{Pt}di + Pie^{Pt}dt = Qe^{Pt}dt$$

$$d(ie^{Pt}) = e^{Pt}di + iPe^{Pt}dt$$

$$d(ie^{Pt}) = Qe^{Pt}dt$$

integration

$$ie^{Pt} = \int Qe^{Pt}dt + A \Rightarrow i = e^{-Pt}\int Qe^{Pt}dt + Ae^{-Pt}$$



$$Ri + L\frac{di}{dt} = V_0u(t)$$

$$P = \frac{R}{L} \quad ,Q = \frac{V_0}{L}$$

source-free case: $Q = 0$

$\Rightarrow i_n = Ae^{-Pt}$ natural response

$t \rightarrow \infty, i_n \rightarrow 0$

After the natural response has disappeared

Q: dc

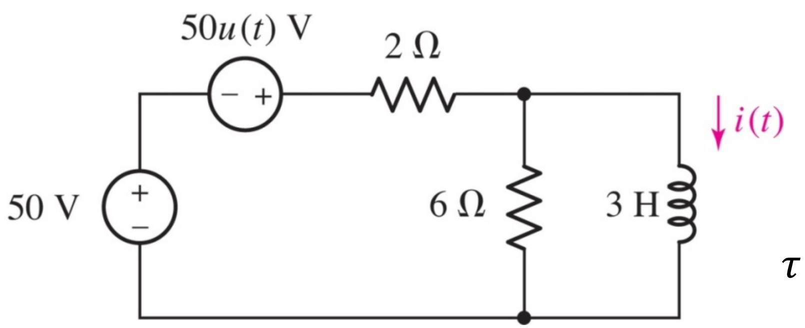
$$i_f = e^{-Pt}Q\int e^{Pt}dt = \frac{Q}{P}e^{-Pt}(e^{Pt}) = \frac{Q}{P}$$

forced response

complete response

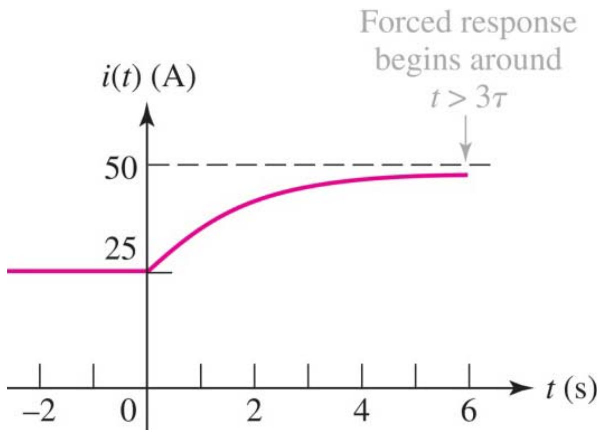
$$i(t) = i_f + i_n = \frac{Q}{P} + Ae^{-Pt}$$

Example 8.8 Find i(t) for all values of time



$$R_{eq} = 2 \parallel 6 = \frac{2 \times 6}{2+6} = 1.5$$

$$\tau = \frac{L}{R_{eq}} = \frac{3}{1.5} = 2\ s$$



$$\frac{100 - 3\frac{di}{dt}}{2} = \frac{3\frac{di}{dt}}{6} + i \;\rightarrow\; \frac{di}{dt} + \frac{1}{2}i = 25 \;\rightarrow\; i(t) = \frac{Q}{P} + Ke^{-Pt} = 50 + Ke^{-\frac{t}{2}} \quad t \geq 0$$

$$i_n = Ke^{-t/\tau} = Ke^{-t/2}\ A, \qquad t > 0,$$

$$i_f = \frac{50+50}{2} = 50\ A$$

$$\Rightarrow i = i_f + i_n = 50 + Ke^{-t/2}\ A, \qquad t > 0$$

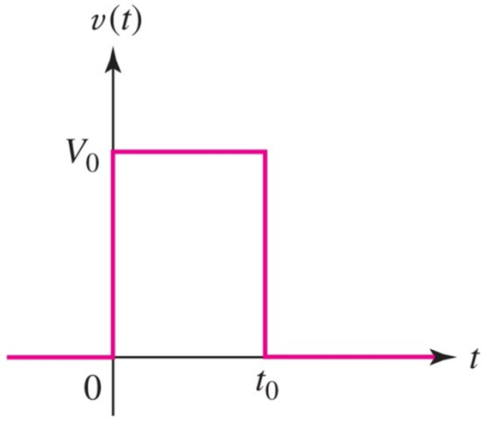
at $t = 0$: $\dfrac{50}{2} = 50 + K \Rightarrow K = -25$

Therefore,

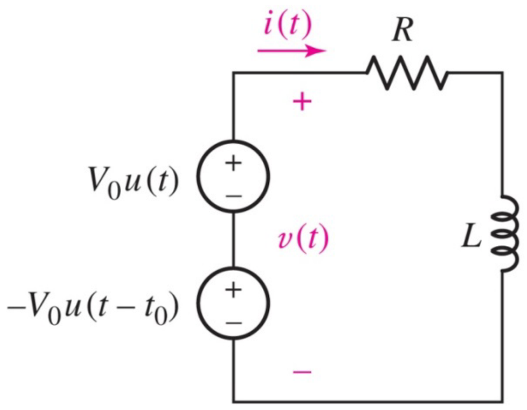
$$i(t) = 50 - 25e^{-\frac{t}{2}}\ A, \qquad t > 0$$
$$i(t) = 25\ A, \qquad t < 0$$

$$\boxed{i(t) = 25 + 25\left(1 - e^{-\frac{t}{2}}\right)u(t)A}$$

Example 8.9 Find i(t) for a rectangular voltage pulse forcing function.

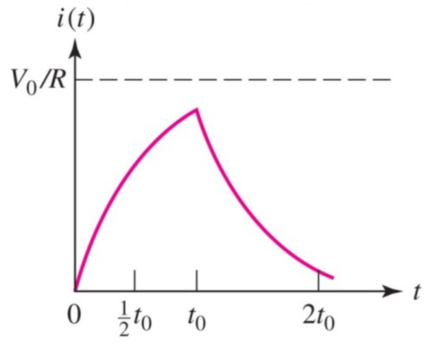


(a)



$$i_1(t) = \frac{V_0}{R}\left(1 - e^{-\frac{R}{L}t}\right), \qquad t > 0$$

$$i_2(t) = -\frac{V_0}{R}\left(1 - e^{-\frac{R}{L}(t-t_0)}\right),$$



$$i(t) = 0, \qquad t < 0$$

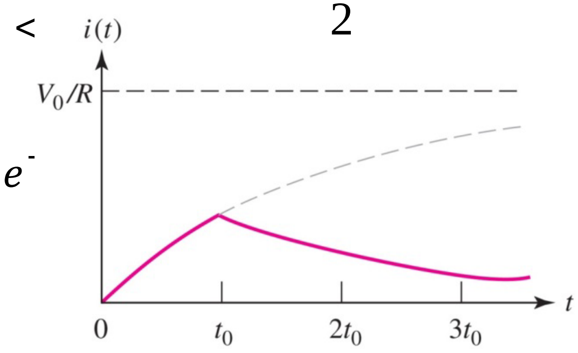
$$\tau = \frac{t_0}{2}$$

$$i(t) = \frac{V_0}{R}\left(1 - e^{-\frac{R}{L}t}\right), \qquad 0 < t <$$

$$i(t) = \frac{V_0}{R}\left(1 - e^{-\frac{R}{L}t}\right) - \frac{V_0}{R}\left(1 - e^{-}\right.$$

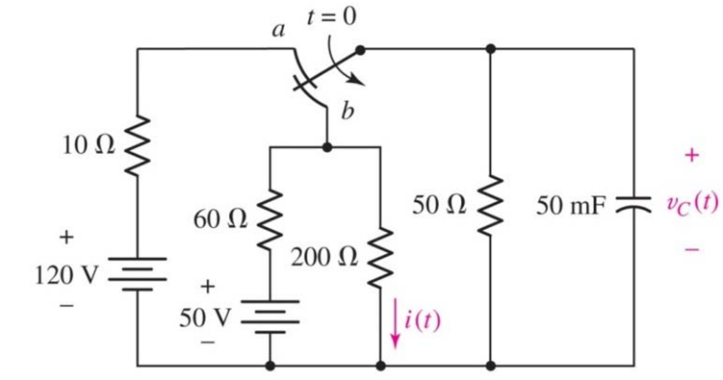
$$= \frac{V_0}{R}e^{-\frac{R}{L}t}\left(e^{\frac{R}{L}t_0} - 1\right)$$

$$t > t_0$$

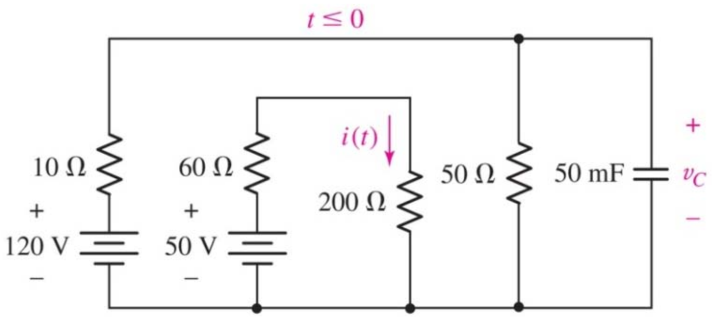


$$\tau = t_0$$

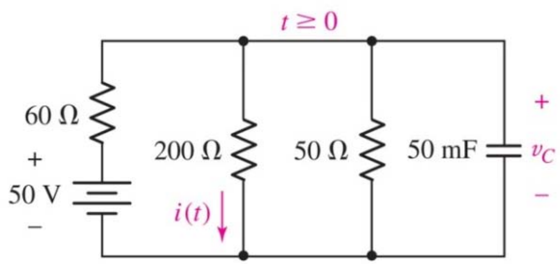
Example 8.10 Find $v_C(t)$ and i(t) for all time



(a)



(b)



$$v_C(0^+) = v_C(0^-) = \frac{50}{10+50}120 = 100\,V$$

$$R_{eq} = 60 \parallel 200 \parallel 50 = 24\,\Omega \quad \rightarrow \tau = R_{eq}C = 1.2\,s$$

$$v_{C_n} = Ae^{-t/\tau} = Ae^{-t/1.2}$$

$$v_{C_f} = v_C(\infty) = \frac{200 \parallel 50}{60 + (200 \parallel 50)}50 = 20\,V$$

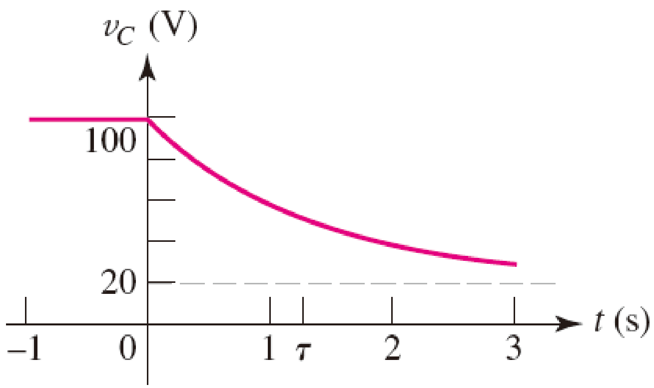
$$v_C = v_{C_n} + v_{C_f} = 20 + Ae^{-t/1.2}$$

$\rightarrow$ at $t = 0$: $v_C(0) = 20 + A = 100$

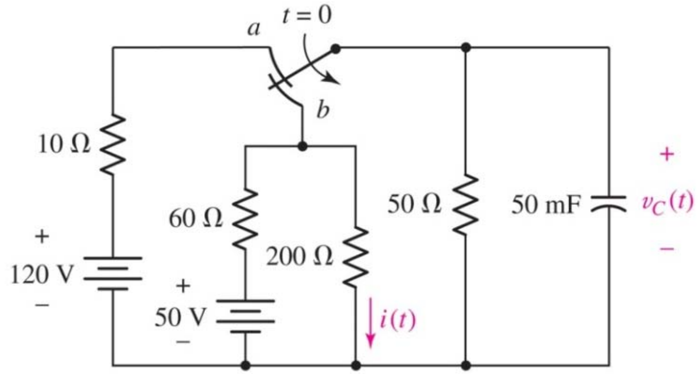
$\Rightarrow A = 80$

$$\therefore v_C = 20 + 80e^{-t/1.2}, t \geq 0$$

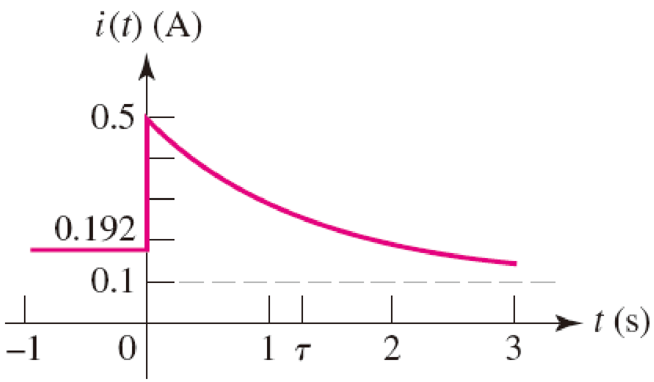
$$v_C = 100,\ t < 0$$



Example 8.10 Find $v_C(t)$ and $i(t)$ for all time

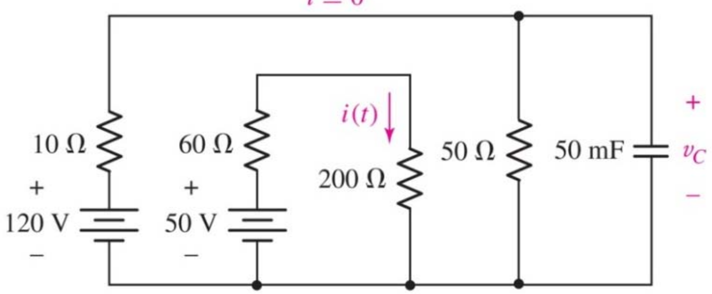


(a)



$t < 0$:

$$i(0^-) = \frac{50}{60+200}$$
$$= 0.1923\ A$$



(b)

$$i_f = i(\infty) = \frac{50}{60 + (200 \parallel 50)}\frac{50}{200+50} = 0.1\ A$$

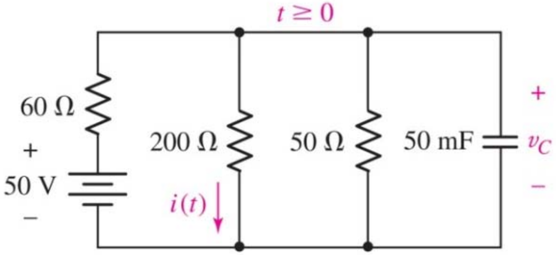
$$i(t) = i_f + i_n = 0.1 + Ae^{-t/1.2} \qquad t > 0$$

from $v_C(0^+) = 100\ V$

$$\Rightarrow i(0^+) = \frac{100}{200} = 0.5\ A$$

$i(0^+) = 0.1 + A = 0.5$

$\rightarrow A = 0.4$

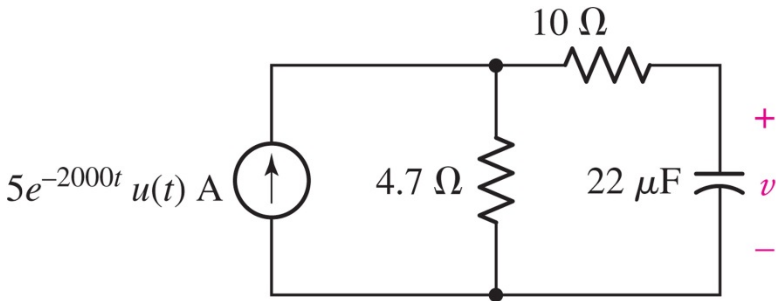


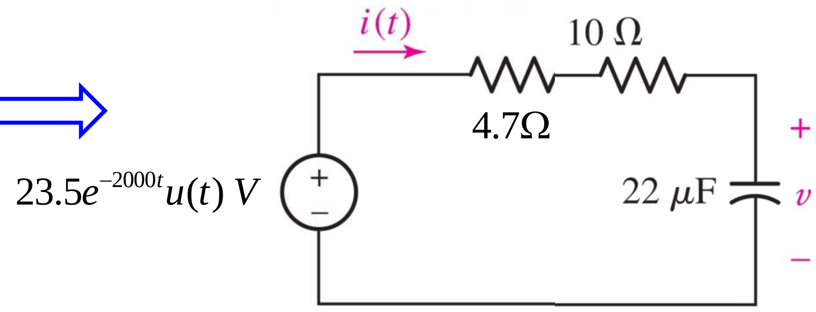
$$i(t) = 0.1923\ A, \qquad t < 0$$
$$i(t) = 0.1 + 0.4e^{-t/1.2}, \qquad t > 0$$

$$i = 0.1923 + \left(-0.1923 + 0.4e^{-\frac{t}{1.2}}\right)u(t)$$

1. With all independent sources zeroed out, simplify the circuit to determine R_{eq}, C_{eq}, and the time constant $\tau = R_{eq}C_{eq}$.
2. Viewing C_{eq} as an open circuit, use dc analysis methods to find $v_C(0^-)$, the capacitor voltage just prior to the discontinuity.
3. Again viewing C_{eq} as an open circuit, use dc analysis methods to find the forced response. This is the value approached by $f(t)$ as $t \to \infty$; we represent it by $f(\infty)$.
4. Write the total response as the sum of the forced and natural responses: $f(t) = f(\infty) + Ae^{-t/\tau}$.
5. Find $f(0^+)$ by using the condition that $v_C(0^+) = v_C(0^-)$. If desired, C_{eq} may be replaced by a voltage source $v_C(0^+)$ [a short circuit if $v_C(0^+) = 0$] for this calculation. With the exception of capacitor voltages (and inductor currents), other voltages and currents in the circuit may change abruptly.
6. $f(0^+) = f(\infty) + A$ and $f(t) = f(\infty) + [f(0^+) - f(\infty)]e^{-t/\tau}$, or total response = final value + (initial value − final value) $e^{-t/\tau}$.

Example 8.11 Determine an expression for v(t) for t > 0





$$R_{eq} = 4.7 + 10 = 14.7\ \Omega \qquad \tau = R_{eq}C = 14.7 \times (22 \times 10^{-6}) = 323.4\ \mu s$$

$t > 0$

$$i(t) = (22 \times 10^{-6})\frac{dv}{dt} \qquad \to 23.55e^{-2000t} = v + 14.7 \times (22 \times 10^{-6})\frac{dv}{dt}$$

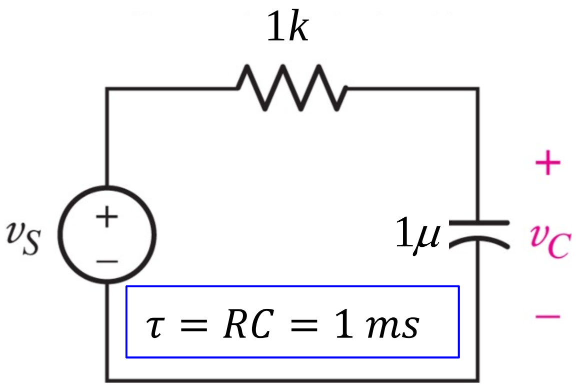
$$\to \frac{dv}{dt} + 3.092 \times 10^3 v = 72.67 \times 10^3 e^{-2000t}$$

$$P = \frac{1}{\tau} = 3.092 \times 10^3$$
$$Q(t) = 72.67 \times 10^3 e^{-2000t}$$

$$v(t) = e^{-Pt}\int Qe^{Pt}dt + Ae^{-Pt} = e^{-3092t}\int 72.67 \times 10^3 e^{-2000t}e^{3092t}dt + Ae^{-3092t} =$$

$$\to \ v(t) = 66.55e^{-2000t} + Ae^{-3092t}\ V \qquad v(0^-) = v(0^+) = 0\ \to\ A = -66.55$$

$$\boxed{v(t) = 66.55(e^{-2000t} - e^{-3092t})u(t)\ V}$$



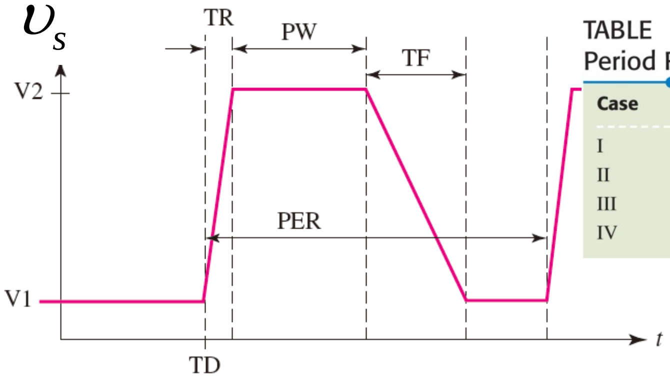
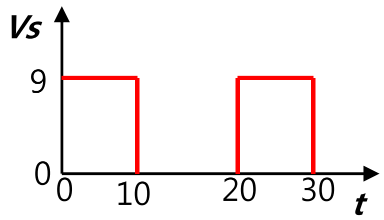


TABLE 8.1 Four Separate Cases of Pulse Width and Period Relative to the Circuit Time Constant of 1 ms

Case	Pulse Width W_p	Period T
I	10 ms ($\tau \ll W_p$)	20 ms ($\tau \ll T - W_p$)
II	10 ms ($\tau \ll W_p$)	10.1 ms ($\tau \gg T - W_p$)
III	0.1 ms ($\tau \gg W_p$)	10.1 ms ($\tau \ll T - W_p$)
IV	0.1 ms ($\tau \gg W_p$)	0.2 ms ($\tau \gg T - W_p$)



- Case I : Time Enough to Fully Charge and Fully Discharge

0 < t < 10 ms

$$v_{C_n} = Ae^{-t/\tau} = Ae^{-1000t}$$

$$v_{C_f} = v_C(\infty) = 9\ V$$

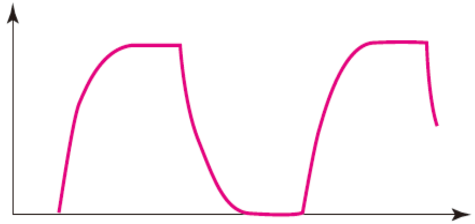
$$v_C(t) = 9(1 - e^{-1000t})$$

$$v_C(0.01) = 9(1 - e^{-10}) = 8.9959 = 9$$

10 < t < 20 ms

$$v_C(t) = 8.9959e^{-1000(t-0.01)} \approx 9e^{-1000(t-0.01)}$$

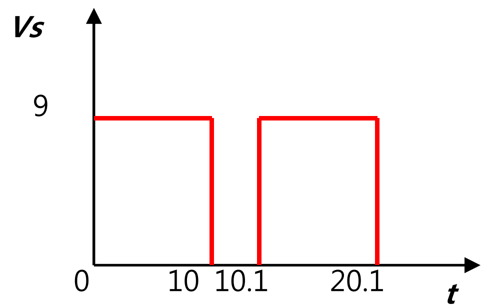
$$v_C(t) = \begin{cases} 9(1 - e^{-1000t}) & 0 \le t \le 10\ ms \\ 9e^{-1000(t-0.01)} & 10 < t \le 20\ ms \\ 9\left(1 - e^{-1000(t-0.02)}\right) & 20 < t \le 30\ ms \\ 9e^{-1000(t-0.03)} & 30 < t \le 40\ ms \\ \vdots & \end{cases}$$



Capacitor has time to fully charge

- Case Ⅱ : Time Enough to Fully Charge but Not Fully Discharge

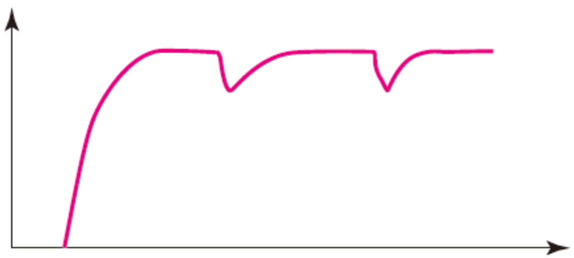
$0 < t < 10$ ms Capacitor has time to fully charge



$$v_C(t) = 9(1 - e^{-1000t})$$

$10 < t < 10.1$ ms $v_C(t) = 9e^{-1000(t-0.01)}$

$$\rightarrow v_C(10.1\,ms) = 9e^{-1000(10.1\times 10^{-3}-0.01)} = 8.144\,V$$



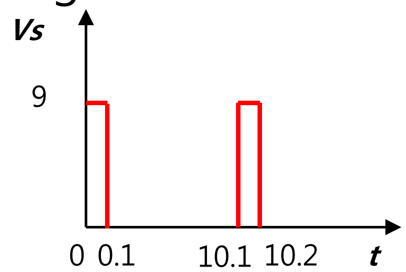
$10.1 < t < 20.1$ ms

$$v_C(t) = 9 + Ae^{-1000(t-10.1\times 10^{-3})}$$

$$v_C(10.1\,ms) = 9 + A = 8.144 \rightarrow A = -0.856$$

$$v_C(t) = 9 - 0.856\ e^{-1000(t-10.1\times 10^{-3})}$$

- Case Ⅲ : No Time to Fully Charge but Time to Fully Discharge

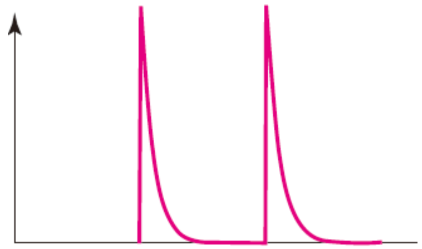


$0 < t < 0.1$ ms $\quad v_C(t) = 9(1 - e^{-1000t})$

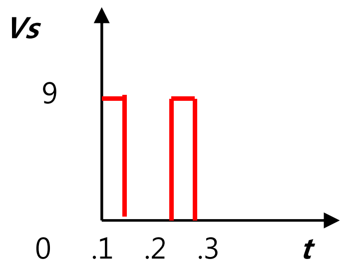
$\rightarrow v_C(0.1 \times 10^{-3}) = 9\left(1 - e^{-1000(0.1\times10^{-3})}\right) = 0.8565$

$0.1 < t < 10.1$ ms $\quad v_C(t) = Be^{-1000(t-0.1\times10^{-3})}$

$v_C(0.1 \times 10^{-3}) = B = 0.8565 \quad \rightarrow \quad v_C(t) = 0.8565e^{-1000(t-0.1\times10^{-3})}$



- Case Ⅳ: No Time to Fully Charge or Even Fully Discharge



$0 < t < 0.1$ ms $\quad v_C(t) = 9(1 - e^{-1000t})$

$0.1 < t < 0.2$ ms $\quad v_C(t) = 0.8565e^{-1000(t-0.1\times10^{-3})}$

$v_C(0.2 \times 10^{-3}) = 0.7750$

$0.2 < t < 0.3$ ms $\quad v_C(t) = 9 + Ce^{-1000(t-0.2\times10^{-3})} \quad C = -8.225$

$v_C(0.3 \times 10^{-3}) = 1.558$

$0.3 < t < 0.4$ ms $\quad v_C(t) = 1.558e^{-1000(t-0.3\times10^{-3})}$

Homework : 8장 Exercises 7의 배수 문제

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