# Lecture 03 – Consumer Preference Theory

1. ***Consumer preferences*** will tell us how an individual would rank (i.e. compare the desirability of) any two consumption bundles (or baskets), *assuming the bundles were available at no cost.* Of course, a consumer's actual choice will ultimately depend on a number of factors in addition to preferences.

**2. Assumptions about the consumer preferences**

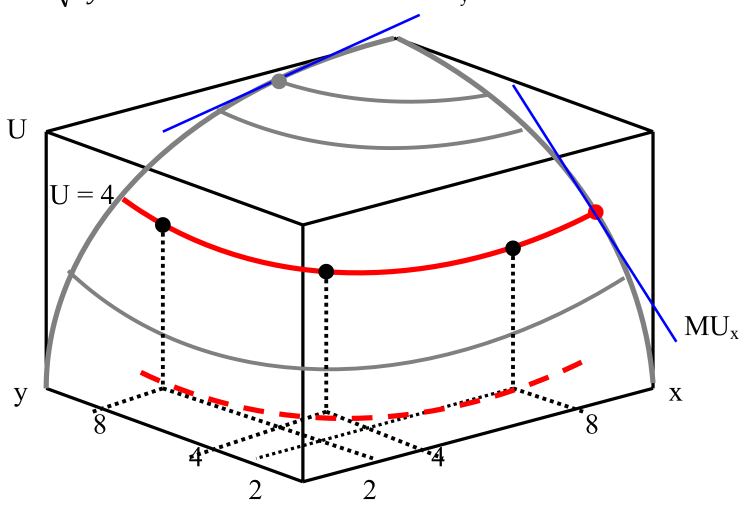
a. completeness: $A \succ B$, $A \prec B$, or $A \approx B$
b. transitivity: If $A \succ B$ and $B \succ C$, then $A \succ C$
c. continuity: gradual change in preference with gradual change in consumption
d. monotonicity: strong non-satiation or "more is better"
$\rightarrow$ continuous utility function

**3. Cardinal vs. Ordinal Utility**

Jeremy Bentham "the greatest happiness of the greatest number"
Jevons, Menger, and Walras "Marginal Revolution"
Pareto's ordinal utility through indifference curve
Hicks and R.G.D. Allen

**4. Utility Surface**

$U = \sqrt{xy}$



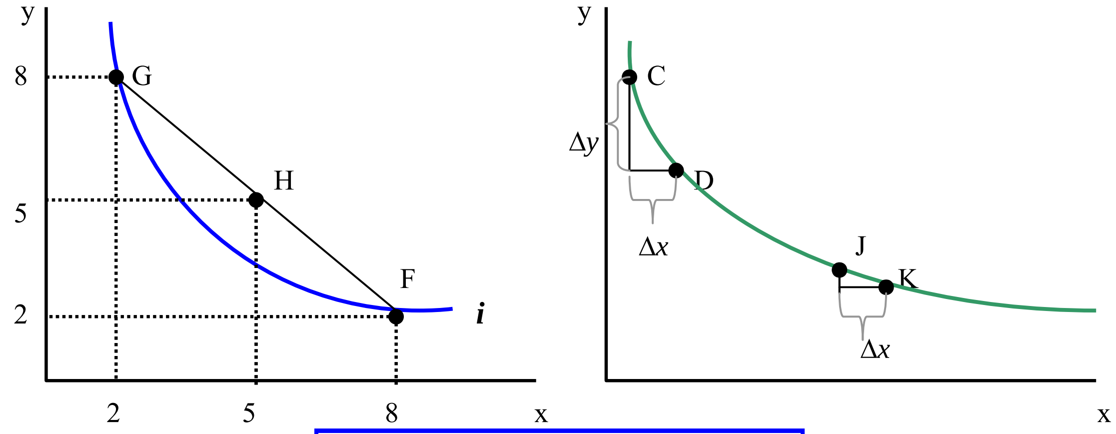

**5. Diminishing MU?**

Ex) $U = \sqrt{H} + R$, where H is number of hamburgers and R is number of root beer.

**6. Properties of Indifference Curves**

a) Downward sloping.
b) Convex to the origin (see MRS).
c) Indifference curves cannot intersect.
d) Every consumption bundle lies on one and only one indifference curve (everywhere dense).
e) Indifference curves are not "thick."
f) The farther from the origin, the more utility it has.

## 7. Marginal Rate of Substitution (MRS)




$$-\text{slope of I.C.} = -\frac{\Delta y}{\Delta x} \text{ or } -\frac{dy}{dx} = MRS_{x,y}$$

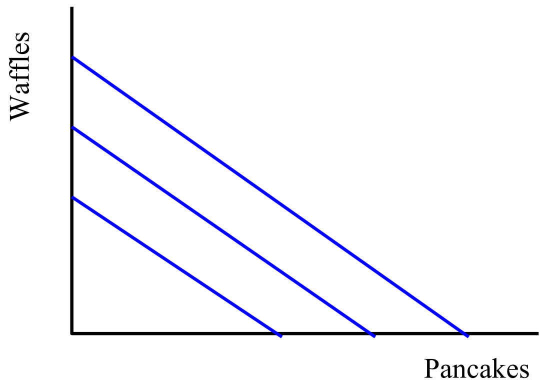
$\overline{U} = U(x, y)$. Totally differentiating to get $d\overline{U} = MU_x dx + MU_y dy = 0$ (why?).

$-\frac{dy}{dx} = \frac{MUx}{MU_y} = MRS_{x,y}$ . $\boxed{\frac{dMRS_{x,y}}{dx} < 0}$ or $\boxed{\frac{d^2y}{dx^2} > 0}$ (MRS is diminishing)

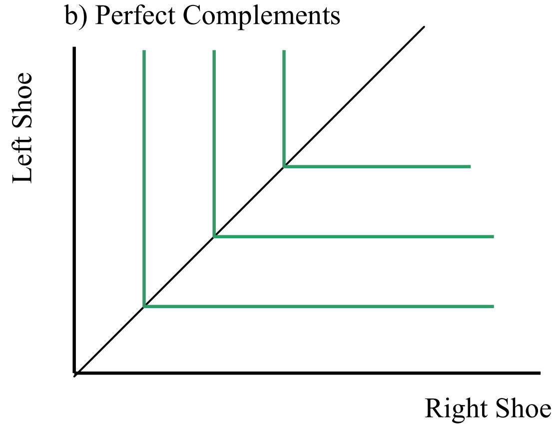
cf) Can you draw an indifference curve with increasing $MRS_{x,y}$ ?

## 8. Special Utility Functions

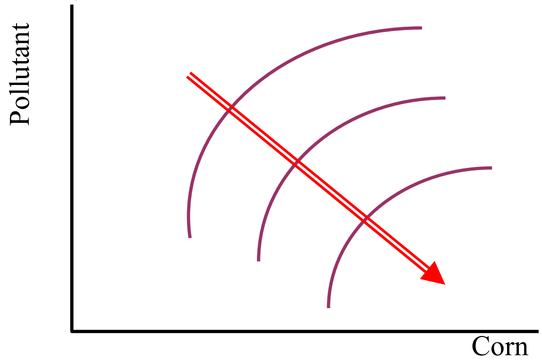
a) Perfect Substitutes



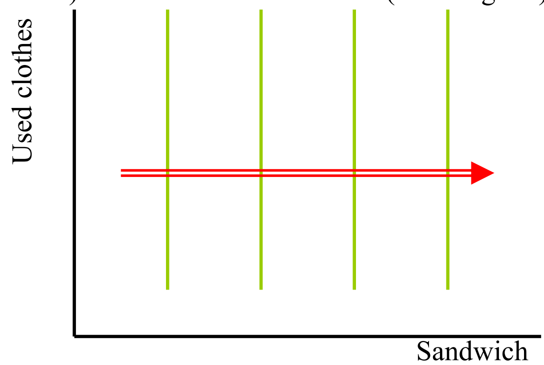

b) Perfect Complements




c) One Good and one Bad




d) One Good and one neuter (neutral good)



e) Cobb-Douglas Utility Function (by Charles Cobb & Paul Douglas)

$U = Ax^{\alpha} y^{\beta}$, where $A$, $\alpha$, and $\beta$ are positive constants.

Cobb-Douglass utility function has three properties that make it of interest in the study of consumer choice.

(1) MU's are positive. Check it out.
(2) Since MU's are all positive, the indifference curves will be downward sloping.
(3) It also exhibits a diminishing MRS

f) Quasi-Linear Utility Function (imperfect substitution)

It can describe preferences for a consumer who purchases the same amount of a commodity regardless of his income. Ex) toothpaste and coffee

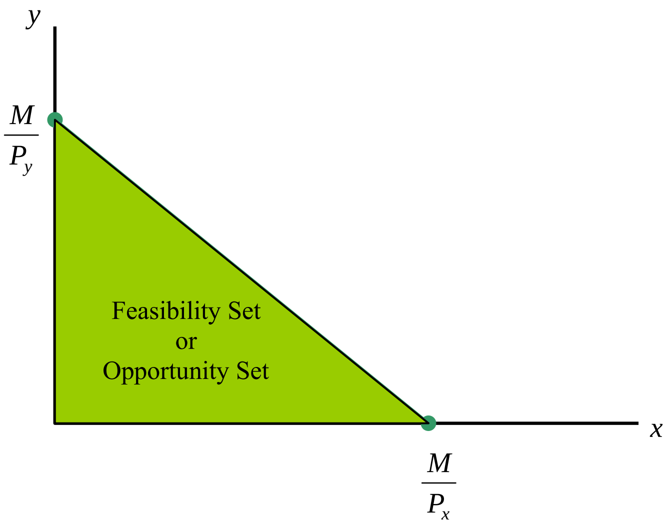
$U(x, y) = v(x) + by$, where $v(x)$ is a function that increases in $x$ and $b$ is positive constant. The indifference curves are parallel, so for any value of $y$, the slopes of I.C. will be the same.

Ex) $U = \sqrt{x} + y$

**9. Budget Constraint**

$P_x x + P_y y \leq M$ From this constraint we can derive the budget line (or price line) to visualize in 2-D space.

$y = -\frac{P_x}{P_y} x + \frac{M}{P_y}$, where $-\frac{P_x}{P_y}$ is slope and $\frac{M}{P_y}$ is vertical intercept.




**10. Change in Income and Change in Price**

Ex) $M_0 = \$800$, $P_x = \$20$, $P_y = \$40$. Draw the budget lines in each case.

1) Income rises from \$800 to \$1,000

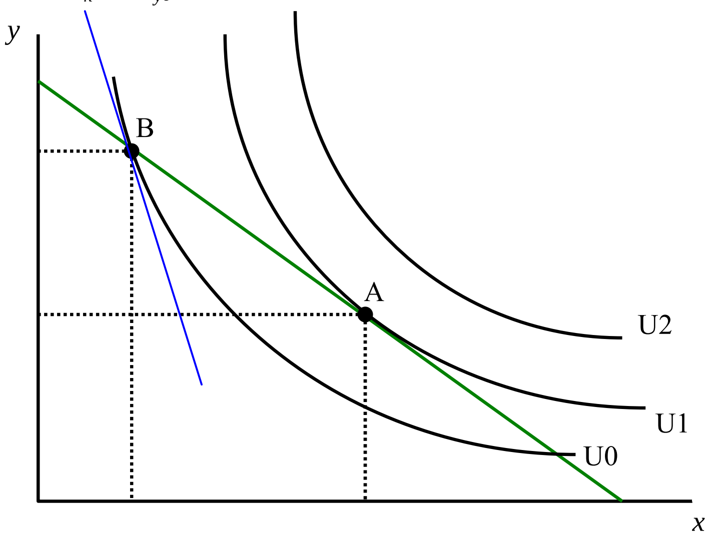
2) $P_x$ rises to \$25, holding initial income and $P_y$ constant.

3) Now, income rises from \$800 to \$1,000 and $P_x$ rises to \$25 and $P_y$ rises to \$50.

**11. Optimal Choice**

$$\begin{cases} \max_{x,y} \ U(x,y) : \text{"choose x and y to maximize utility"} \\ \text{subject to: } P_x x + P_y y \le M : \text{"expenditures on x and y must not exceed the consumer's income"} \end{cases}$$

If the consumer likes more of both goods, the marginal utilities of good x and y are both positive. At an optimal basket all income will be spent. So, the consumer will choose a basket on the budget line $P_x x + P_y y = M$ .




At point A,

$$-\frac{dy}{dx} = \frac{MU_x}{MU_y} = MRS_{x,y} = \frac{P_x}{P_y}$$

At point B,

$$\frac{MU_x}{MU_y} > \frac{P_x}{P_y} \text{ or } \frac{MU_x}{P_x} > \frac{MU_y}{P_y}$$

**12. Lagrangean Function (**by Joseph-Louis Lagrange**)**

$$\begin{cases} \max_{x,y} \ U(x,y) \\ \text{subject to: } P_x x + P_y y \le M \end{cases}$$

We define the Lagrangean ( $L$ ) as $L(x,y,\lambda) = U(x,y) + \lambda(M - P_x x - P_y y)$ , where $\lambda$ is a Lagrange multiplier. The first-order necessary condition (FOC) for an interior optimum (with $x > 0$ and $y > 0$ ) are

$$\begin{cases} \frac{\partial L}{\partial x} = 0 \Rightarrow \frac{\partial U(x,y)}{\partial x} = \lambda P_x & (1) \\ \frac{\partial L}{\partial y} = 0 \Rightarrow \frac{\partial U(x,y)}{\partial y} = \lambda P_y & (2) \\ \frac{\partial L}{\partial \lambda} = 0 \Rightarrow M - P_x x - P_y y = 0 & (3) \end{cases}$$

We can combine (1) and (2) to eliminate the Lagrange multiplier, so FOCs reduce to:

$$\begin{cases} \frac{MU_x}{MU_y} = \frac{P_x}{P_y} \text{ or } \frac{MU_x}{P_x} = \frac{MU_y}{P_y} & (4) \\ P_x x + P_y y = M & (5) \end{cases}$$

From the above equations, we can derive demand function of *x* and *y*.

Ex) You are given $U(x,y) = \sqrt{x} + \sqrt{y} = x^{1/2} + y^{1/2}$ and $P_x x + P_y y = M$ . Now derive demand functions of *x* and *y* when the consumer is maximizing his utility.

$MU_x = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$ and $MU_y = \frac{1}{2}y^{-1/2} = \frac{1}{2\sqrt{y}}$. So $\boxed{MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{\sqrt{y}}{\sqrt{x}}}$

And FOC of utility maximization is $\boxed{\frac{MU_x}{MU_y} = \frac{\sqrt{y^*}}{\sqrt{x^*}} = \frac{P_x}{P_y}}$, where $x^*$ and $y^*$ are utility-maximizing quantities demanded.

By squaring both sides of above equation and rearranging terms, we find that $y^* = x^* \frac{P_x^2}{P_y^2}$.

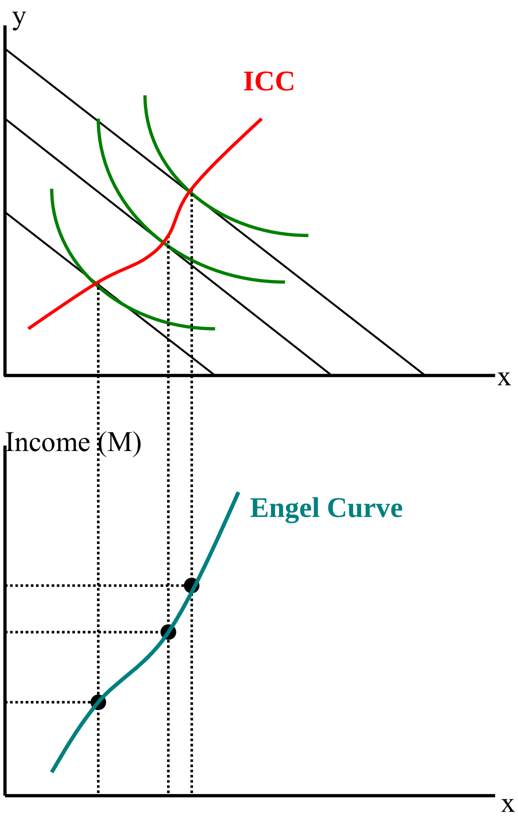
Plugging the last expression into budget equation and simplifying, we get

$$P_x x^* + P_y \left( x^* \frac{P_x^2}{P_y^2} \right) = M$$

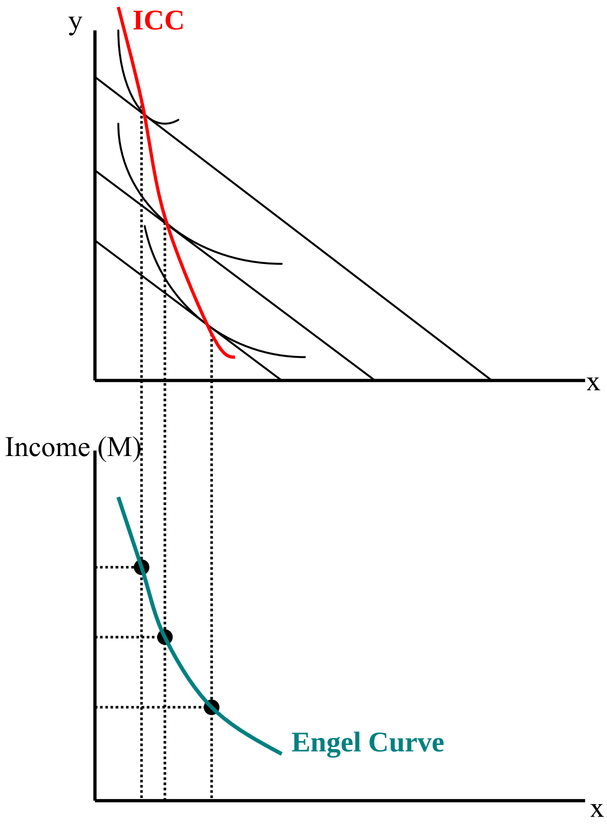
$$P_x x^* + x^* \frac{P_x^2}{P_y} = x^* \left( P_x + \frac{P_x^2}{P_y} \right) = x^* \left( \frac{P_x P_y + P_x^2}{P_y} \right) = M \text{. So, } \boxed{x^* = \frac{P_y M}{P_x P_y + P_x^2}} \quad (1)$$

$$y^* = x^* \frac{P_x^2}{P_y^2} = \left( \frac{P_y M}{P_x P_y + P_x^2} \right) \left( \frac{P_x^2}{P_y^2} \right) = \frac{P_y P_x^2 M}{P_x P_y^3 + P_x^2 P_y^2} \text{. So, } \boxed{y^* = \frac{P_x M}{P_x P_y + P_y^2}} \quad (2)$$

**13. Change in Income and Optimal Choice**

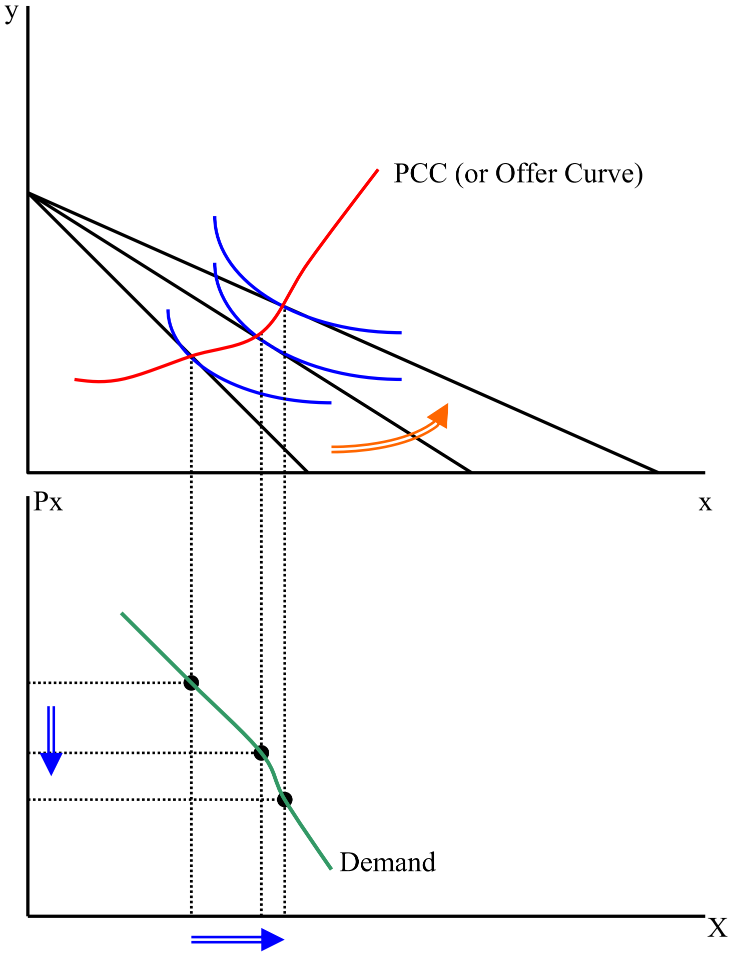



(Normal Good, $\eta > 0$)




(Inferior Good, $\eta < 0$)

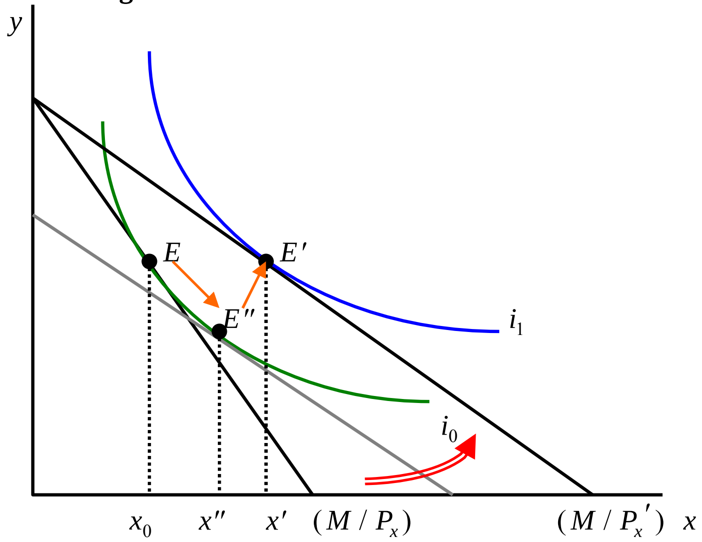
## 14. Change in Price and Optimal Choice




cf) What would be the shape of PCC if the price elasticity of demand is 1 (unitary elastic)?

cf) Think about the shape of PCC if the quantity is reduced with price hike. And also think about the shape of demand for that good.

## 15. Change in Price: Revisited




* Price Effect $(E \rightarrow E')$
$(E \rightarrow E'')$: Substitution Effect due to price change
$(E'' \rightarrow E')$: Income Effect

In this case, good *x* is a *normal* good.

cf) Derive substitution and income effect if good x is neither a normal nor an inferior good.

cf) Derive substitution and income effect if good x is an inferior good. Maybe you can think of two different cases.

## 16. Slutzky Equation (Slutzky Decomposition)

$x' - x_0 = (x'' - x_0) + (x' - x'')$ (1)

Dividing both sides by $\Delta P_x$, we get the following equation,

$$\frac{x' - x_0}{\Delta P_x} = \frac{x'' - x_0}{\Delta P_x} + \frac{x' - x''}{\Delta P_x} \quad (2)$$

*l.h.s.* can be explained $\left.\frac{\Delta x}{\Delta P_x}\right|_M$. And the first term on *r.h.s.* is equal to $\left.\frac{\Delta x}{\Delta P_x}\right|_U$.

And the second term on *r.h.s.* is now rewritten as follows,

$\frac{x' - x''}{\Delta P_x} = \frac{\Delta x}{\Delta R} \cdot \frac{\Delta R}{\Delta P_x}$, where $R$ means *real income*. (3)

$\frac{\Delta R}{\Delta P_x}$ means the change in real income *w.r.t.* change in price.

So, $\frac{\Delta R}{\Delta P_x} = -x$ (Shepard's lemma) (4)

And $\frac{\Delta x}{\Delta R} = \frac{\Delta x}{\Delta M}$ (why? Think about the definition of income effect!) (5)

Plugging (4) and (5) in (3) and then in (2), we can get

$$\left.\frac{\Delta x}{\Delta P_x}\right|_M = \left.\frac{\Delta x}{\Delta P_x}\right|_U - x \cdot \frac{\Delta x}{\Delta M} \quad \text{(Slutzky Equation)}$$

## 17. Labor Supply and Leisure (Application)

$H + L = 24$, where $H$ is hours worked and $L$ is amount of leisure activities.

And hourly wage is given at $w_0$ and non-labor income is given at $V_0$.

The utility function is now $U = U(L, M)$ (1)

The budget equation is $M = w_0 H + V_0 = w_0(24 - L) + V_0 = 24w_0 - w_0 L + V_0$ (2)

Rewriting (2), we can get $M + w_0 L = 24w_0 + V_0$ (full-income constraint) (3)

If we assume the price level of goods (composite good) is $P_C = \$1$, then M on *l.h.s* would equal total number of goods that this consumer buys.

Can you interpret terms in equation (3)?

1) Now, can you draw the budget line?

2) Find the utility-maximizing point.

3) Suppose wage rises. Overlap new optimal points on the original budget line.

4) Connect those optimization points to get PCC. And labor supply curve.

5) Why do you think is it a backward bending supply curve?