

## Lecture 4

# How to Value Bonds and Stocks

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How to Value Bonds

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Growth Opportunities

The Dividend Growth Model and the NPVGO Model (Advanced)

Price Earnings Ratio

Stock Market Reporting

# Valuation of Bonds and Stock

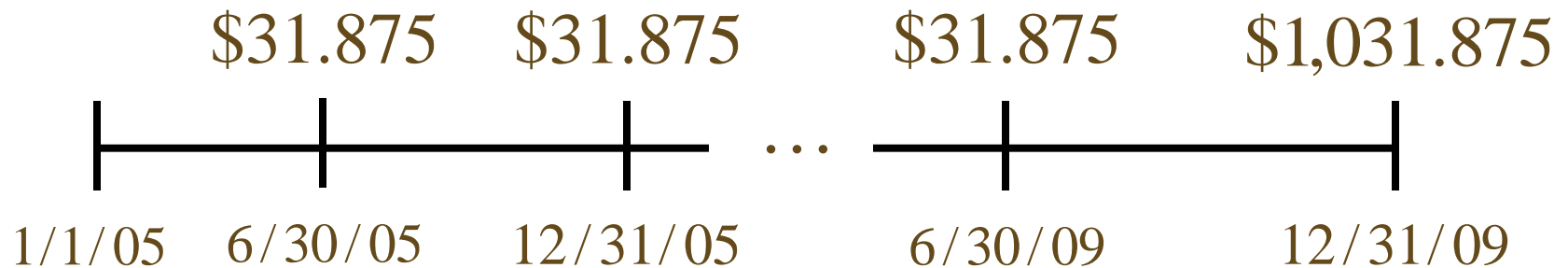
- First Principles:
  - ▣ Value of financial securities = PV of expected future cash flows
- To value bonds and stocks we need to:
  - ▣ Estimate future cash flows:
    - Size (how much) and
    - Timing (when)
  - ▣ Discount future cash flows at an appropriate rate:
    - The rate should be appropriate to the risk presented by the security.

# Definition and Example of a Bond

- A bond is a legally binding agreement between a borrower and a lender:
  - Specifies the principal amount of the loan.
  - Specifies the size and timing of the cash flows:
    - In dollar terms (fixed-rate borrowing)
    - As a formula (adjustable-rate borrowing)

# Definition and Example of a Bond

- Consider a U.S. government bond listed as 6 3/8 of December 2009.
  - The *Par Value* of the bond is \$1,000.
  - *Coupon payments* are made semi-annually (June 30 and December 31 for this particular bond).
  - Since the *coupon rate* is 6 3/8 the payment is \$31.875.
  - On January 1, 2005 the size and timing of cash flows are:



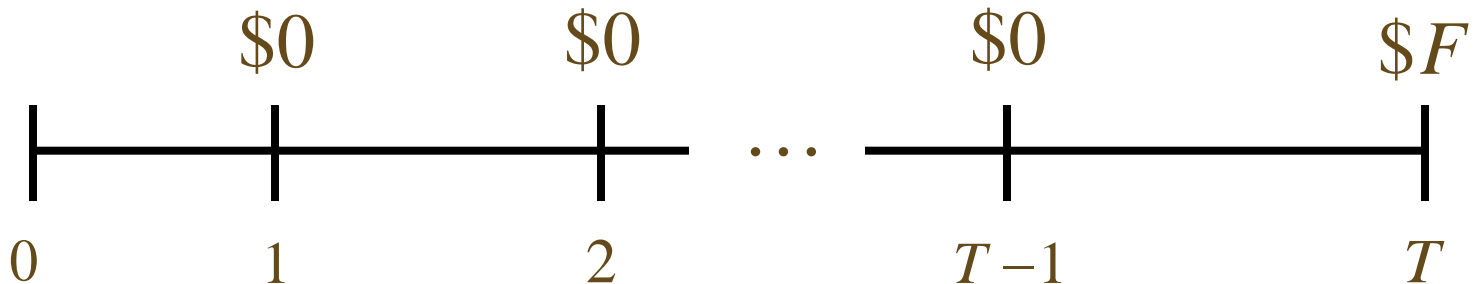
## 5.2 How to Value Bonds

- Identify the size and timing of cash flows.
- Discount at the correct discount rate.
  - ▣ If you know the price of a bond and the size and timing of cash flows, the *yield to maturity* is the discount rate.

# Pure Discount Bonds

## Information needed for valuing pure discount bonds:

- ▣ Time to maturity ( $T$ ) = Maturity date - today's date
- ▣ Face value ( $F$ )
- ▣ Discount rate ( $r$ )

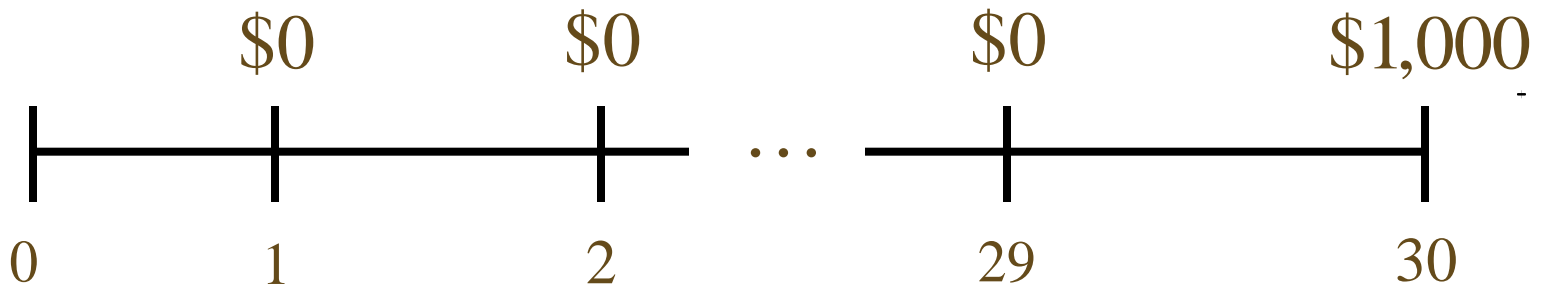


Present value of a pure discount bond at time 0:

$$PV = \frac{F}{(1+r)^T}$$

# Pure Discount Bonds: Example

Find the value of a 30-year zero-coupon bond with a \$1,000 par value and a YTM of 6%.



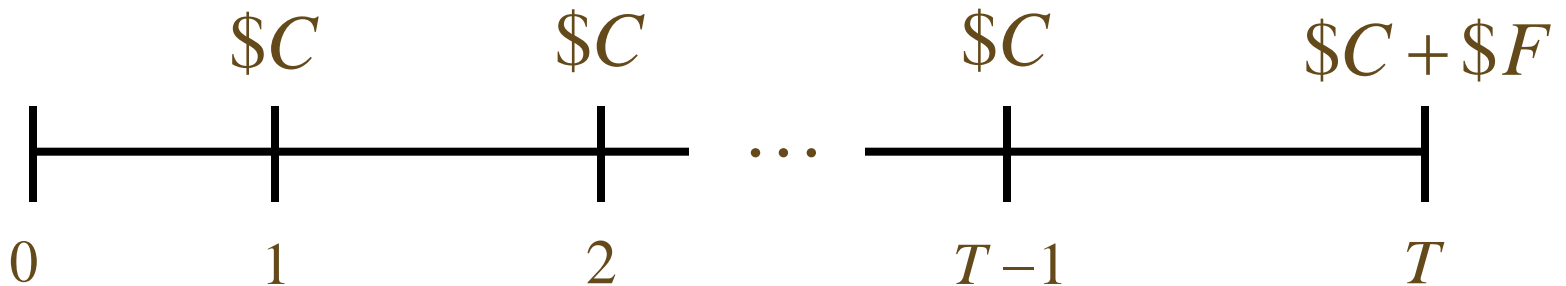
$$PV = \frac{F}{(1+r)^T} = \frac{\$1,000}{(1.06)^{30}} = \$174.11$$



# Level-Coupon Bonds

## Information needed to value level-coupon bonds:

- Coupon payment dates and time to maturity ( $T$ )
- Coupon payment ( $C$ ) per period and Face value ( $F$ )
- Discount rate



Value of a Level-coupon bond

= PV of coupon payment annuity + PV of face value

$$PV = \frac{C}{r} \left[ 1 - \frac{1}{(1+r)^T} \right] + \frac{F}{(1+r)^T}$$

# Level-Coupon Bonds: Example

Find the present value (as of January 1, 2004), of a 6-3/8 coupon T-bond with semi-annual payments, and a maturity date of December 2009 if the YTM is 5-percent.

- On January 1, 2004 the size and timing of cash flows are:

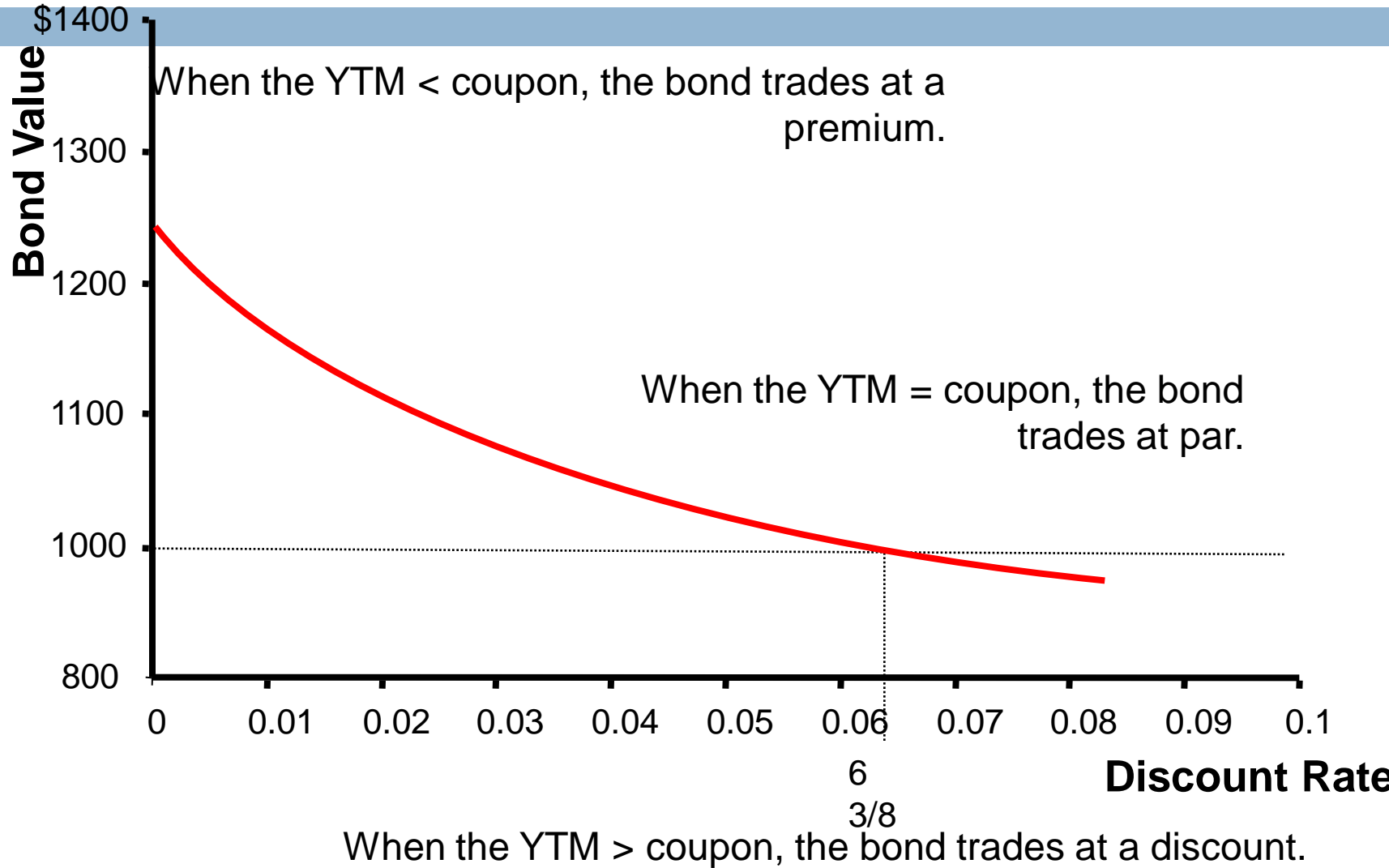


$$PV = \frac{\$31.875}{.05/2} \left[ 1 - \frac{1}{(1.025)^{12}} \right] + \frac{\$1,000}{(1.025)^{12}} = \$1,070.52$$

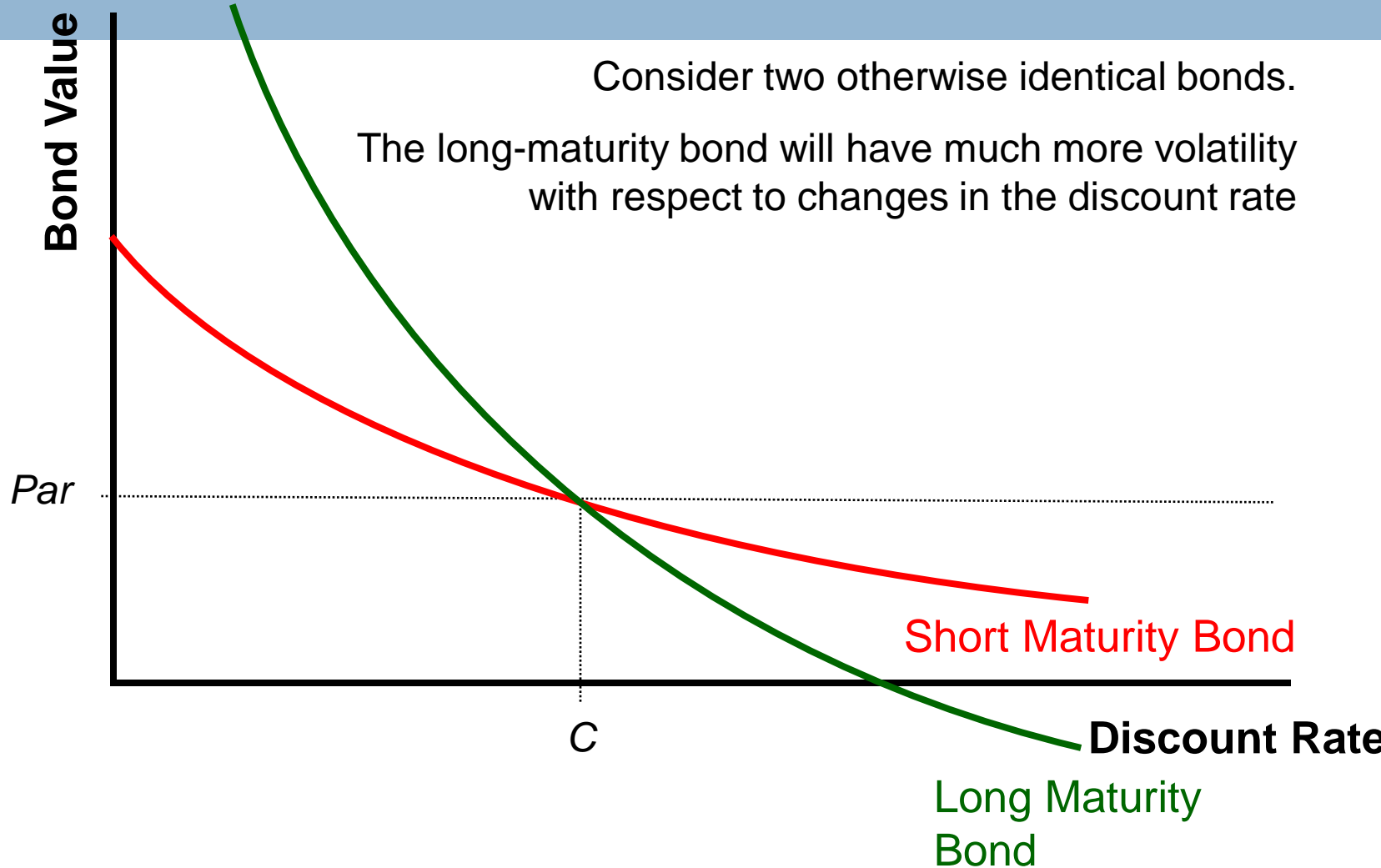
# Bond Concepts

1. Bond prices and market interest rates move in opposite directions.
2. When coupon rate = YTM, price = par value.  
When coupon rate  $>$  YTM, price  $>$  par value (premium bond)  
When coupon rate  $<$  YTM, price  $<$  par value (discount bond)
3. A bond with longer maturity has higher relative (%) price change than one with shorter maturity when interest rate (YTM) changes. All other features are identical.
4. A lower coupon bond has a higher relative price change than a higher coupon bond when YTM changes. All other features are identical.

# YTM and Bond Value

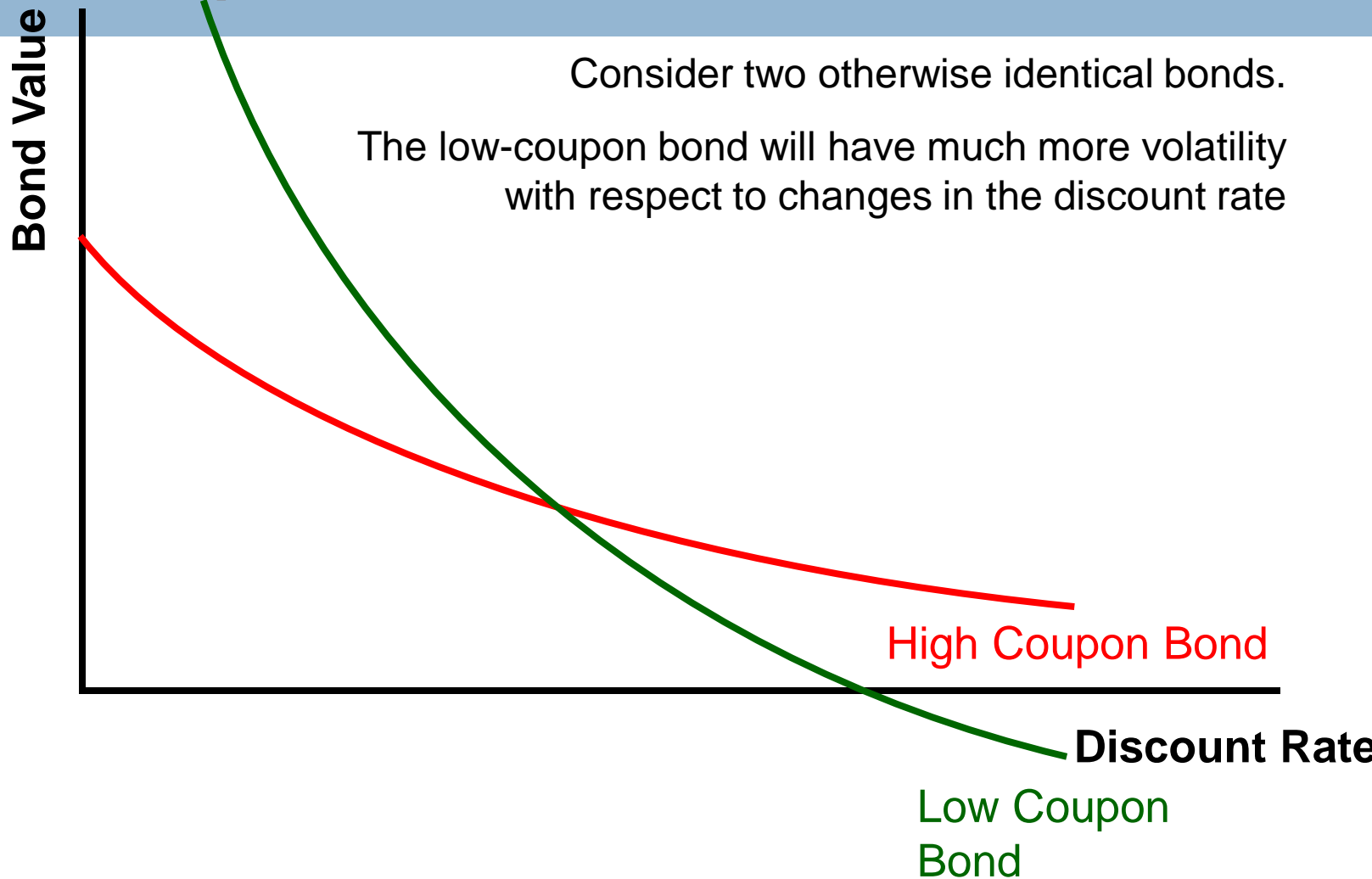


# Maturity and Bond Price Volatility



# Coupon Rate and Bond Price

## Volatility



# The Present Value of Common Stocks

- Dividends versus Capital Gains
- Valuation of Different Types of Stocks
  - ▣ Zero Growth
  - ▣ Constant Growth
  - ▣ Differential Growth

# Case 1: Zero Growth

- Assume that dividends will remain at the same level forever

$$\text{Div}_1 = \text{Div}_2 = \text{Div}_3 = \dots$$

- Since future cash flows are constant, the value of a zero growth stock is the present value of a perpetuity:

$$P_0 = \frac{\text{Div}_1}{(1+r)^1} + \frac{\text{Div}_2}{(1+r)^2} + \frac{\text{Div}_3}{(1+r)^3} + \dots$$

$$P_0 = \frac{\text{Div}}{r}$$



## Case 2: Constant Growth

Assume that dividends will grow at a constant rate,  $g$ , forever. *i.e.*

$$\text{Div}_1 = \text{Div}_0(1 + g)$$

$$\text{Div}_2 = \text{Div}_1(1 + g) = \text{Div}_0(1 + g)^2$$

$$\text{Div}_3 = \text{Div}_2(1 + g) = \text{Div}_0(1 + g)^3$$

⋮

Since future cash flows grow at a constant rate forever, the value of a constant growth stock is the present value of a growing perpetuity:

$$P_0 = \frac{\text{Div}_1}{r - g}$$

# Case 3: Differential Growth

- Assume that dividends will grow at different rates in the foreseeable future and then will grow at a constant rate thereafter.
- To value a Differential Growth Stock, we need to:
  - ▣ Estimate future dividends in the foreseeable future.
  - ▣ Estimate the future stock price when the stock becomes a Constant Growth Stock (case 2).
  - ▣ Compute the total present value of the estimated future dividends and future stock price at the appropriate discount rate.

# Case 3: Differential Growth

- Assume that dividends will grow at rate  $g_1$  for  $N$  years and grow at rate  $g_2$  thereafter

$$\text{Div}_1 = \text{Div}_0(1 + g_1)$$

$$\text{Div}_2 = \text{Div}_1(1 + g_1) = \text{Div}_0(1 + g_1)^2$$

⋮

$$\text{Div}_N = \text{Div}_{N-1}(1 + g_1) = \text{Div}_0(1 + g_1)^N$$

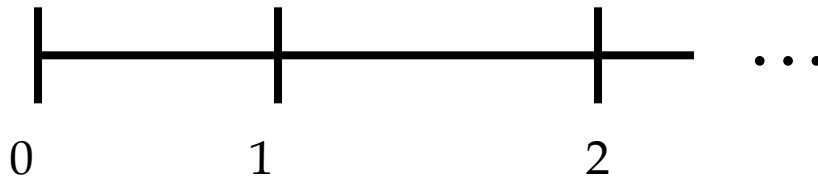
$$\text{Div}_{N+1} = \text{Div}_N(1 + g_2) = \text{Div}_0(1 + g_1)^N(1 + g_2)$$

⋮

# Case 3: Differential Growth

- Dividends will grow at rate  $g_1$  for  $N$  years and grow at rate  $g_2$  thereafter

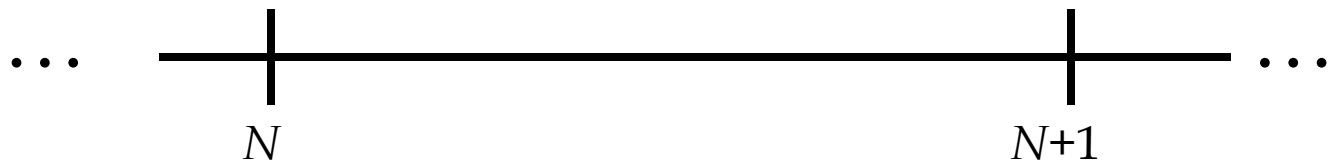
$$\text{Div}_0(1+g_1) \quad \text{Div}_0(1+g_1)^2$$



$$\text{Div}_0(1+g_1)^N$$

$$\text{Div}_N(1+g_2)$$

$$= \text{Div}_0(1+g_1)^N(1+g_2)$$



# Case 3: Differential Growth

We can value this as the sum of:

an  $N$ -year annuity growing at rate  $g_1$

$$P_A = \frac{C}{r - g_1} \left[ 1 - \frac{(1 + g_1)^T}{(1 + r)^T} \right]$$

plus the discounted value of a perpetuity growing at rate  $g_2$  that starts in year  $N+1$

$$P_B = \frac{\left( \frac{\text{Div}_{N+1}}{r - g_2} \right)}{(1 + r)^N}$$

# Case 3: Differential Growth

To value a Differential Growth Stock, we can use

$$P = \frac{C}{r - g_1} \left[ 1 - \frac{(1 + g_1)^T}{(1 + r)^T} \right] + \frac{\left( \frac{\text{Div}_{N+1}}{r - g_2} \right)}{(1 + r)^N}$$

- Or we can cash flow it out.

# A Differential Growth Example

*A common stock just paid a dividend of \$2. The dividend is expected to grow at 8% for 3 years, then it will grow at 4% in perpetuity.*

*What is the stock worth? The discount rate is 12%.*

# With the Formula

$$P = \frac{C}{r - g_1} \left[ 1 - \frac{(1 + g_1)^T}{(1 + r)^T} \right] + \frac{\left( \frac{\text{Div}_{N+1}}{r - g_2} \right)}{(1 + r)^N}$$

$$P = \frac{\$2 \times (1.08)}{.12 - .08} \left[ 1 - \frac{(1.08)^3}{(1.12)^3} \right] + \frac{\left( \frac{\$2(1.08)^3(1.04)}{.12 - .04} \right)}{(1.12)^3}$$

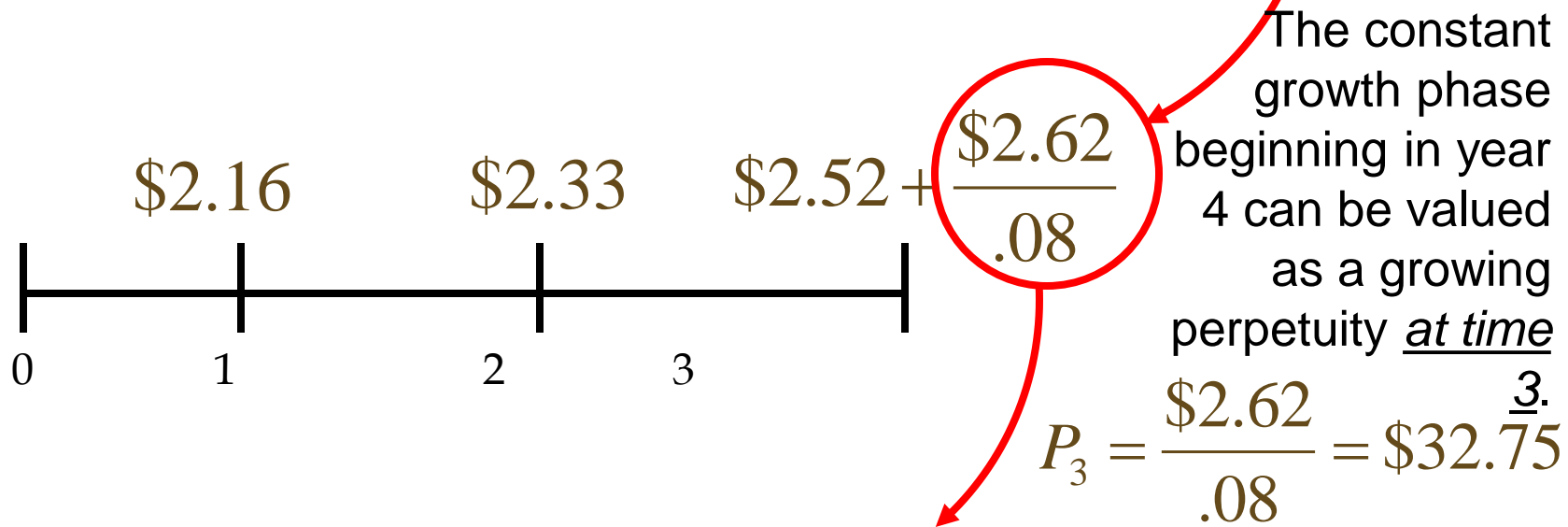
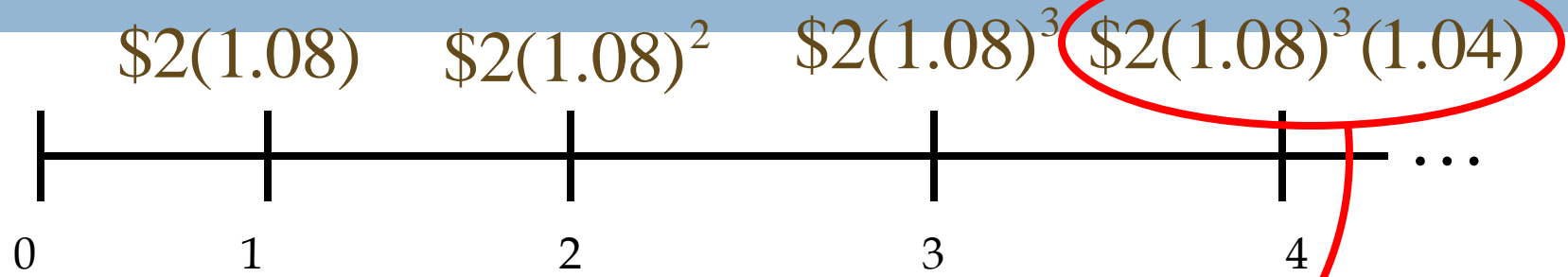
$$P = \$54 \times [1 - .8966] + \frac{(\$32.75)}{(1.12)^3}$$

$$P = \$5.58 + \$23.31$$

$$P = \$28.89$$



# A Differential Growth Example (continued)



$$P_0 = \frac{\$2.16}{1.12} + \frac{\$2.33}{(1.12)^2} + \frac{\$2.52 + \$32.75}{(1.12)^3} = \$28.89$$

# Estimates of Parameters in the Dividend-Discount Model

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- The value of a firm depends upon its growth rate,  $g$ , and its discount rate,  $r$ .
  - Where does  $g$  come from?
  - Where does  $r$  come from?

# Where does $g$ come from?

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$g = \text{Retention ratio} \times \text{Return on retained earnings}$

# Where does $r$ come from?

- The discount rate can be broken into two parts.
  - ▣ The dividend yield
  - ▣ The growth rate (in dividends)
- In practice, there is a great deal of estimation error involved in estimating  $r$ .

# Growth Opportunities

- Growth opportunities are opportunities to invest in positive NPV projects.
- The value of a firm can be conceptualized as the sum of the value of a firm that pays out 100-percent of its earnings as dividends and the net present value of the growth opportunities.

$$P = \frac{EPS}{r} + NPVGO$$

# The Dividend Growth Model and the NPVGO Model (Advanced)

- We have two ways to value a stock:
  - ▣ The dividend discount model.
  - ▣ The price of a share of stock can be calculated as the sum of its price as a cash cow plus the per-share value of its growth opportunities.

# The Dividend Growth Model and the NPVGO Model

Consider a firm that has EPS of \$5 at the end of the first year, a dividend-payout ratio of 30%, a discount rate of 16-percent, and a return on retained earnings of 20-percent.

- The dividend at year one will be  $\$5 \times .30 = \$1.50$  per share.
- The retention ratio is .70 ( $= 1 - .30$ ) implying a growth rate in dividends of  $14\% = .70 \times 20\%$

From the dividend growth model, the price of a share is:

$$P_0 = \frac{\text{Div}_1}{r - g} = \frac{\$1.50}{.16 - .14} = \$75$$

# The NPVGO Model

First, we must calculate the value of the firm as a cash cow.

$$P_0 = \frac{\text{Div}_1}{r} = \frac{\$5}{.16} = \$31.25$$

Second, we must calculate the value of the growth opportunities.

$$P_0 = \frac{\left[ -3.50 + \frac{3.50 \times .20}{.16} \right]}{r - g} = \frac{\$.875}{.16 - .14} = \$43.75$$

Finally,  $P_0 = 31.25 + 43.75 = \$75$



# 5.8 Price Earnings Ratio

- Many analysts frequently relate earnings per share to price.
- The price earnings ratio is a.k.a. the *multiple*
  - ▣ Calculated as current stock price divided by annual EPS
  - ▣ *The Wall Street Journal* uses last 4 quarter's earnings

$$\text{P/E ratio} = \frac{\text{Price per share}}{EPS}$$

- Firms whose shares are “in fashion” sell at high multiples. *Growth stocks* for example.
- Firms whose shares are out of favor sell at low multiples. *Value stocks* for example.

# Other Price Ratio Analysis

- Many analysts frequently relate earnings per share to variables other than price, e.g.:

- Price/Cash Flow Ratio

- cash flow = net income + depreciation = cash flow from operations or operating cash flow

- Price/Sales

- current stock price divided by annual sales per share

- Price/Book (a.k.a. Market to Book Ratio)

- price divided by book value of equity, which is measured as assets – liabilities

# Stock Market Reporting

52 WEEKS				YLD		VOL				NET
HI	LO	STOCKSYM	DIV	%	PE	100s	HI	LO	CLOSE	CHG
52.75	19.06	Gap Inc GPS	0.09	0.5	15	65172	20.50	19	19.25	-1.75

Gap has been as high as \$52.75 in the last year.

Gap has been as low as \$19.06 in the last year.

Gap pays a dividend of 9 cents/share

Given the current price, the dividend yield is  $\frac{1}{2}$  %

Given the current price, the PE ratio is 15 times earnings

Gap ended trading at \$19.25, down \$1.75 from yesterday's close

6,517,200 shares traded hands in the last day's trading

# Stock Market Reporting

52 WEEKS					YLD		VOL				NET
HI	LO	STOCKSYM	DIV	%	PE	100s	HI	LO	CLOSE	CHG	
52.75	19.06	Gap Inc	GPS	0.09	0.5	15	65172	20.50	19	19.25	-1.75

Gap Incorporated is having a tough year, trading near their 52-week low.

Imagine how you would feel if within the past year you had paid \$52.75 for a share of Gap and now had a share worth \$19.25! That 9-cent dividend wouldn't go very far in making amends.

Yesterday, Gap had another rough day in a rough year. Gap “opened the day down” beginning trading at \$20.50, which was down from the previous close of \$21.00 = \$19.25 + \$1.75

Looks like cargo pants aren't the only things on sale at Gap.