Lecture 9

The Capital Asset Pricing Model (CAPM)

The Capital Asset Pricing Model (CAPM)

Individual Securities

Expected Return, Variance, and Covariance

The Return and Risk for Portfolios

The Efficient Set for Two Assets

The Efficient Set for Many Securities

Diversification: An Example

Riskless Borrowing and Lending

Market Equilibrium

Relationship between Risk and Expected Return (CAPM)

Individual Securities

- The characteristics of individual securities that are of interest are the:
  - Expected Return
  - Variance and Standard Deviation
  - Covariance and Correlation

Expected Return, Variance, and Covariance

		Rate of Return	
Scenario	**Probability**	**Stock fund**	**Bond fund**
Recession	33.3%	-7%	17%
Normal	33.3%	12%	7%
Boom	33.3%	28%	-3%

Consider the following two risky asset world. There is a 1/3 chance of each state of the economy and the only assets are a stock fund and a bond fund.

Expected Return, Variance, and Covariance

	Stock fund		Bond Fund	
Scenario	**Rate of Return**	**Squared Deviation**	**Rate of Return**	**Squared Deviation**
Recession	-7%	3.24%	17%	1.00%
Normal	12%	0.01%	7%	0.00%
Boom	28%	2.89%	-3%	1.00%
Expected return	11.00%		7.00%	
Variance	0.0205		0.0067	
Standard Deviation	14.3%		8.2%	

Expected Return, Variance, and Covariance

	Stock fund		Bond Fund	
Scenario	***Rate of Return***	***Squared Deviation***	***Rate of Return***	***Squared Deviation***
Recession	-7%	3.24%	17%	1.00%
Normal	12%	0.01%	7%	0.00%
Boom	28%	2.89%	-3%	1.00%
Expected return	11.00%		7.00%	
Variance	0.0205		0.0067	
Standard Deviation	14.3%		8.2%	

$$E(r_S) = \tfrac{1}{3} \times (-7\%) + \tfrac{1}{3} \times (12\%) + \tfrac{1}{3} \times (28\%)$$

$$E(r_S) = 11\%$$

Expected Return, Variance, and Covariance

Scenario	***Stock fund***		***Bond Fund***	
	Rate of Return	***Squared Deviation***	***Rate of Return***	***Squared Deviation***
Recession	-7%	3.24%	17%	1.00%
Normal	12%	0.01%	7%	0.00%
Boom	28%	2.89%	-3%	1.00%
Expected return	11.00%		7.00%	
Variance	0.0205		0.0067	
Standard Deviation	14.3%		8.2%	

$$E(r_B) = \frac{1}{3} \times (17\%) + \frac{1}{3} \times (7\%) + \frac{1}{3} \times (-3\%)$$

$$E(r_B) = 7\%$$

Expected Return, Variance, and Covariance

	Stock fund		Bond Fund	
Scenario	***Rate of Return***	***Squared Deviation***	***Rate of Return***	***Squared Deviation***
Recession	-7%	3.24%	17%	1.00%
Normal	12%	0.01%	7%	0.00%
Boom	28%	2.89%	-3%	1.00%
Expected return	11.00%		7.00%	
Variance	0.0205		0.0067	
Standard Deviation	14.3%		8.2%	

$$(11\% - -7\%)^2 = 3.24\%$$

Expected Return, Variance, and Covariance

	Stock fund		Bond Fund	
Scenario	**Rate of Return**	**Squared Deviation**	**Rate of Return**	**Squared Deviation**
Recession	-7%	3.24%	17%	1.00%
Normal	12%	0.01%	7%	0.00%
Boom	28%	2.89%	-3%	1.00%
Expected return	11.00%		7.00%	
Variance	0.0205		0.0067	
Standard Deviation	14.3%		8.2%	

$$(11\% - 12\%)^2 = .01\%$$

Expected Return, Variance, and Covariance

Scenario	Stock fund Rate of Return	Stock fund Squared Deviation	Bond Fund Rate of Return	Bond Fund Squared Deviation
Recession	-7%	3.24%	17%	1.00%
Normal	12%	0.01%	7%	0.00%
Boom	28%	2.89%	-3%	1.00%
Expected return	11.00%		7.00%	
Variance	0.0205		0.0067	
Standard Deviation	14.3%		8.2%	

$$(11\% - 28\%)^2 = 2.89\%$$

Expected Return, Variance, and Covariance

	Stock fund		Bond Fund	
Scenario	***Rate of Return***	***Squared Deviation***	***Rate of Return***	***Squared Deviation***
Recession	-7%	3.24%	17%	1.00%
Normal	12%	0.01%	7%	0.00%
Boom	28%	2.89%	-3%	1.00%
Expected return	11.00%		7.00%	
Variance	0.0205		0.0067	
Standard Deviation	14.3%		8.2%	

$$2.05\% = \frac{1}{3}(3.24\% + 0.01\% + 2.89\%)$$

Expected Return, Variance, and Covariance

	Stock fund		Bond Fund	
Scenario	**Rate of Return**	**Squared Deviation**	**Rate of Return**	**Squared Deviation**
Recession	-7%	3.24%	17%	1.00%
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Expected return	11.00%		7.00%	
Variance	0.0205		0.0067	
Standard Deviation	14.3%		8.2%	

$$14.3\% = \sqrt{0.0205}$$

The Return and Risk for Portfolios

Scenario	Stock fund Rate of Return	Stock fund Squared Deviation	Bond Fund Rate of Return	Bond Fund Squared Deviation
Recession	-7%	3.24%	17%	1.00%
Normal	12%	0.01%	7%	0.00%
Boom	28%	2.89%	-3%	1.00%
Expected return	11.00%		7.00%	
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Standard Deviation	14.3%		8.2%	

Note that stocks have a higher expected return than bonds and higher risk. Let us turn now to the risk-return tradeoff of a portfolio that is 50% invested in bonds and 50% invested in stocks.

The Return and Risk for Portfolios

	Rate of Return			
Scenario	***Stock fund***	***Bond fund***	***Portfolio***	***squared deviation***
Recession	-7%	17%	5.0%	0.160%
Normal	12%	7%	9.5%	0.003%
Boom	28%	-3%	12.5%	0.123%
Expected return	11.00%	7.00%	9.0%	
Variance	0.0205	0.0067	0.0010	
Standard Deviation	14.31%	8.16%	3.08%	

The rate of return on the portfolio is a weighted average of the returns on the stocks and bonds in the portfolio:

$$r_P = w_B r_B + w_S r_S$$

$$5\% = 50\% \times (-7\%) + 50\% \times (17\%)$$

10.3 The Return and Risk for Portfolios

	Rate of Return			
Scenario	***Stock fund***	***Bond fund***	***Portfolio***	***squared deviation***
Recession	-7%	17%	5.0%	0.160%
Normal	12%	7%	9.5%	0.003%
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The rate of return on the portfolio is a weighted average of the returns on the stocks and bonds in the portfolio:

$$r_P = w_B r_B + w_S r_S$$

$$9.5\% = 50\% \times (12\%) + 50\% \times (7\%)$$

10.3 The Return and Risk for Portfolios

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The rate of return on the portfolio is a weighted average of the returns on the stocks and bonds in the portfolio:

$$r_P = w_B r_B + w_S r_S$$

$$12.5\% = 50\% \times (28\%) + 50\% \times (-3\%)$$

The Return and Risk for Portfolios

	Rate of Return			
Scenario	***Stock fund***	***Bond fund***	***Portfolio***	***squared deviation***
Recession	-7%	17%	5.0%	0.160%
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Expected return	11.00%	7.00%	9.0%	
Variance	0.0205	0.0067	0.0010	
Standard Deviation	14.31%	8.16%	3.08%	

The *expected* rate of return on the portfolio is a weighted average of the *expected* returns on the securities in the portfolio.

$$E(r_P) = w_B E(r_B) + w_S E(r_S)$$

$$9\% = 50\% \times (11\%) + 50\% \times (7\%)$$

The Return and Risk for Portfolios

	Rate of Return			
Scenario	***Stock fund***	***Bond fund***	***Portfolio***	***squared deviation***
Recession	-7%	17%	5.0%	0.160%
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Standard Deviation	14.31%	8.16%	3.08%	

The variance of the rate of return on the two risky assets portfolio is

$$\sigma_P^2 = (w_B\sigma_B)^2 + (w_S\sigma_S)^2 + 2(w_B\sigma_B)(w_S\sigma_S)\rho_{BS}$$

where ρ_{BS} is the correlation coefficient between the returns on the stock and bond funds.

The Return and Risk for Portfolios

	Rate of Return			
Scenario	***Stock fund***	***Bond fund***	***Portfolio***	***squared deviation***
Recession	-7%	17%	5.0%	0.160%
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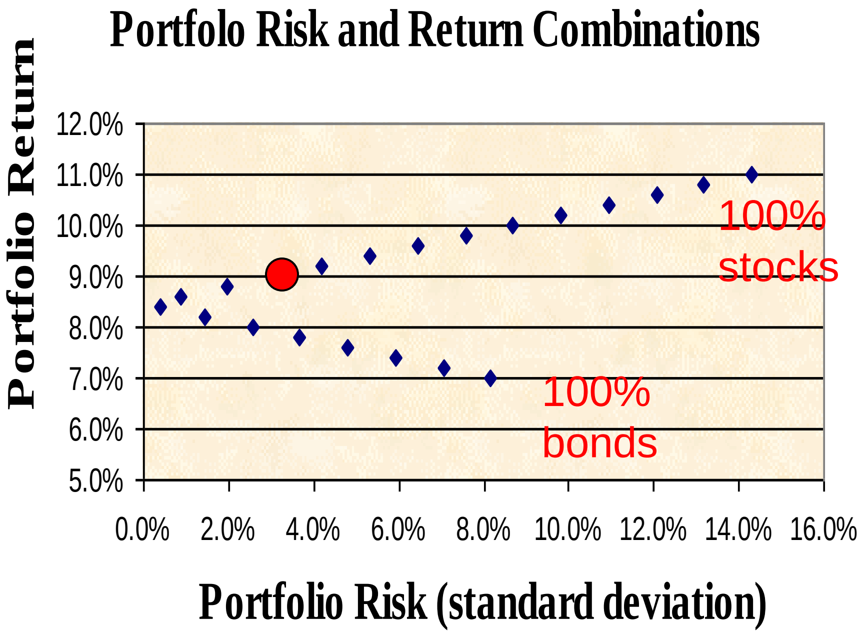
Expected return	11.00%	7.00%	9.0%	
Variance	0.0205	0.0067	0.0010	
Standard Deviation	14.31%	8.16%	3.08%	

Observe the decrease in risk that diversification offers.

An equally weighted portfolio (50% in stocks and 50% in bonds) has less risk than stocks or bonds held in isolation.

The Efficient Set for Two Assets

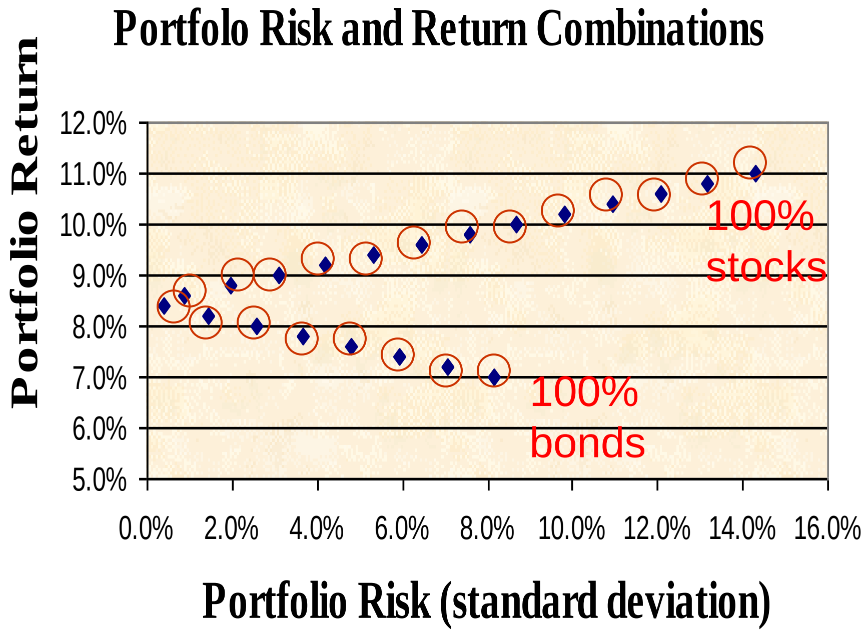
% in stocks	Risk	Return
0%	8.2%	7.0%
5%	7.0%	7.2%
10%	5.9%	7.4%
15%	4.8%	7.6%
20%	3.7%	7.8%
25%	2.6%	8.0%
30%	1.4%	8.2%
35%	0.4%	8.4%
40%	0.9%	8.6%
45%	2.0%	8.8%
50.00%	**3.08%**	**9.00%**
55%	4.2%	9.2%
60%	5.3%	9.4%
65%	6.4%	9.6%
70%	7.6%	9.8%
75%	8.7%	10.0%
80%	9.8%	10.2%
85%	10.9%	10.4%
90%	12.1%	10.6%
95%	13.2%	10.8%
100%	14.3%	11.0%



We can consider other portfolio weights besides 50% in stocks and 50% in bonds …

The Efficient Set for Two Assets

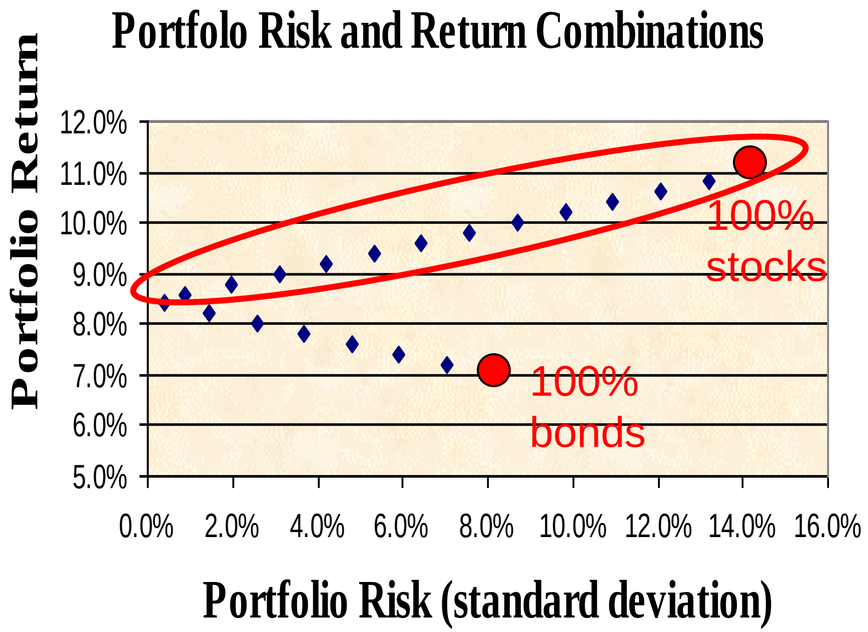
% in stocks	Risk	Return
0%	8.2%	7.0%
5%	7.0%	7.2%
10%	5.9%	7.4%
15%	4.8%	7.6%
20%	3.7%	7.8%
25%	2.6%	8.0%
30%	1.4%	8.2%
35%	0.4%	8.4%
40%	0.9%	8.6%
45%	2.0%	8.8%
50%	3.1%	9.0%
55%	4.2%	9.2%
60%	5.3%	9.4%
65%	6.4%	9.6%
70%	7.6%	9.8%
75%	8.7%	10.0%
80%	9.8%	10.2%
85%	10.9%	10.4%
90%	12.1%	10.6%
95%	13.2%	10.8%
100%	**14.3%**	**11.0%**



We can consider other portfolio weights besides 50% in stocks and 50% in bonds …

The Efficient Set for Two Assets

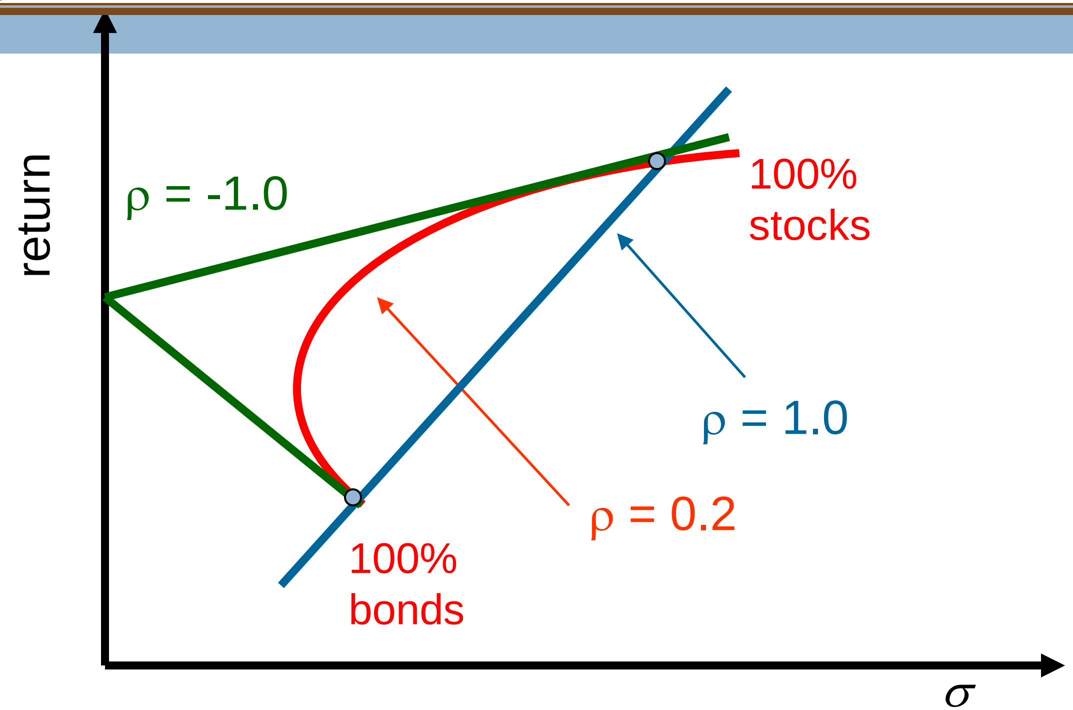
% in stocks	*Risk*	*Return*
0%	8.2%	7.0%
5%	7.0%	7.2%
10%	5.9%	7.4%
15%	4.8%	7.6%
20%	3.7%	7.8%
25%	2.6%	8.0%
30%	1.4%	8.2%
35%	0.4%	8.4%
40%	0.9%	8.6%
45%	2.0%	8.8%
50%	3.1%	9.0%
55%	4.2%	9.2%
60%	5.3%	9.4%
65%	6.4%	9.6%
70%	7.6%	9.8%
75%	8.7%	10.0%
80%	9.8%	10.2%
85%	10.9%	10.4%
90%	12.1%	10.6%
95%	13.2%	10.8%
100%	14.3%	11.0%



Note that some portfolios are "better" than others. They have higher returns for the same level of risk or less.

These compromise the *efficient frontier*.

Two-Security Portfolios with Various Correlations



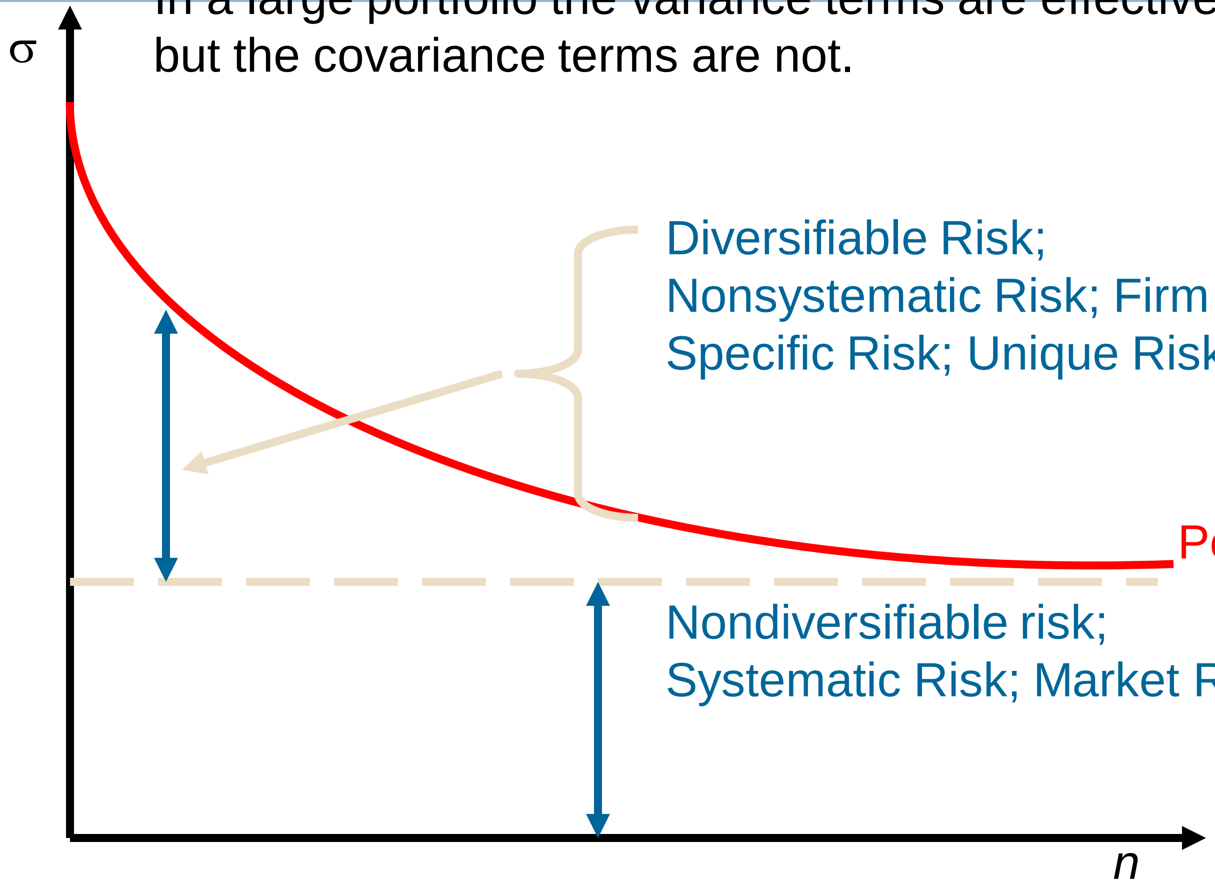
- Relationship depends on correlation coefficient

$$-1.0 \leq \rho \leq +1.0$$

- If $\rho = +1.0$, no risk reduction is possible
- If $\rho = -1.0$, complete risk reduction is possible

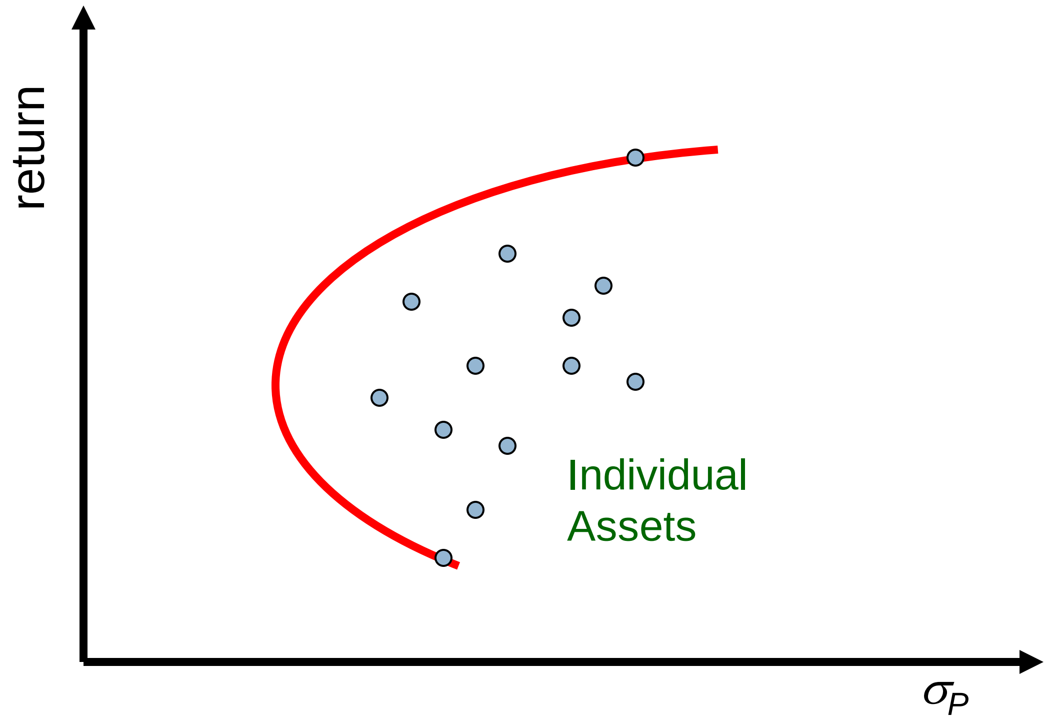
Portfolio Risk as a Function of the Number of Stocks in the Portfolio

In a large portfolio the variance terms are effectively diversified away, but the covariance terms are not.



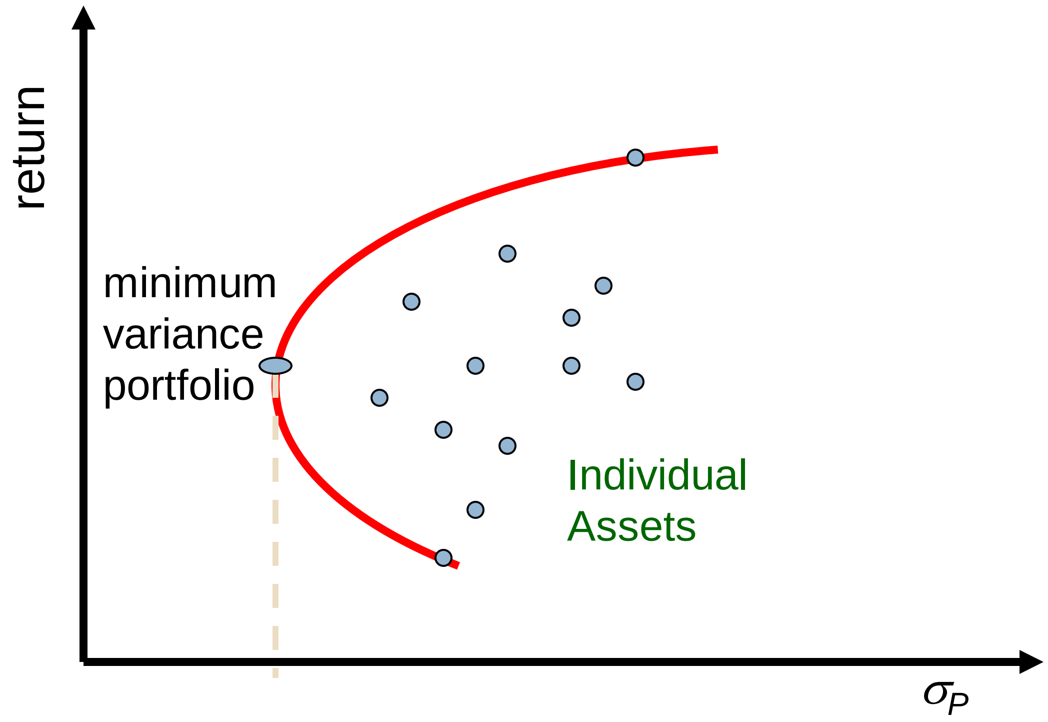
Thus diversification can eliminate some, but not all of the risk of individual securities.

The Efficient Set for Many Securities



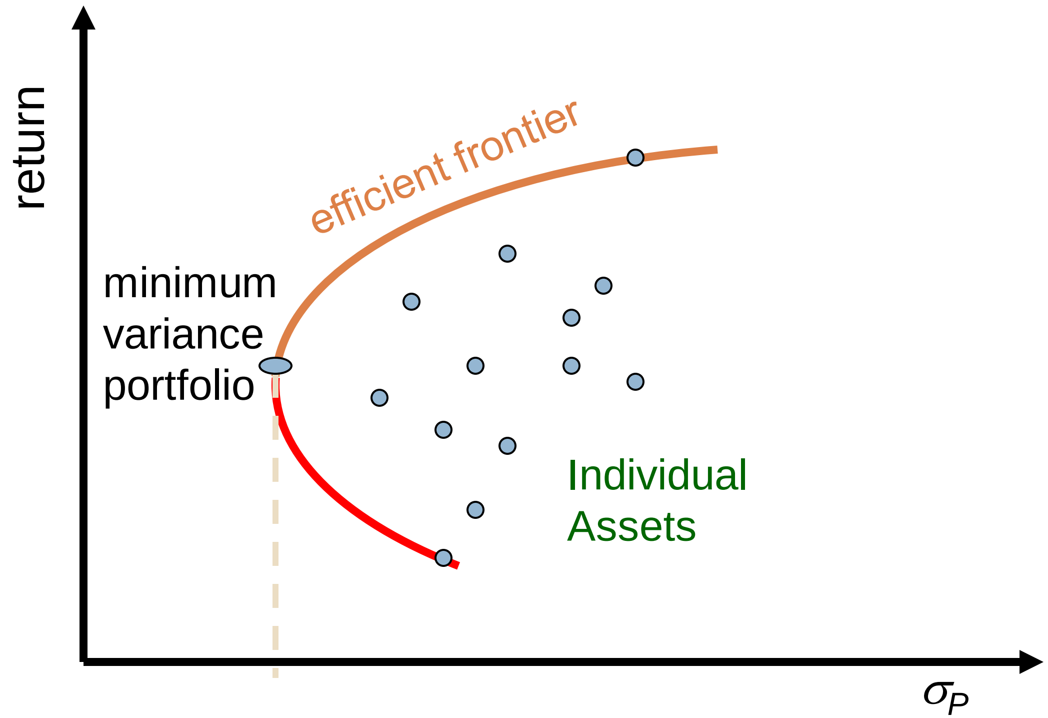
Consider a world with many risky assets; we can still identify the *opportunity set* of risk-return combinations of various portfolios.

The Efficient Set for Many Securities



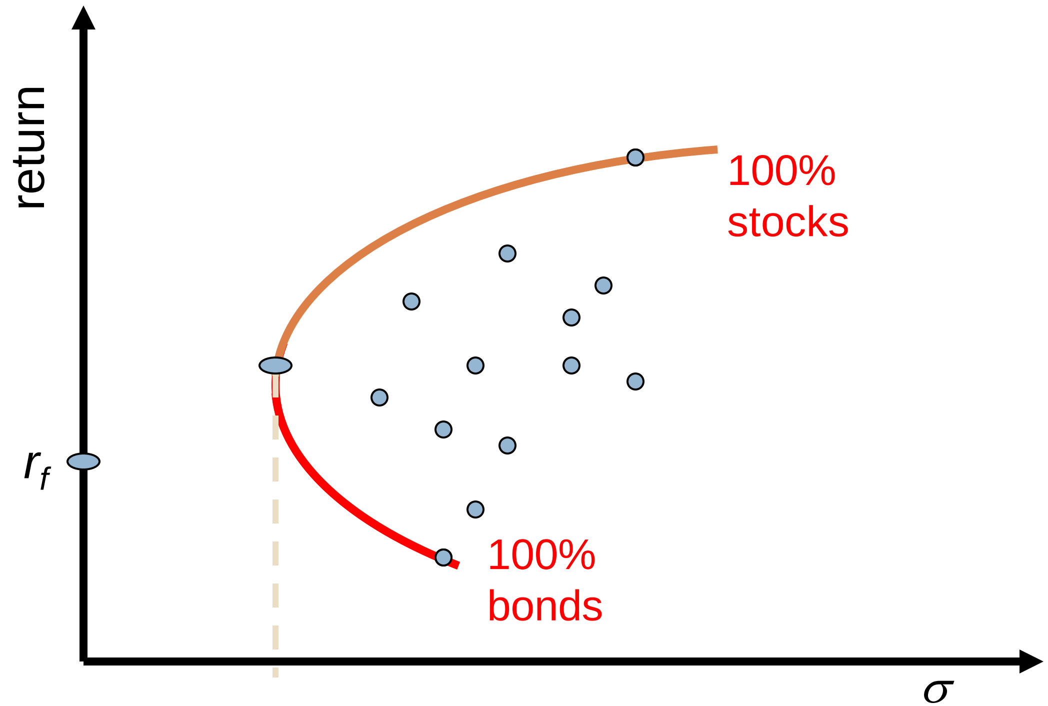
Given the *opportunity set* we can identify the **minimum variance portfolio.**

The Efficient Set for Many Securities



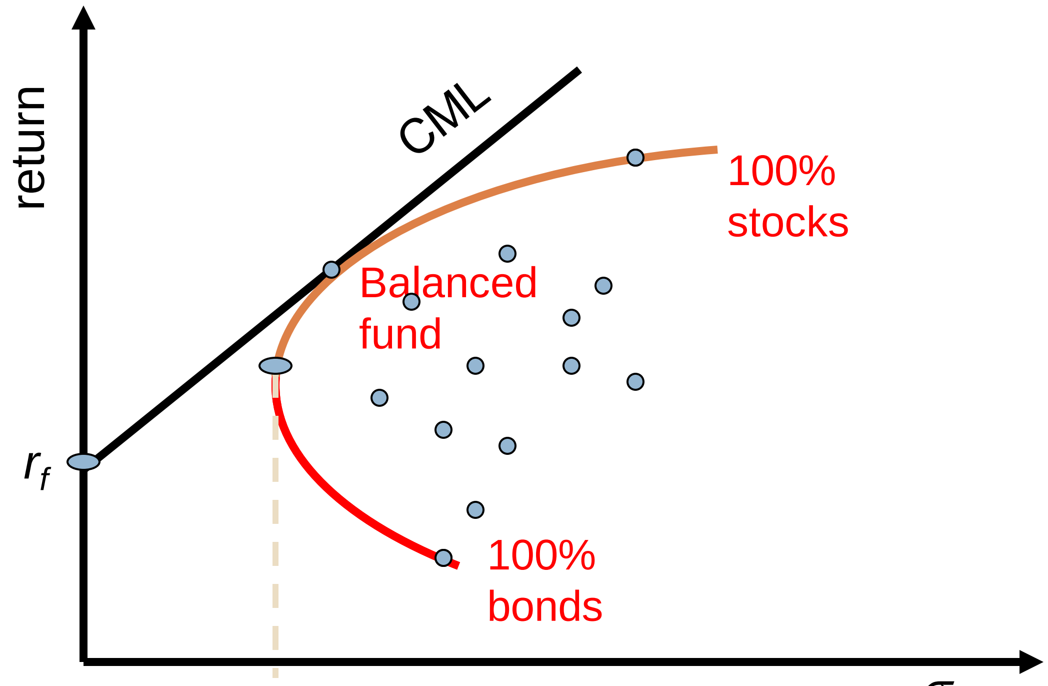
The section of the opportunity set above the minimum variance portfolio is the efficient frontier.

Optimal Risky Portfolio with a Risk-Free Asset



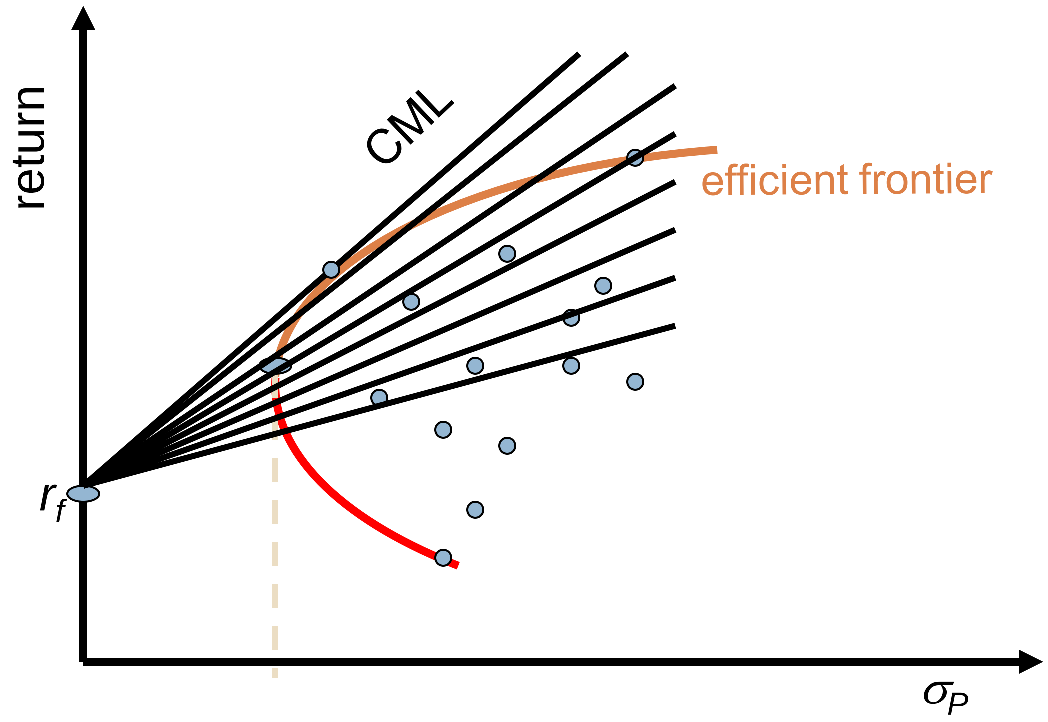
In addition to stocks and bonds, consider a world that also has risk-free securities like T-bills

Riskless Borrowing and Lending



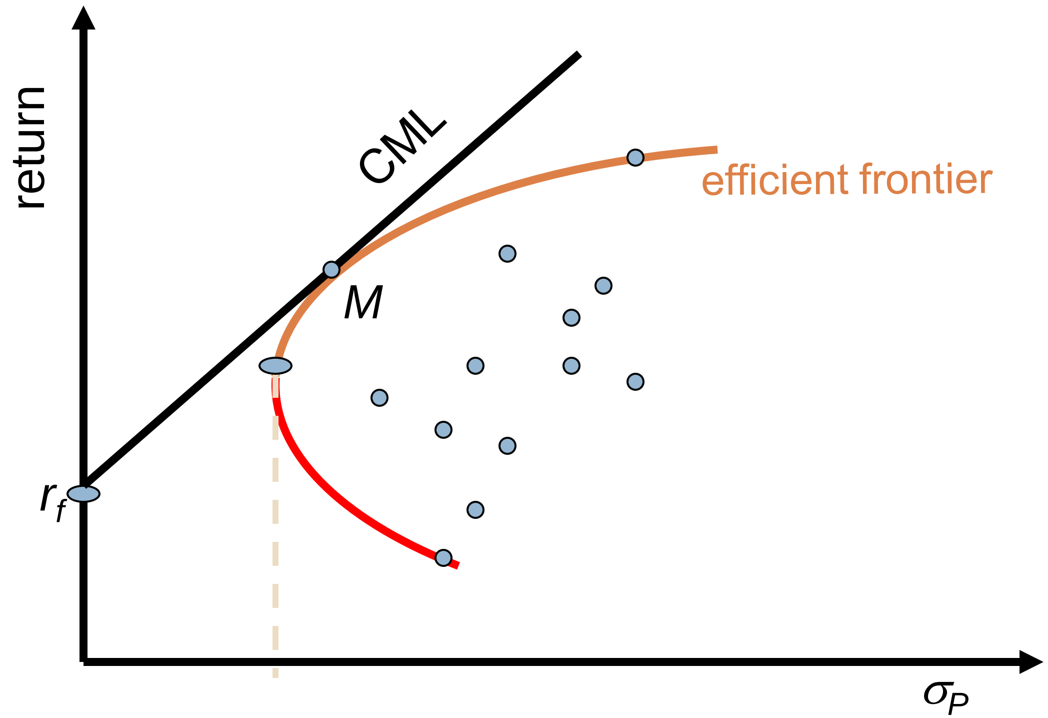
Now investors can allocate their money across the T-bills and a balanced mutual fund

Riskless Borrowing and Lending



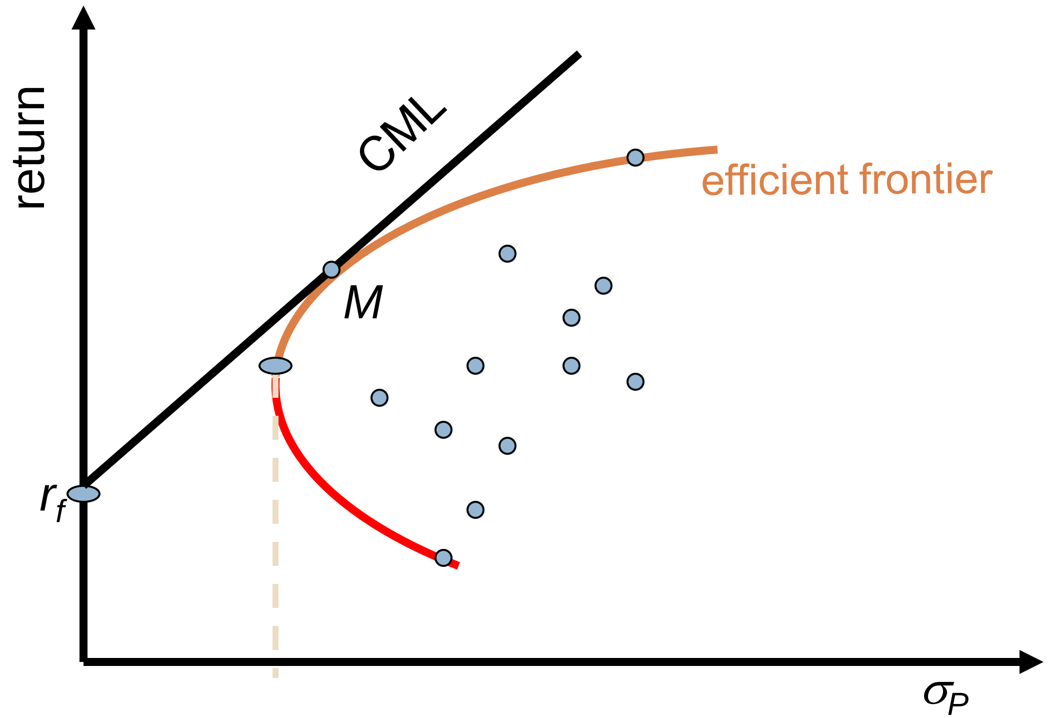
With a risk-free asset available and the efficient frontier identified, we choose the capital allocation line with the steepest slope

10.8 Market Equilibrium



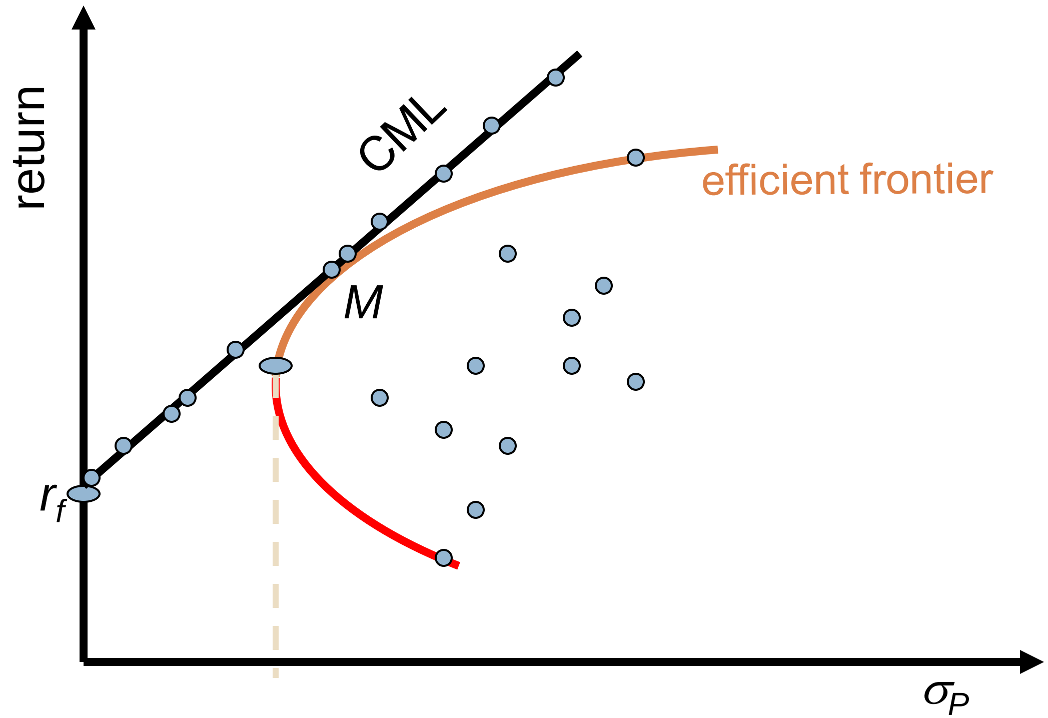
With the capital allocation line identified, all investors choose a point along the line—some combination of the risk-free asset and the market portfolio *M*. In a world with homogeneous expectations, *M* is the same for all investors.

The Separation Property



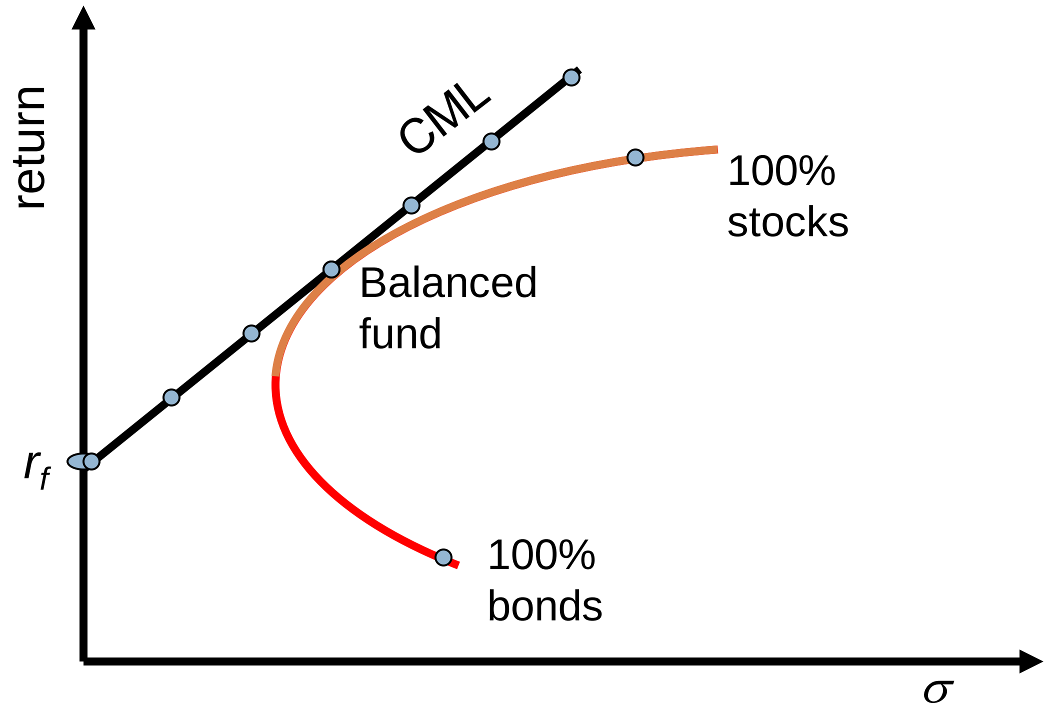
The Separation Property states that the market portfolio, *M*, is the same for all investors—they can *separate* their risk aversion from their choice of the market portfolio.

The Separation Property



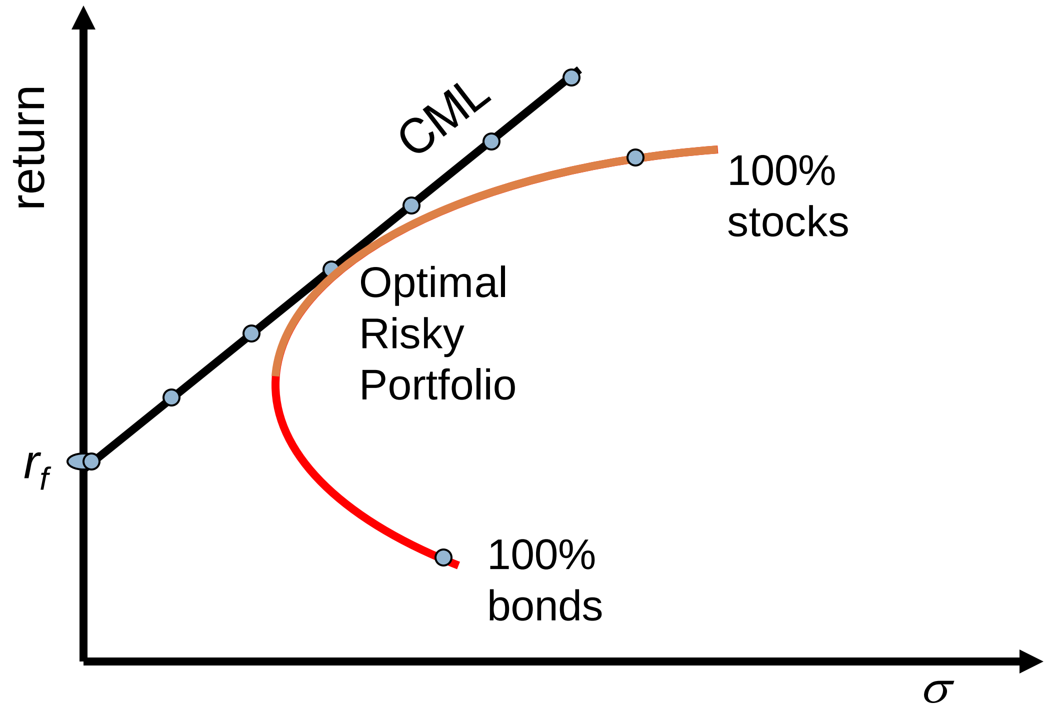
Investor risk aversion is revealed in their choice of where to stay along the capital allocation line—not in their choice of the line.

Market Equilibrium



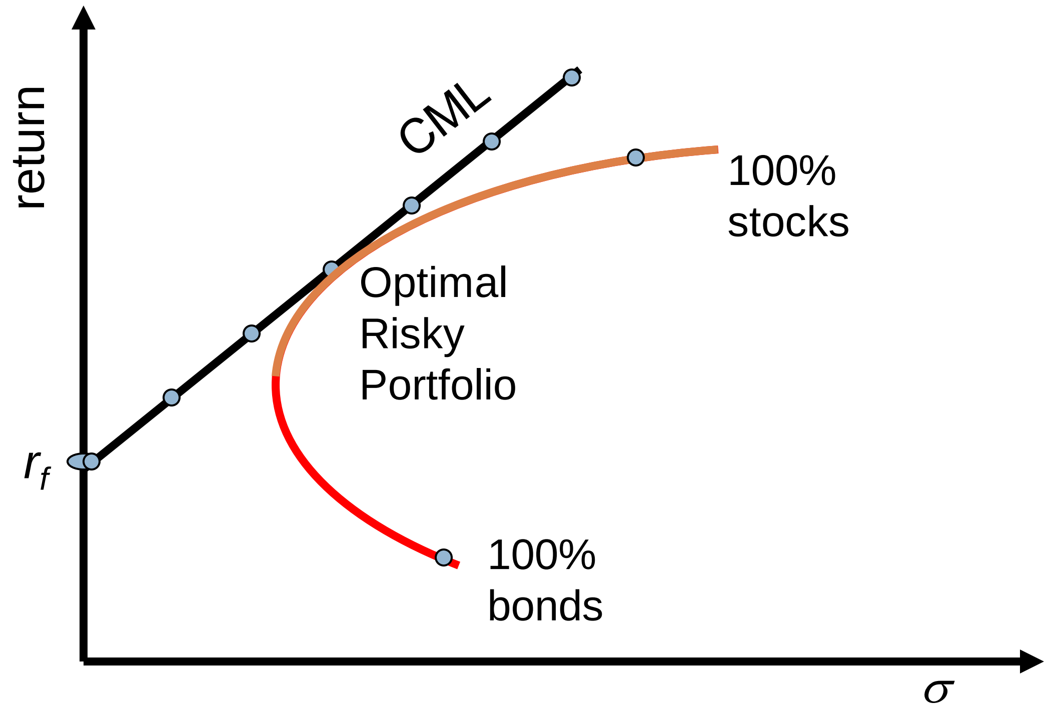
Just where the investor chooses along the Capital Asset Line depends on his risk tolerance. The big point though is that all investors have the same CML

Market Equilibrium



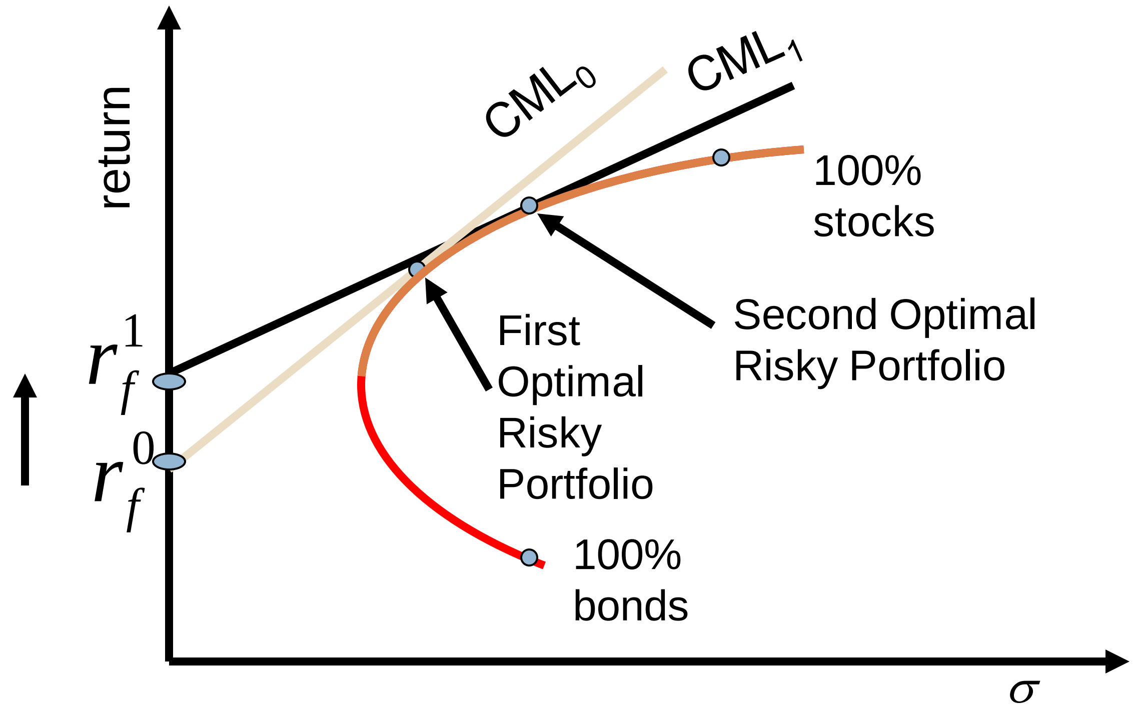
All investors have the same CML because they all have the same optimal risky portfolio given the risk-free rate.

The Separation Property



The separation property implies that portfolio choice can be separated into two tasks: (1) determine the optimal risky portfolio, and (2) selecting a point on the CML

Optimal Risky Portfolio with a Risk-Free Asset



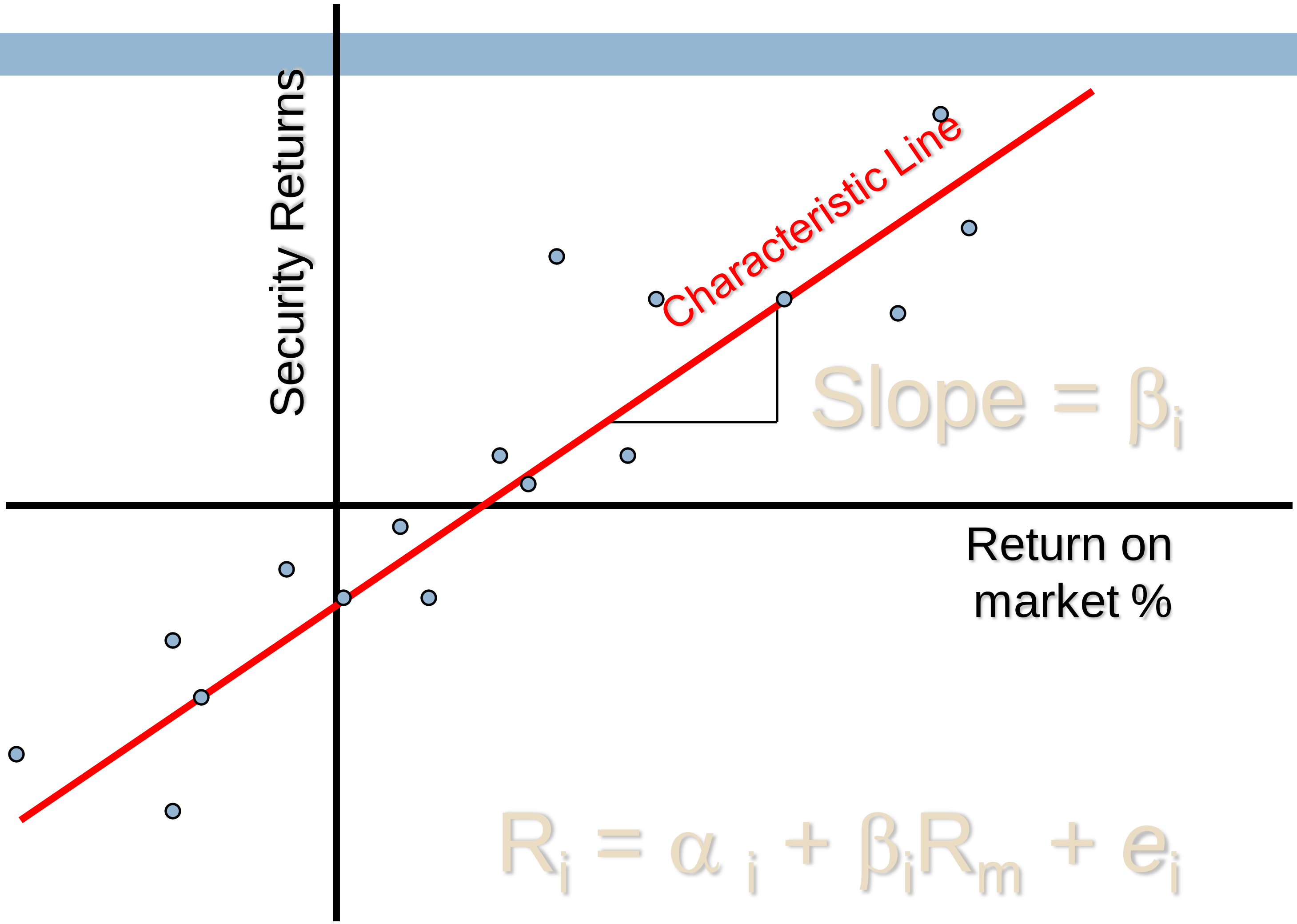
By the way, the optimal risky portfolio depends on the risk-free rate as well as the risky assets.

Definition of Risk When Investors Hold the Market Portfolio

- Researchers have shown that the best measure of the risk of a security in a large portfolio is the *beta* (β)of the security.
- Beta measures the responsiveness of a security to movements in the market portfolio.

$$\beta_i = \frac{Cov(R_{i,} R_M)}{\sigma^2(R_M)}$$

Estimating β with regression



Estimates of β for Selected Stocks

Stock	Beta
Bank of America	1.55
Borland International	2.35
Travelers, Inc.	1.65
Du Pont	1.00
Kimberly-Clark Corp.	0.90
Microsoft	1.05
Green Mountain Power	0.55
Homestake Mining	0.20
Oracle, Inc.	0.49

The Formula for Beta

$$\beta_i = \frac{Cov(R_{i,} R_M)}{\sigma^2(R_M)}$$

Clearly, your estimate of beta will depend upon your choice of a proxy for the market portfolio.

Relationship between Risk and Expected Return (CAPM)

- Expected Return on the Market:

$$\overline{R}_M = R_F + \text{Market Risk Premium}$$

- Expected return on an individual security:

$$\overline{R}_i = R_F + \beta_i \times \underbrace{(\overline{R}_M - R_F)}_{\text{Market Risk Premium}}$$

This applies to individual securities held within well-diversified portfolios.

Expected Return on an Individual Security

- This formula is called the Capital Asset Pricing Model (CAPM)

$$\bar{R}_i = R_F + \beta_i \times (\bar{R}_M - R_F)$$

Expected return on a security	=	Risk-free rate	+	Beta of the security	×	Market risk premium

- Assume $\beta_i = 0$, then the expected return is R_F.
- Assume $\beta_i = 1$, then $\bar{R}_i = \bar{R}_M$

Relationship Between Risk & Expected Return

$$\overline{R}_i = R_F + \beta_i \times (\overline{R}_M - R_F)$$