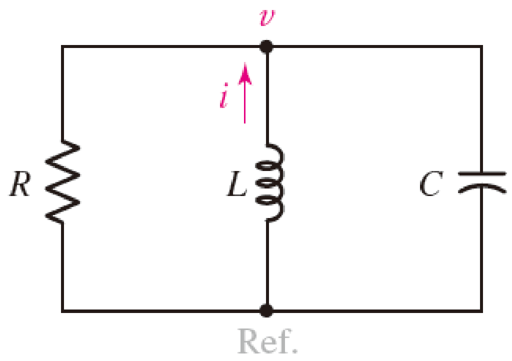
Chapter 9
The *RLC* Circuits

- Natural response of parallel *RLC* circuit



$$\frac{v}{R}+\frac{1}{L}\int_{t_0}^{t} v\,dt' - i(t_0) + C\frac{dv}{dt} = 0$$

$$\rightarrow C\frac{d^2v}{dt^2}+\frac{1}{R}\frac{dv}{dt}+\frac{1}{L}v = 0$$

Assume solution :

$$v = Ae^{st}$$

Second-order homogeneous differential equation

$$\rightarrow\ CAs^2e^{st}+\frac{1}{R}Ase^{st}+\frac{1}{L}Ae^{st} = 0 \qquad \rightarrow\ Ae^{st}\left(Cs^2+\frac{1}{R}s+\frac{1}{L}\right) = 0$$

$$s_1 = -\frac{1}{2RC}+\sqrt{\left(\frac{1}{2RC}\right)^2-\frac{1}{LC}}$$

$$s_2 = -\frac{1}{2RC}-\sqrt{\left(\frac{1}{2RC}\right)^2-\frac{1}{LC}}$$

Real or Complex numbers

$$v_1 = A_1e^{s_1t}\ \Rightarrow\ C\frac{d^2v_1}{dt^2}+\frac{1}{R}\frac{dv_1}{dt}+\frac{1}{L}v_1 = 0$$

$$v_2 = A_2e^{s_2t}\ \Rightarrow\ C\frac{d^2v_2}{dt^2}+\frac{1}{R}\frac{dv_2}{dt}+\frac{1}{L}v_2 = 0$$

$$\Rightarrow\ C\frac{d^2(v_1+v_2)}{dt^2}+\frac{1}{R}\frac{d(v_1+v_2)}{dt}+\frac{1}{L}(v_1+v_2) = 0 \qquad v(t) = v_1+v_2 = A_1e^{s_1t}+A_2e^{s_2t}$$

General form of the natural response

- Definition of Frequency Terms

$$s = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

s **: real numbers of complex conjugates**

unit: $\boldsymbol{sec^{-1}}$ **: frequency**

$$\alpha = \frac{1}{2RC}$$

exponential damping coefficient

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

resonant frequency

$$\rightarrow s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

complex frequency

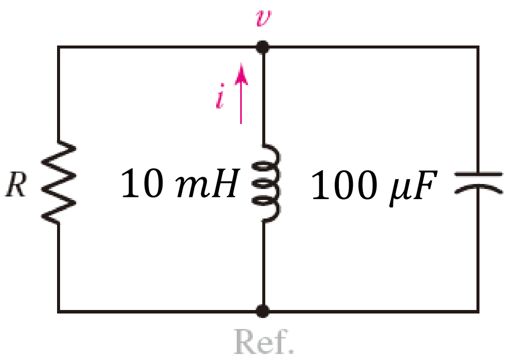
$$\rightarrow v = Ae^{st} = Ae^{-\alpha \pm \sqrt{\alpha^2 - \omega_0^2}}$$
$$= Ae^{-\alpha} e^{\pm \sqrt{\alpha^2 - \omega_0^2}}$$

$\alpha > \omega_0 \rightarrow \sqrt{\alpha^2 - \omega_0^2} > 0 \Rightarrow s\text{: } \textbf{real roots}$ overdamped response

$\alpha = \omega_0 \rightarrow \sqrt{\alpha^2 - \omega_0^2} = 0 \Rightarrow s\text{: } \textbf{multiple real roots}$ critically damped response

$\alpha < \omega_0 \rightarrow \sqrt{\alpha^2 - \omega_0^2} < 0 \Rightarrow s\text{: } \textbf{complex roots}$ underdamped response

Example 9.1 Determine the resistor value for overdamped and underdamped responses.



$$\alpha = \frac{1}{2RC} \qquad \omega_0 = \frac{1}{\sqrt{RC}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{\frac{1}{(10 \times 10^{-3})(100 \times 10^{-6})}} = 10^3\ rad/s$$

overdamped case : $\alpha > \omega_0 \quad \rightarrow \frac{1}{2RC} > 10^3 \Rightarrow R < \frac{1}{2 \times 10^3 \times 100 \times 10^{-6}} = 5\ \Omega$

underdamped case : $R > 5\ \Omega$

- Overdamped parallel *RLC* circuit

$$\alpha > \omega_0 \;\Rightarrow\; \frac{1}{2RC} > \frac{1}{\sqrt{LC}} \;\Rightarrow\; LC > 4R^2C^2$$

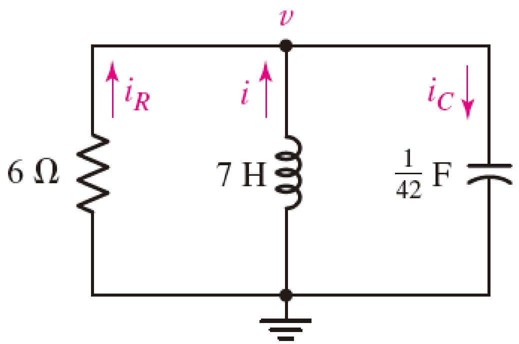
$\boldsymbol{s_1}, \boldsymbol{s_2}$: negative real number

$$\sqrt{\alpha^2 - \omega_0} < \alpha$$

$$\Rightarrow \underbrace{\left(-\alpha - \sqrt{\alpha^2 - \omega_0}\right)}_{\boldsymbol{s_1}} < \underbrace{\left(-\alpha + \sqrt{\alpha^2 - \omega_0}\right)}_{\boldsymbol{s_2}} < 0$$

$$v(t)\Big|_{t\to\infty} = A_1 e^{s_1 t} + A_2 e^{s_2 t}\Big|_{t\to\infty} \to 0$$

The constants A_1 and A_2 are determined by the initial conditions.



$$v(0) = 0\,V, i(0) = 10\,A$$

$$v(t) = A_1 e^{-t} + A_2 e^{-6t}$$

$$v(0) = A_1 + A_2 = 0$$

$$i_C = C\frac{dv}{dt}$$

$$\to \frac{dv}{dt}\Big|_{t=0} = \frac{i_C(0)}{C} = \frac{i_R(0) + i(0)}{C}$$

$$= \frac{10}{\frac{1}{42}} = 420\,V/s$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 6 \times \frac{1}{42}} = 3.5\,s^{-1},$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{7 \times \frac{1}{42}}} = \sqrt{6}\,s^{-1},$$

$$s_1 = -1, s_2 = -6$$

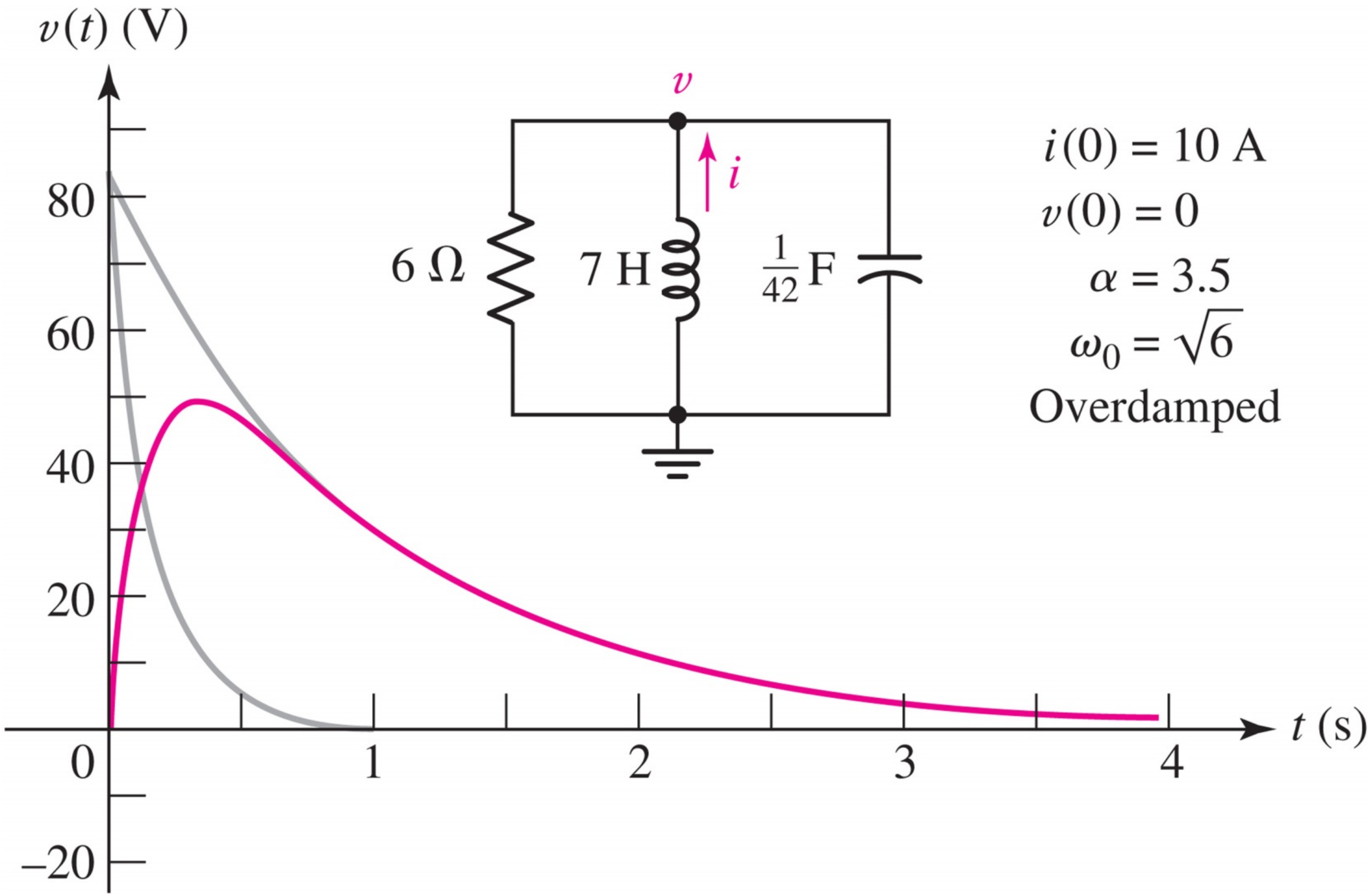
$$\frac{dv}{dt} = -A_1 e^{-t} - 6A_2 e^{-6t} \;\Rightarrow\; \frac{dv}{dt}\Big|_{t=0} = -A_1 - 6A_2$$

$$A_1 + A_2 = 0$$
$$-A_1 - 6A_2 = 420$$

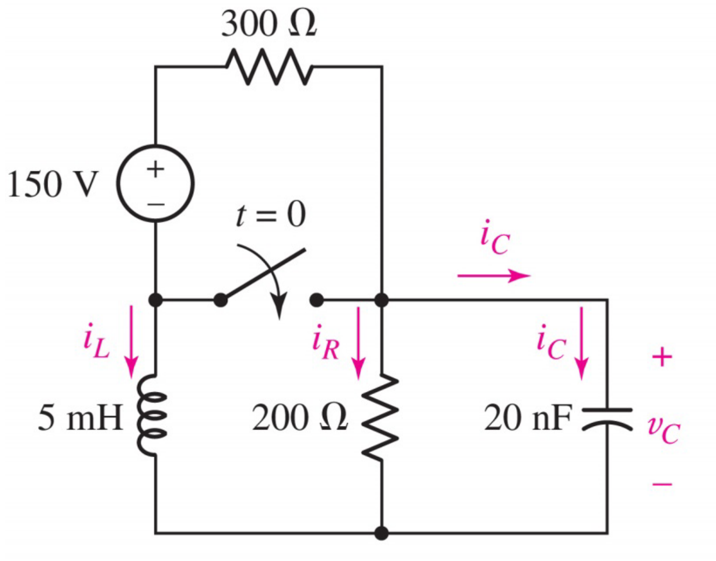
$$\Rightarrow A_1 = 84, A_2 = -84$$

$$\therefore v(t) = 84(e^{-t} - e^{-6t})$$

- Natural response of Overdamped parallel *RLC* circuit



Example 9.2 Find an expression for $v_C(t)$ for t >0.



$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 200 \times 20 \times 10^{-9}} = 125{,}000\ s^{-1}$$

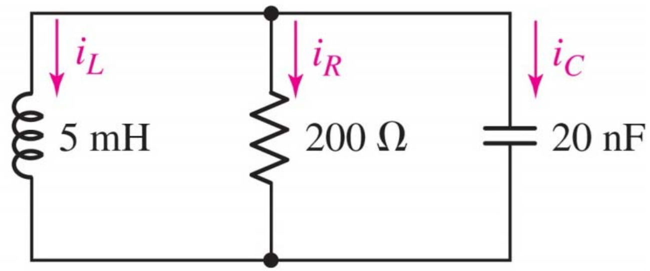
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 10^{-3} \times 20 \times 10^{-9}}} = 100{,}000\ rad/s$$

$\alpha > \omega_0$: overdamped response

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -50{,}000\ s^{-1},$$

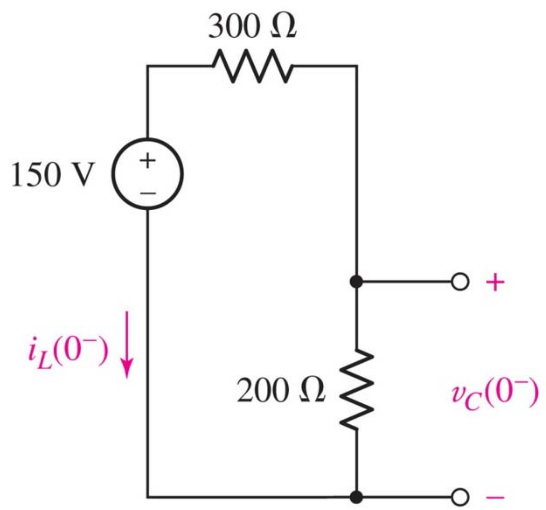
$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -200{,}000\ s^{-1},$$

$t > 0$



$$\Rightarrow\ v_C(t) = A_1 e^{-50000t} + A_2 e^{-200000t}$$

Example 9.2 continued...



$$\boxed{v_C(t) = A_1 e^{-50000t} + A_2 e^{-200000t}} \qquad t > 0$$

$$i_L(0^-) = -\frac{150}{300+200} = -0.3\ A$$

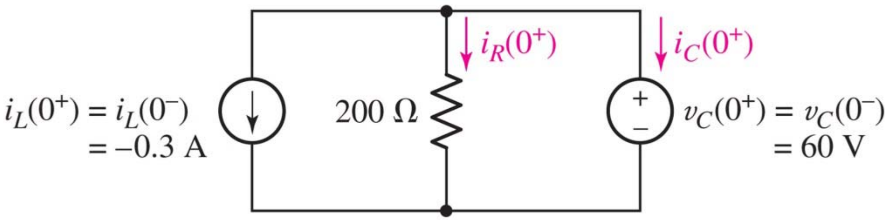
$$\Rightarrow v_C(0^-) = \frac{200}{300+200} 150 = 60\ V$$

$$\Rightarrow v_C(0^+) = A_1 + A_2 = 60$$

$$i_C = C\frac{dv_C}{dt}$$

$$\rightarrow \left.\frac{dv_C}{dt}\right|_{t=0^+} = \frac{i_C(0^+)}{C} = \frac{-i_R(0^+) - i_L(0^+)}{C}$$

$$= \frac{-\frac{v_C(0^+)}{200} - (-0.3)}{20\times 10^{-9}} = \frac{-\frac{60}{200} + 0.3}{20\times 10^{-9}} = 0$$

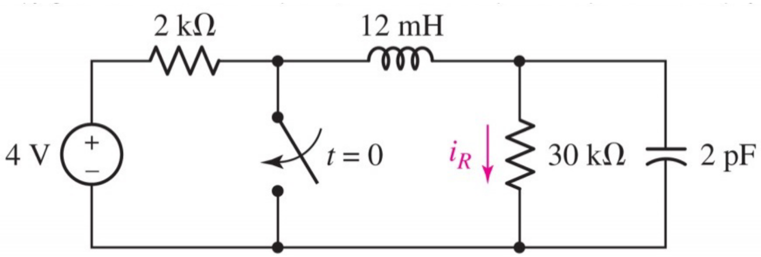


$$\therefore -50000A_1 - 200000A_2 = 0$$

$$\Rightarrow A_1 = 80, A_2 = -20$$

$$\therefore v_C(t) = 80e^{-50000t} - 20e^{-200000t}\ V, \qquad t > 0$$

Example 9.3 Express i_R for all time



$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 30 \times 10^3 \times 2 \times 10^{-12}} = 8.33 \times 10^6\ s^{-1}$$

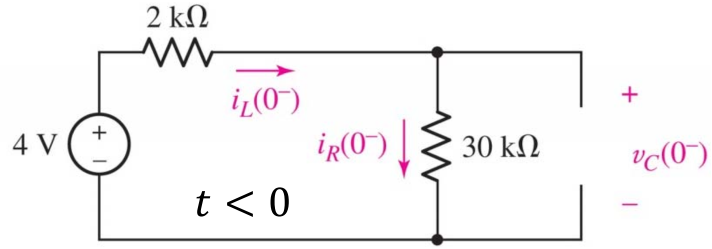
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{12 \times 10^{-3} \times 2 \times 10^{-12}}} = 6.45 \times 10^6\ rad/s$$

$\alpha > \omega_0$: overdamped response

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -3.063 \times 10^6\ s^{-1},$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -13.60 \times 10^6\ s^{-1},$$

Let $i_R(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$



$$i_R(0^-) = i_L(0^-) = \frac{4}{32 \times 10^3} = 125 \times 10^{-6}\ A \quad \rightarrow \quad v_C(0^-) = (30 \times 10^3)(125 \times 10^{-6}) = 3.75\ V$$

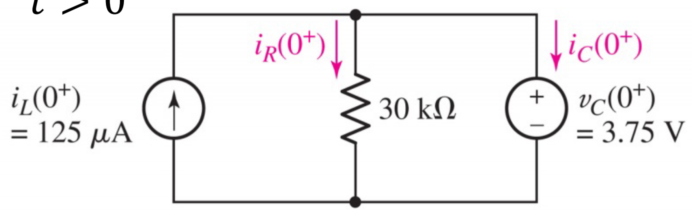
$$i_R(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \ \Rightarrow\ i_R(0^+) = A_1 + A_2 = 125 \times 10^{-6} \quad \therefore v_C(0^-) = v_C(0^+)$$

$$v_C(t) = 30k \times i_R(t)$$

$$i_C = C\frac{dv_C}{dt} \rightarrow \left.\frac{dv_C}{dt}\right|_{t=0^+} = \frac{i_C(0^+)}{C} = \frac{-i_R(0^+) + i_L(0^+)}{C} = 0$$

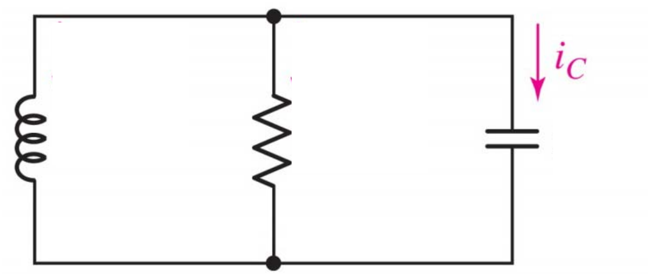
$$\therefore -3.063 \times 10^6 A_1 - 13.60 \times 10^6 A_2 = 0$$

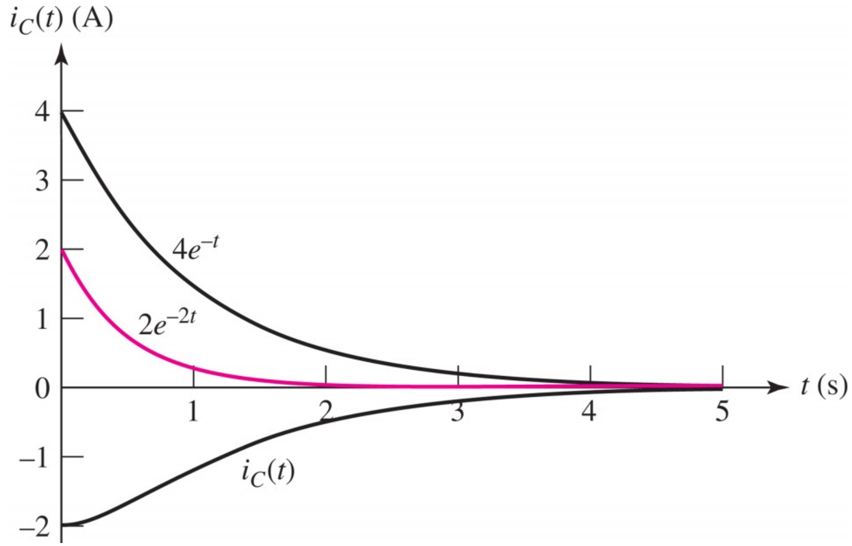
$t > 0$



$$i_R(t) = \begin{cases} 125 \times 10^{-6}\ A, & t < 0 \\ 161.3 \times 10^{-6} e^{-3.063 \times 10^6 t} \\ -36.34 \times 10^{-6} e^{-13.6 \times 10^6 t}\ A, & t > 0 \end{cases}$$

Example 9.4 For t>0, $i_C(t)$ =2e^{-2t}-4e^{-t} A. Sketch the current 0<t<5s and find settling time.





$$i_C(t) = 2e^{-2t} - 4e^{-t}\ A$$

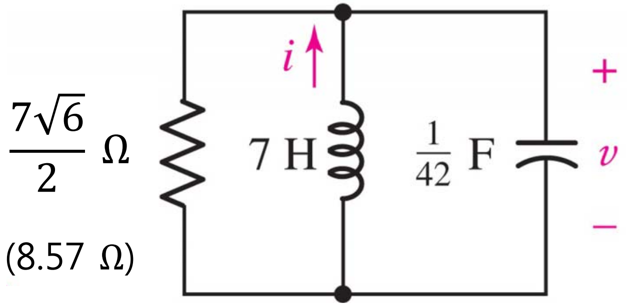
Maximum current: $i_C(t_m = 0) = -2\ A$

Settling time : 10% of maximum value.

$$i_C(t_s) = 2e^{-2t_s} - 4e^{-t_s} = -0.02$$

$$t_s = 5.296\ s$$

$$v(0) = 0\,V, i(0) = 10\,A$$



$$\alpha = \frac{1}{2\frac{7\sqrt{6}}{2}\frac{1}{42}} = \frac{1}{\sqrt{7\frac{1}{42}}} = \sqrt{6} \qquad \omega_0 = \frac{1}{\sqrt{7\frac{1}{42}}} = \sqrt{6}$$

$$s_1 = s_2 = -\alpha = -\sqrt{6}\,s^{-1}$$

$$v(t) = ?$$

$$v(t) = A_1 e^{-\sqrt{6}t} + A_2 e^{-\sqrt{6}t}\ ?$$

$$\boxed{\begin{aligned} v(t) &= A_1 t e^{-\alpha t} + A_2 e^{-\alpha t} \\ &= A_1 t e^{-\sqrt{6}t} + A_2 e^{-\sqrt{6}t} \end{aligned}}$$

$$v(0) = 0 + A_2 = 0 \rightarrow A_2 = 0$$

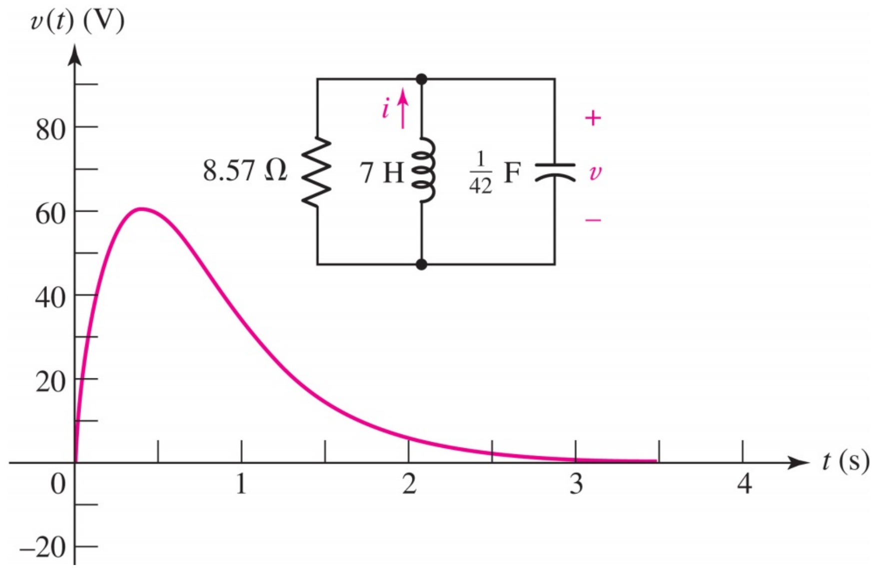
$$\left.\frac{dv}{dt}\right|_{t=0} = -\sqrt{6}A_1 t e^{-\sqrt{6}t} + A_1 e^{-\sqrt{6}t} = A_1$$

$$i_C = C\frac{dv_C}{dt} \rightarrow \left.\frac{dv_C}{dt}\right|_{t=0}$$

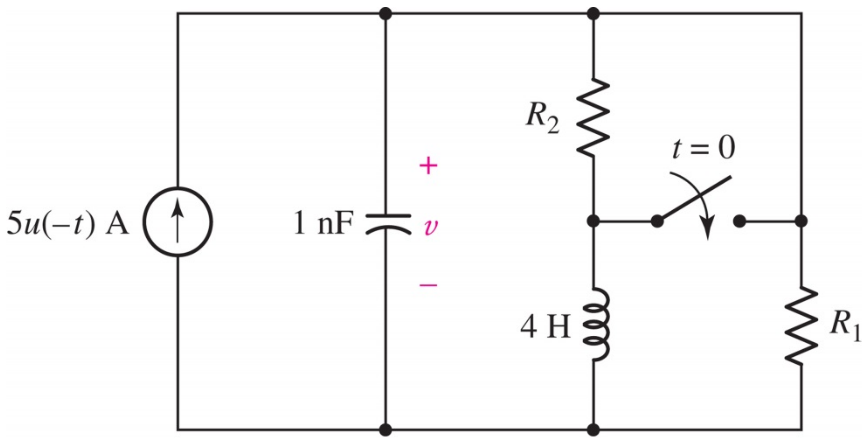
$$= \frac{i_C(0)}{C} = \frac{i_R(0) + i\,(0)}{C} = \frac{10}{\frac{1}{42}} = A_1$$

$$\therefore A_1 = 420$$

$$\boxed{v(t) = 420te^{-\sqrt{6}t}\ V}$$



Example 9.5 Find R_1 such that the circuit is critically damped for $t>0$ and R_2 so that $v(0)=2V$



At $t = 0^-$

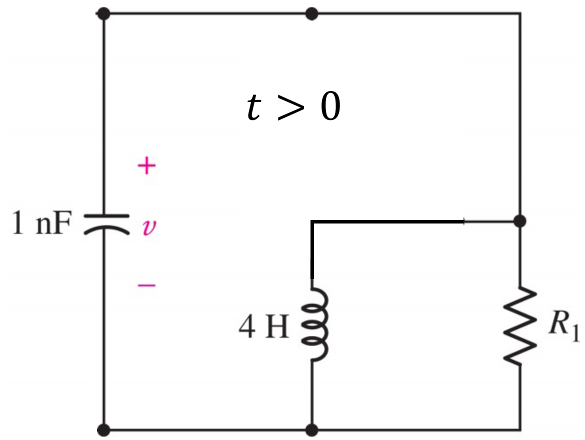
$$v(0^-) = 5(R_1 \parallel R_2) = 2$$

$$\rightarrow (R_1 \parallel R_2) = \frac{2}{5} = 0.4\ \Omega$$

At $t > 0$: soure off, R_2 is shorted

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 10^{-9} R_1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4 \times 10^{-9}}} = 15{,}810\ rad/s$$



For critical damping case

$$\alpha = \omega_0$$

$$\rightarrow R_1 = \frac{1}{2 \times 10^{-9} \times 15810} = 31.63\ k\Omega$$

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \qquad s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

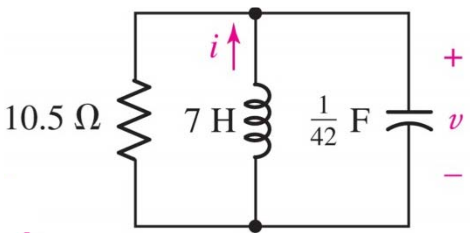
$$\alpha < \omega_0 \qquad \sqrt{\alpha^2 - \omega_0^2} = \sqrt{-1}\sqrt{\omega_0^2 - \alpha^2} = j\sqrt{\omega_0^2 - \alpha^2} = j\omega_d$$ imaginary

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} = A_1 e^{-\alpha + j\omega_d} + A_2 e^{-\alpha - j\omega_d} = e^{-\alpha}(A_1 e^{j\omega_d} + A_2 e^{-j\omega_d})$$

$$v(t) = e^{-\alpha}(A_1 e^{j\omega_d} + A_2 e^{-j\omega_d}) = e^{-\alpha}[A_1(\cos\omega_d + j\sin\omega_d) + A_2(\cos\omega_d - j\sin\omega_d)]$$

$$= e^{-\alpha}[(A_1 + A_2)\cos\omega_d + j(A_1 - A_2)\sin\omega_d] = e^{-\alpha}(B_1\cos\omega_d + B_2\sin\omega_d)$$

$$v(0) = 0\,V, i(0) = 10\,A$$



$$\alpha = \frac{1}{2 \times 10.5 \times \frac{1}{42}} = 2\,s^{-1} \qquad \omega_0 = \frac{1}{\sqrt{7 \times \frac{1}{42}}} = \sqrt{6}\,rad/s$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{2}\,rad/s \qquad v(t) = e^{-2t}(B_1\cos\sqrt{2}t + B_2\sin\sqrt{2}\,t)$$

$$v(0) = B_1 = 0 \qquad \rightarrow \quad v(t) = e^{-2t}B_2\sin\sqrt{2}t$$

$$i_C = C\frac{dv}{dt} \rightarrow \left.\frac{dv}{dt}\right|_{t=0} = \sqrt{2}B_2 e^{-2t}\cos\sqrt{2}t - 2B_2 e^{-2t}\sin\sqrt{2}t\Big|_{t=0} = \sqrt{2}B_2 = \frac{i(0)}{C} = \frac{10}{\frac{1}{42}} = 420$$

$$\therefore B_2 = 210\sqrt{2}$$

$$\boxed{\rightarrow v(t) = 210\sqrt{2}e^{-2t}\sin\sqrt{2}t}$$

- Graphical Representation of Critically Damped Response

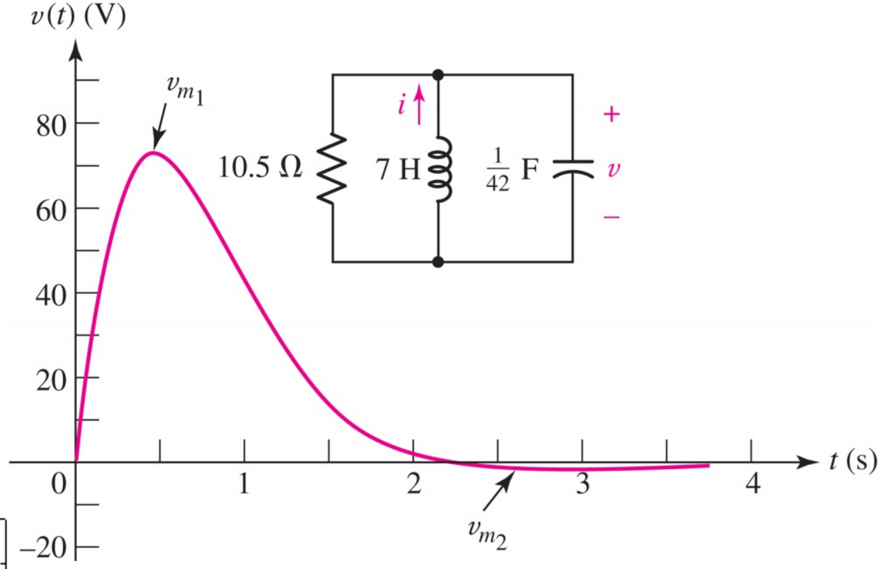
$$v(t) = 210\sqrt{2}e^{-2t}\sin\sqrt{2}t$$

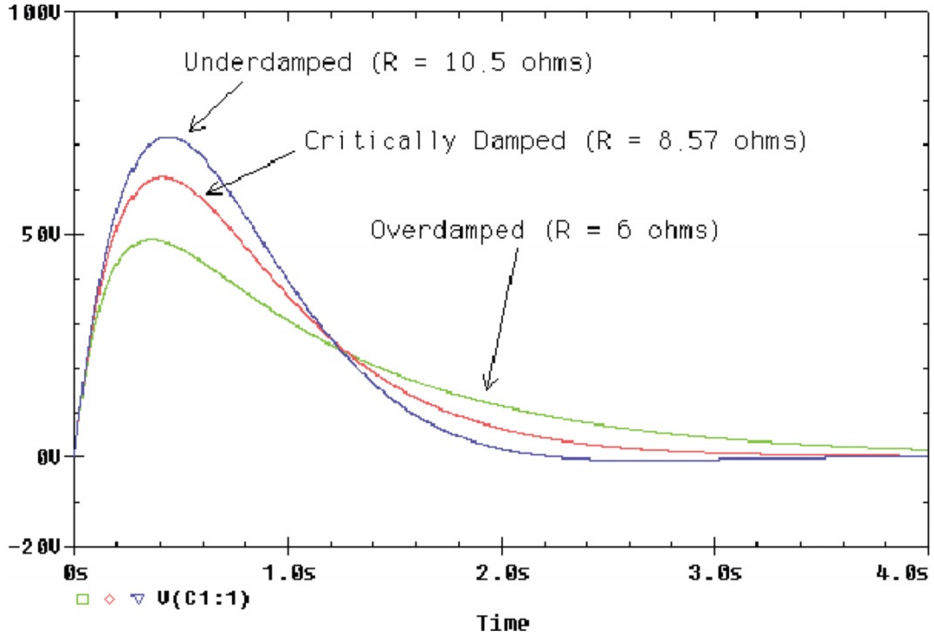
decreasing nature　　oscillating nature

$$\sqrt{2}t = n\pi \;\rightarrow\; v(t) = 0,$$

$$t = \frac{n\pi}{\sqrt{2}} \rightarrow v(t) = 0$$

Damping Oscillation!!





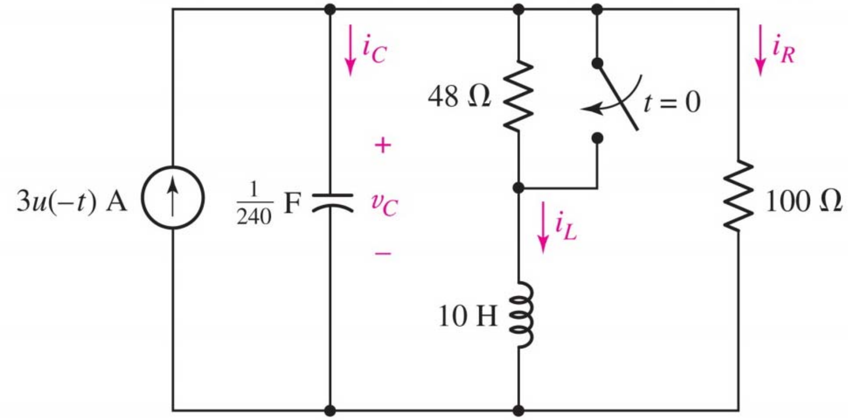
- The Role of Finite Resistance

Example 9.6 Determine $i_L(t)$ and plot the waveform.

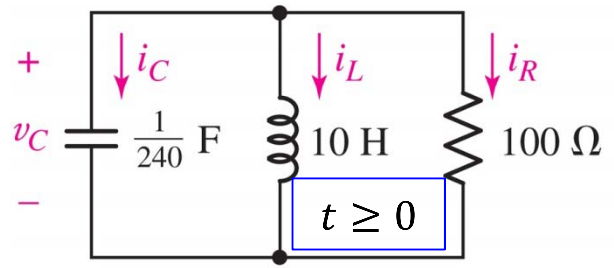
$$\alpha = \frac{1}{2RC} = 1.2\ s^{-1} \qquad \omega_0 = \frac{1}{\sqrt{LC}} = 4.899\ rad/s$$

$$\alpha < \omega_0 \rightarrow \text{underdamped}$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 4.750\ rad/s$$



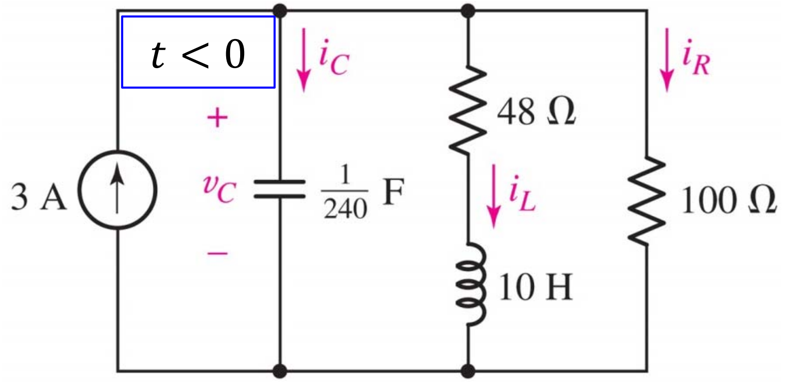
$$\boxed{i_L(t) = e^{-1.2t}(B_1 \cos 4.75t + B_2 \sin 4.75\,t) \quad t > 0}$$



$$i_L(0^-) = \frac{100}{48+100}3 = 2.027\ A$$

$$v_C(0^-) = (48 \parallel 100) \times 3 = \frac{48 \times 100}{48+100}3 = 97.297\ V$$

$$i_L(0^-) = i_L(0^+) = B_1 = 2.027\ A$$



$$v_C(0^+) = L\frac{di_L(0^+)}{dt} = 10\frac{di_L(0^+)}{dt}$$

$$\rightarrow \frac{di_L(0^+)}{dt} = \frac{v_C(0^+)}{10} = \frac{v_C(0^-)}{10} = \frac{97.297}{10} = 9.73$$

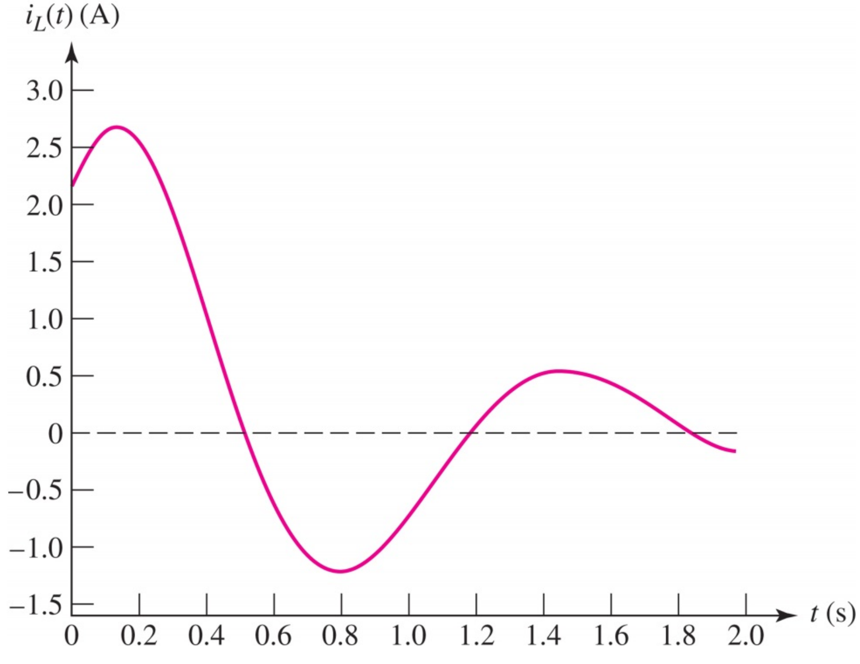
$$\boxed{i_L(t) = e^{-1.2t}(B_1 \cos 4.75t + B_2 \sin 4.75\,t) \quad t > 0}$$

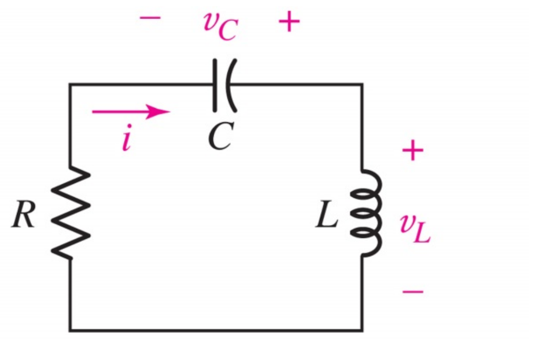
$$\frac{di_L}{dt} = e^{-1.2t}(-4.75B_1 \sin 4.75t + 4.75B_2 \cos 4.75t) - 1.2e^{-1.2t}(B_1 \cos 4.75t + B_2 \sin 4.75\,t)$$

$$\left.\frac{di_L}{dt}\right|_{t=0} = 4.75B_2 - 1.2B_1 = 4.75B_2 - 1.2 \times 2.027 = 9.73$$

$$\Rightarrow B_2 = \frac{9.73 + 1.2 \times 2.027}{4.75} = 2.5605$$

$$i_L(t) = e^{-1.2t}(2.027 \cos 4.75t + 2.5605 \sin 4.75\,t)$$

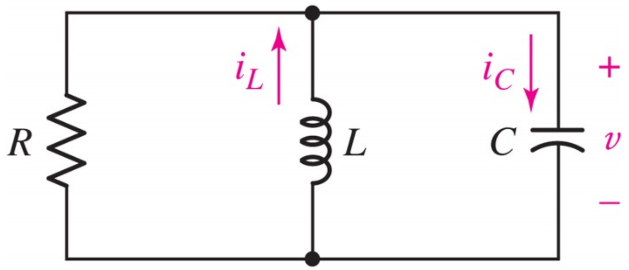




Series *RLC* Circuit

$$L\frac{di}{dt}+Ri+\frac{1}{C}\int_{t_0}^{t} idi' - v_C(t_0)=0$$

$$\rightarrow L\frac{d^2i}{dt^2}+R\frac{di}{dt}+\frac{1}{C}i=0$$



Parallel *RLC* Circuit

$$\frac{v}{R}+\frac{1}{L}\int_{t_0}^{t} vdt' - i(t_0)+C\frac{dv}{dt}=0$$

$$\rightarrow C\frac{d^2v}{dt^2}+\frac{1}{R}\frac{dv}{dt}+\frac{1}{L}v=0$$

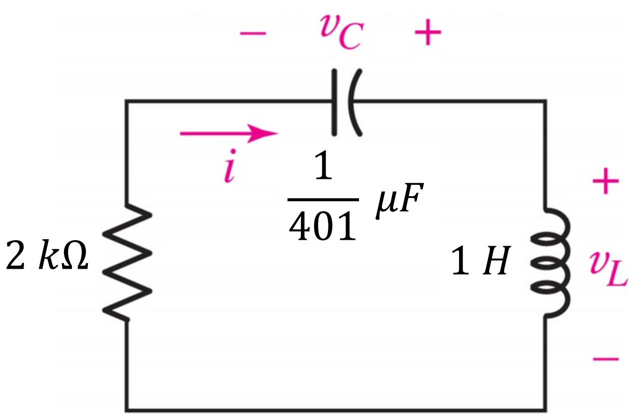
Let $i(t)=Ae^{st}$ $\qquad LAs^2e^{st}+RAse^{st}+\frac{1}{C}Ae^{st}=0 \qquad \rightarrow Ae^{st}\left(Ls^2+Rs+\frac{1}{C}\right)=0$

$$s_1=-\frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}} \qquad s_2=-\frac{R}{2L}-\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}} \qquad \alpha=\frac{R}{2L}, \qquad \omega_0=\frac{1}{\sqrt{LC}}$$

TABLE 9.1 Summary of Relevant Equations for Source-Free *RLC* Circuits

Type	Condition	Criteria	α	ω_0	Response
Parallel Series	Overdamped	$\alpha > \omega_0$	$\frac{1}{2RC}$ $\frac{R}{2L}$	$\frac{1}{\sqrt{LC}}$	$A_1 e^{s_1 t} + A_2 e^{s_2 t}$, where $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega^2}$
Parallel Series	Critically damped	$\alpha = \omega_0$	$\frac{1}{2RC}$ $\frac{R}{2L}$	$\frac{1}{\sqrt{LC}}$	$e^{-\alpha t}(A_1 t + A_2)$
Parallel Series	Underdamped	$\alpha < \omega_0$	$\frac{1}{2RC}$ $\frac{R}{2L}$	$\frac{1}{\sqrt{LC}}$	$e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$, where $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

Example 9.7 Determine i(t)



$i(0) = 2\ mA, \quad v_C(0) = 2\ V$

$\alpha < \omega_0 \rightarrow$ underdamped

$$\alpha = \frac{R}{2L} = 1000\ s^{-1} \quad \omega_0 = \frac{1}{\sqrt{1 \times \frac{1}{401} \times 10^{-6}}} = 20025\ rad/s$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 20000\ rad/s$$

$$i(t) = e^{-1000t}(B_1 \cos 20000t + B_2 \sin 20000t) \quad \rightarrow \quad i(0) = B_1 = 2 \times 10^{-3}\ A$$

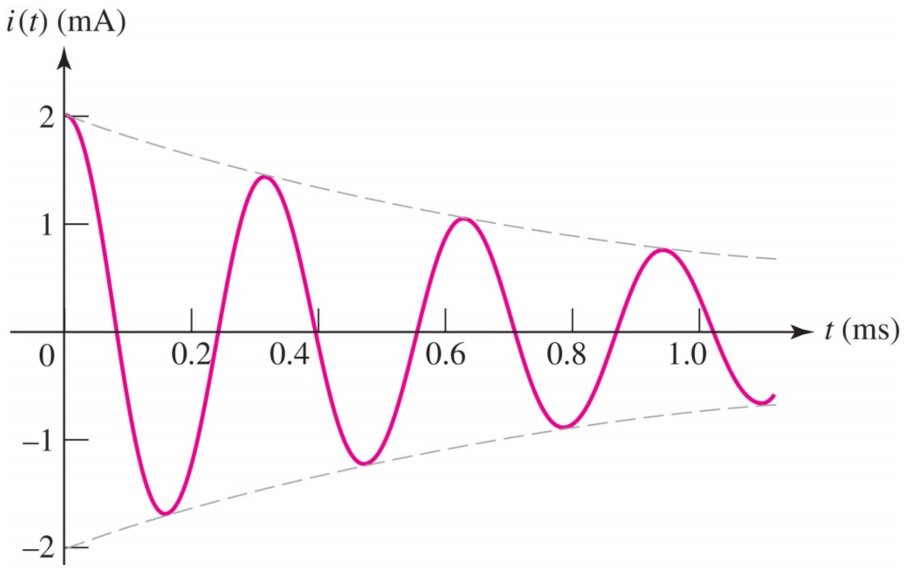
$$L\frac{di}{dt} + Ri - v_C = 0 \rightarrow \frac{di}{dt} = \frac{v_C - 2000i}{L}$$

$$\Rightarrow \frac{di}{dt}\bigg|_{t=0} = \frac{v_C(0) - 2000 \times i(0)}{1} = -2$$

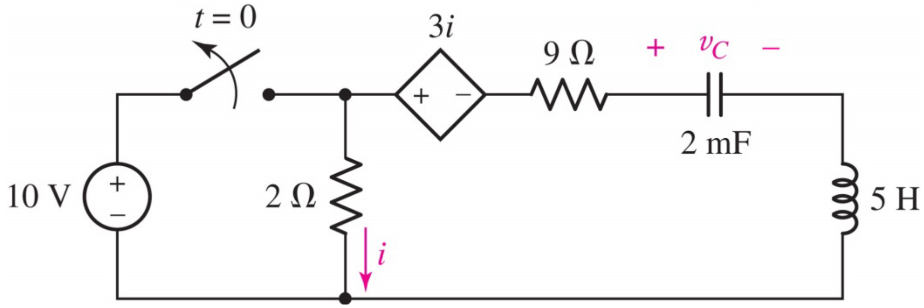
$$\frac{di_L}{dt}\bigg|_{t=0} = 20000B_2 - 1000 \times B_1$$

$$= 20000B_2 - 2 = -2 \quad \Rightarrow B_2 = \frac{0}{20000} = 0$$

$$\boxed{i(t) = 2 \times 10^{-3} e^{-1000t} \cos 20000t\ A \quad t > 0}$$

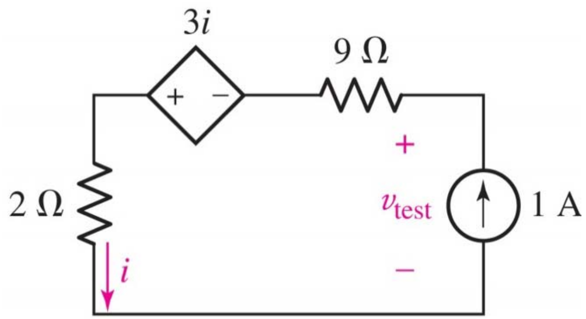


Example 9.8 Determine $v_C(t)$ for $t > 0$.



At $t = 0^-$

$$i_L(0^-) = 0\ A,$$
$$i(0^-) = \frac{10}{2} = 5\ A,$$
$$10 = 3i(0^-) + v_C(0^-)$$
$$\rightarrow v_C(0^-) = 10 - 3 \times 5 = -5\ V$$



$$R_{eq} = \frac{v_{test}}{1\ A} = \frac{11 \times 1 - 3i}{1\ A} = 8\ \Omega$$

$$L = 5,$$
$$C = 2 \times 10^{-3},$$

$$L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0 \quad \Rightarrow \quad 5\frac{d^2 i}{dt^2} + 8\frac{di}{dt} + \frac{1}{2 \times 10^{-3}}i = 0$$

$$\alpha = \frac{R}{2L} = \frac{8}{2 \times 5} = 0.8\ \mathrm{s}^{-1},$$
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 2 \times 10^{-3}}} = 10\ rad/s$$

$$\alpha < \omega_0 \rightarrow \text{underdamped}$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 9.968\ rad/s$$

$$v_C(t) = e^{-0.8t}(B_1 \cos 9.968t + B_2 \sin 9.968t)$$

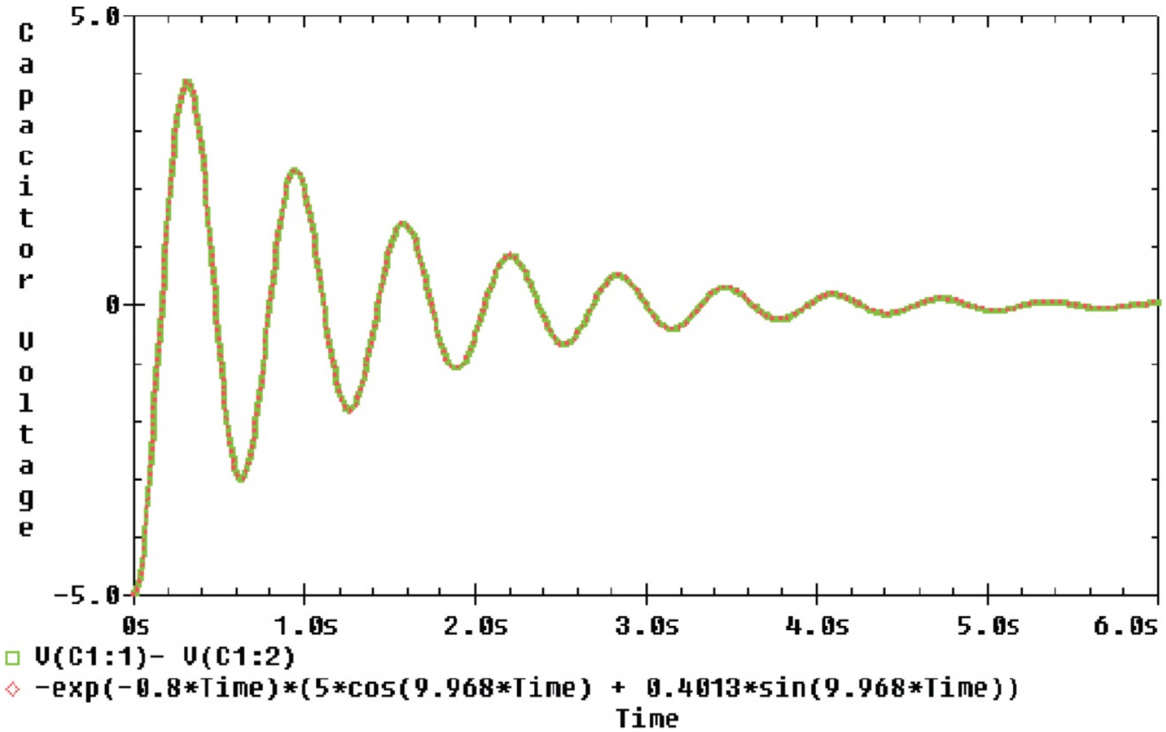
$$v_C(0^-) = v_C(0^+) = B_1 = -5\ V$$

$$i = -C\frac{dv_C}{dt} \rightarrow \frac{dv_C}{dt} = -\frac{i}{C} \quad \Rightarrow \quad \left.\frac{dv_C}{dt}\right|_{t=0} = -\frac{i(0^+)}{C}$$
$$= -\frac{i_L(0^-)}{C} = -\frac{0}{2 \times 10^{-3}} = 0$$

$$\frac{dv_C}{dt} = e^{-0.8t}(-9.968B_1 \sin 9.968t + 9.968B_2 \cos 9.968t) - 0.8e^{-0.8t}(B_1 \cos 9.968t + B_2 \sin 9.968\,t)$$

$$\left.\frac{dv_C}{dt}\right|_{t=0} = 9.968B_2 - 0.8 \times B_1 = 9.968B_2 + 4 = 0 \qquad \Rightarrow \quad B_2 = \frac{-4}{9.968} = -0.4013\,V$$

$$\boxed{v_C(t) = -e^{-0.8t}(5 \cos 9.968t + 0.4013 \sin 9.968t)}$$



- General Solution

1. Determine the **initial conditions**.
2. Obtain a numerical value for the **forced response**.
3. Write the appropriate form of the **natural response** with the necessary number of arbitrary constants.
4. **Add the forced response and natural response** to form the complete response.
5. **Evaluate** the response and its derivative at t = 0, and employ the initial conditions to solve for the values of the unknown constants.

Forced response $v_f(t) = V_f$

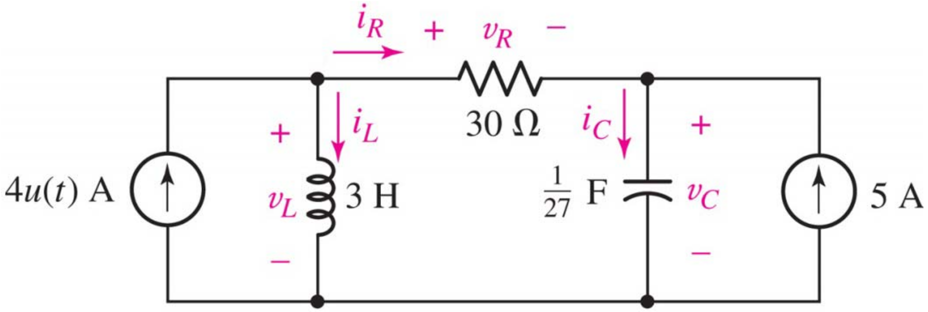
Natural response $v_n(t) = Ae^{s_1 t} + Be^{s_2 t}$ $\Rightarrow$ $v(t) = V_f + Ae^{s_1 t} + Be^{s_2 t}$

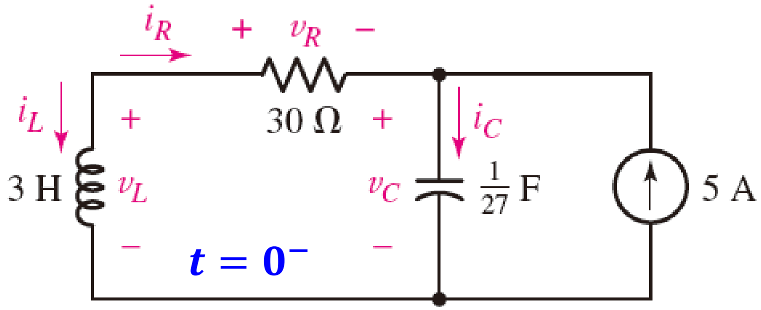
$$v_n(t) = e^{-\alpha}(B_1 \cos \omega_d + B_2 \sin \omega_d)$$

$$v_n(t) = A_1 te^{-\alpha t} + A_2 e^{-\alpha t}$$

$$\frac{dv}{dt} = 0 + s_1 Ae^{s_1 t} + s_2 Be^{s_2 t}$$

Example 9.9 Find the values of these six quantities at both t=0⁻ and t=0⁺.



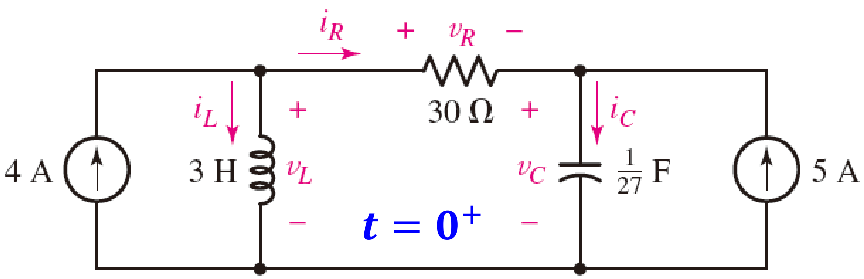


$v_L(0^-) = 0, \quad i_C(0^-) = 0$ for dc current soruce

$i_R(0^-) = -5\ A, \quad v_R(0^-) = 30 \times (-5) = -150\ V,$

$v_C(0^-) = -v_R(0^-) = 150\ V,$

$i_L(0^-) = -i_R(0^-) = 5\ A$



$i_L(0^+) = i_L(0^-) = 5\ A, \quad v_C(0^+) = v_C(0^-) = 150\ V$

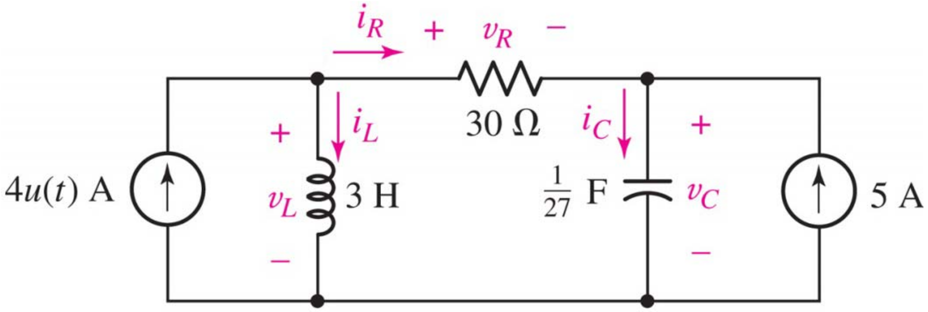
$i_R(0^+) = 4 - i_L(0^+) = 4 - 5 = -1\ A,$

$v_R(0^+) = 30 \times i_R(0^+) = 30 \times (-1) = -30\ V,$

$i_C(0^+) = 5 + i_R(0^+) = 5 - 1 = 4\ A,$

$v_L(0^+) = v_R(0^+) + v_C(0^+) = -30 + 150 = 120\ V$

Example 9.10 Find the determination of the initial conditions



$$v_L = L\frac{di_L}{dt} \rightarrow \left.\frac{di_L}{dt}\right|_{t=0^+} = \frac{v_L(0^+)}{L} = \frac{120}{3} = 40,$$

$$i_C = C\frac{dv_C}{dt} \rightarrow \left.\frac{dv_C}{dt}\right|_{t=0^+} = \frac{i_C(0^+)}{C} = \frac{4}{1/27} = 108,$$

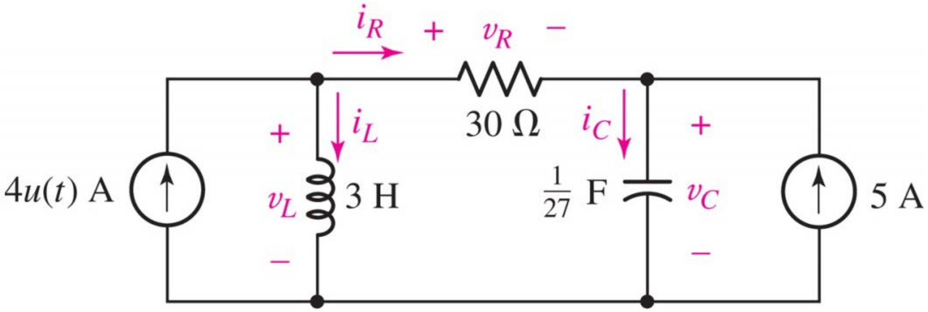
$$i_R = 4 - i_L \rightarrow \left.\frac{di_R}{dt}\right|_{t=0^+} = -\left.\frac{di_L}{dt}\right|_{t=0^+} = -40,$$

$$v_R = 30 \times i_R \rightarrow \left.\frac{dv_R}{dt}\right|_{t=0^+} = 30\left.\frac{di_R}{dt}\right|_{t=0^+} = -1200,$$

$$v_L = v_R + v_C \rightarrow \left.\frac{dv_L}{dt}\right|_{t=0^+} = \left.\frac{dv_R}{dt}\right|_{t=0^+} + \left.\frac{dv_c}{dt}\right|_{t=0^+} = -1200 + 108 = -1092,$$

$$i_C = 5 - (-i_R) \rightarrow \left.\frac{di_C}{dt}\right|_{t=0^+} = \left.\frac{di_R}{dt}\right|_{t=0^+} = -40$$

Find vC(t) for t > 0.



$$\alpha = \frac{R}{2L} = \frac{30}{2\times 3} = 5\ \mathrm{s}^{-1},$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3\times\frac{1}{27}}} = 3\ rad/s$$

$$\alpha > \omega_0 \rightarrow \text{overdamped}$$

$$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -5 \pm 4$$

$$\rightarrow s_1 = -1, s_2 = -9$$

Forced response

$$v_{C_f}(t) = v_C(\infty) = 150\ V$$

Complete response

$$v_c(t) = 150 + Ae^{-t} + Be^{-9t}$$

$$v_c(0^+) = 150 + A + B = 150$$
$$\rightarrow A + B = 0$$

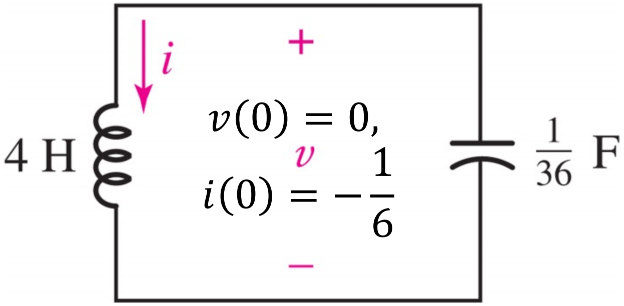
$$\frac{dv_C}{dt} = -Ae^{-t} - 9Be^{-9t} = \frac{i_C(t)}{C}$$

$$\Rightarrow \left.\frac{dv_C}{dt}\right|_{t=0^+} = -A - 9B = \frac{4}{\frac{1}{27}} = 108$$

$$A = 13.5, \qquad B = -13.5$$

$$v_c(t) = 150 + 13.5(e^{-t} - Be^{-9t}) \quad t > 0$$

- The resistor in the RLC circuit serves to dissipate initial stored energy.
- When this resistor becomes 0 in the series RLC or infinite in the parallel RLC, the circuit will oscillate.



$$\alpha = \frac{R}{2L} = 0\ \mathrm{s}^{-1}, \qquad \omega_0 = \frac{1}{\sqrt{4 \times \frac{1}{36}}} = 3\ rad/s$$

$$\alpha < \omega_0 \rightarrow \text{underdamped}$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 3$$

$$v = A\cos 3t + B\sin 3t\,,$$

$$i = -C\frac{dv}{dt} \rightarrow \left.\frac{dv}{dt}\right|_{t=0} = -\frac{i(0)}{C} = -\frac{-\frac{1}{6}}{\frac{1}{36}} = 6$$

$$v(0) = A = 0$$

$$\frac{dv}{dt} = 3B\cos 3t \rightarrow \left.\frac{dv}{dt}\right|_{t=0} = 3B = 6$$

$$\Rightarrow B = 2$$

$$v(t) = 2\sin 3t \quad t > 0$$

Does not decay

$$\frac{1}{4}\int_0^t v\,dt' - i(0) + \frac{1}{36}\frac{dv}{dt} = 0 \rightarrow \frac{1}{4}v + \frac{1}{36}\frac{d^2v}{dt^2} = 0 \quad \Rightarrow \frac{d^2v}{dt^2} + 9v = 0 \quad \Rightarrow v(t) = \sin 3t$$

General solution

Homework : 9장 Exercises 6의 배수 문제

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