Lecture 04 – Production Theory

1. How to define a 'Firm'? – roles, objectives, and more

a) *Specialization* in production.
b) *Team production* is more efficient than individual production in many aspects.
   Less transaction costs but *more monitoring costs* in firm.
c) *Internal control* than *external market transactions* with much transaction cost.
   Opportunistic behaviors due to *asset specificity*.
   But, is internal control always reasonable? Not really. Why?
   Ex) Vehicle brand owner
d) Profit-maximizers?
e) Ownership and management are separated.

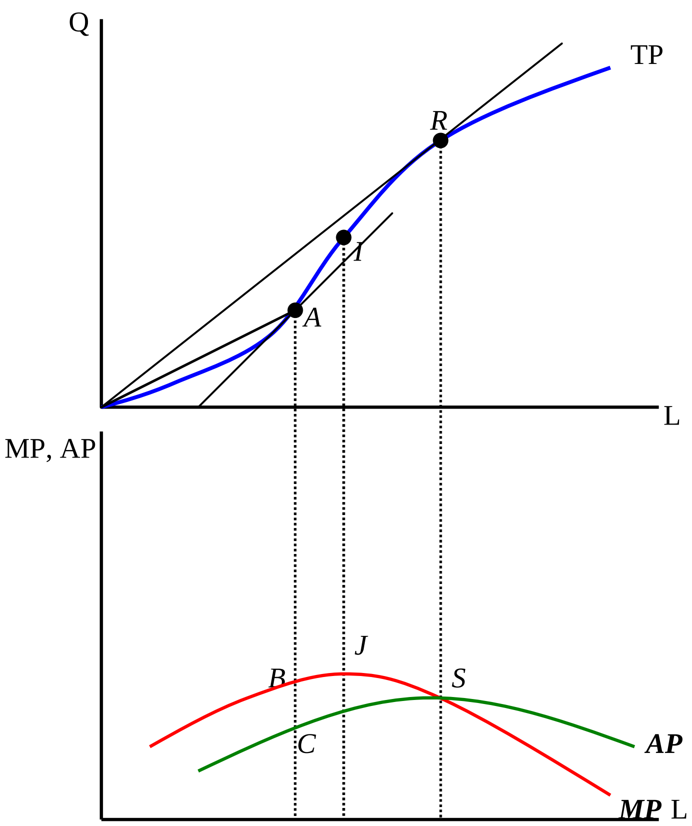
2. Production Technology: Basic Analysis

a) Time periods
   SR with at least one fixed input / LR with all variable inputs
b) Production function (total product curve)

$$\boxed{Q = f(L, \overline{K})} \quad (1)$$

Q: is *maximum amount of output* a firm could get from a given combination of inputs.
So, inefficient management could reduce output from what is technologically possible.
If we invert the equation (1) to get $L = g(Q)$, which tells us the minimum amount of labor L required to produce a given amount of output Q, this function is *labor requirement function.*

3. TP_L, MP_L, and AP_L



$$\boxed{AP_L = \frac{Q}{L}} \quad \boxed{MP_L = \frac{dQ}{dL}}$$

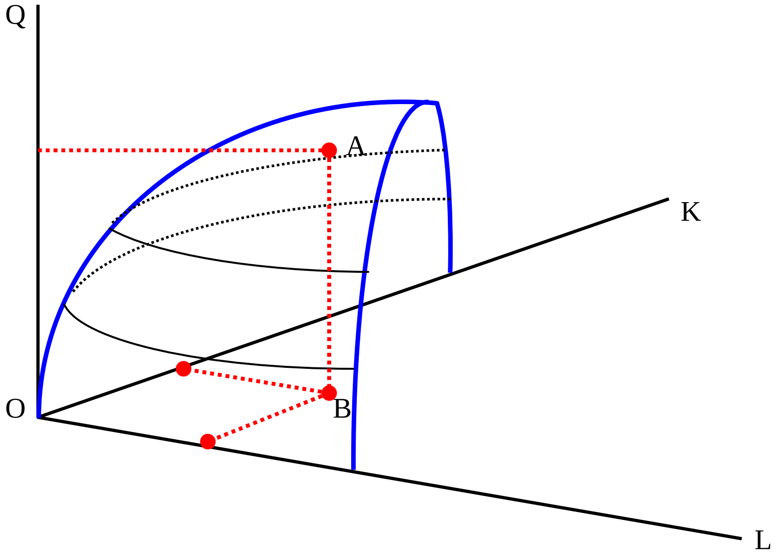
Up to I (inflection point), MP_L is increasing (increasing marginal returns to labor), and after that MP_L starts diminishing.

Up to S, AP_L is increasing, so $AP_L < MP_L$.
After S, AP_L is decreasing, $AP_L > MP_L$

Law of Diminishing Marginal Returns: Principle that as the usage of one input increases, the quantities of other inputs being held fixed, a point will be reached beyond which the marginal product of the variable input will decrease.

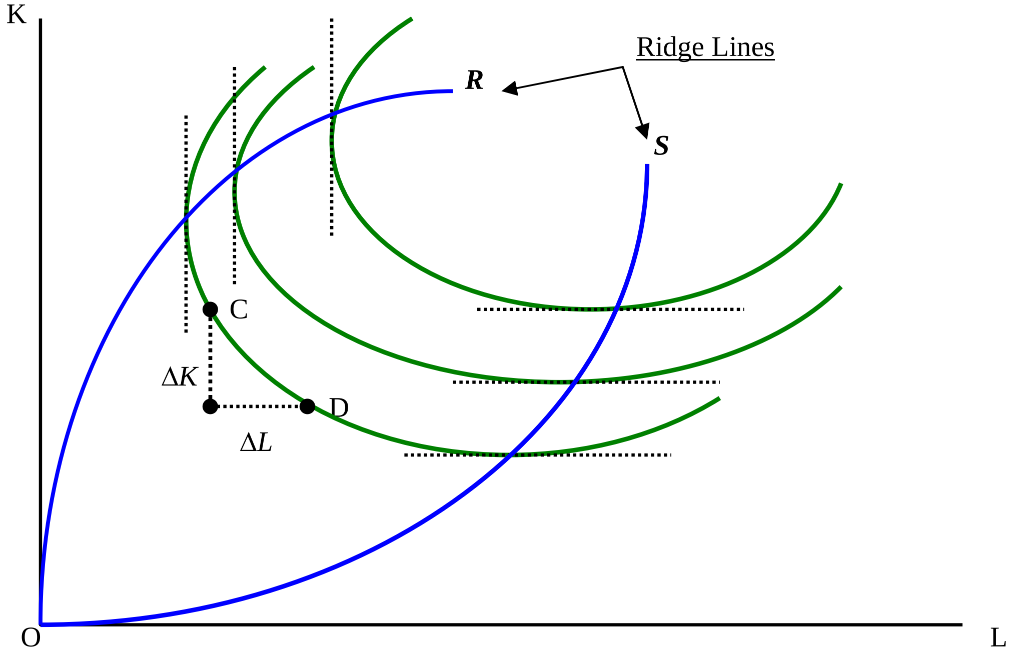
4. Production Technology: Two Variable Inputs

Production Surface (or Total Product Hill)



5. Isoquants

A curve that shows all of the combinations of labor and capital that can produce a given level of output.



6. MRTS (Marginal rate of Technical Substitution)

$$MRTS_{L,K} = -\frac{\Delta K}{\Delta L}$$

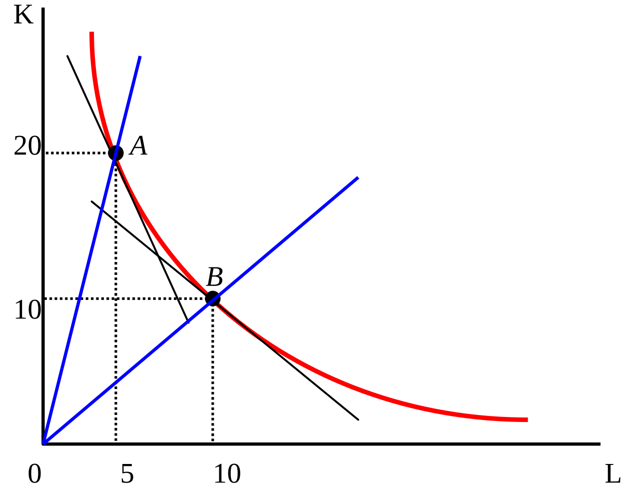
From $Q = f(L,K)$, we can get the following equation by totally differentiating the production function. $\Delta Q = MP_L \cdot \Delta L + MP_K \cdot \Delta K = 0$.

Finally, we can get

$\boxed{MRTS_{L,K} = -\dfrac{\Delta K}{\Delta L} = \dfrac{MP_L}{MP_K}}$ and $\dfrac{d^2K}{dL^2} > 0$ implies the <u>Law of Diminishing MRTS</u>.

7. Elasticity of Substitution

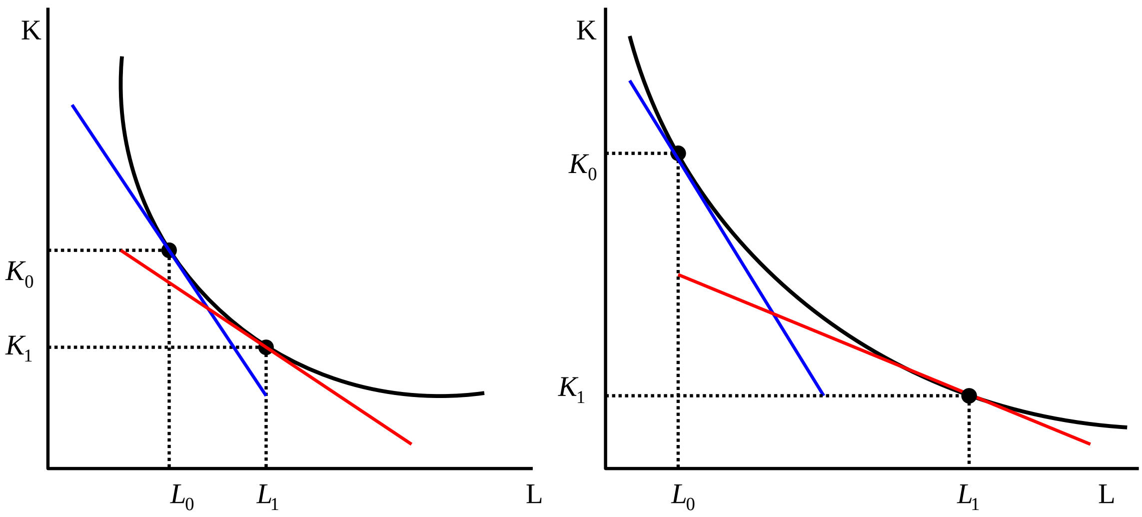
$$\sigma = \frac{\text{percentage change in capital} - \text{labor ratio}}{\text{percentage change in } \mathrm{MRTS}_{L,K}} = \frac{\Delta\left(\dfrac{K}{L}\right) \Big/ \left(\dfrac{K}{L}\right)}{\Delta MRTS / MRTS}$$



K/L at A = slope of ray OA = 4
MRTS at A = 4

K/L at B = slope of ray OB = 1
MRTS at B = 1

As the firm moves from A to B, the elasticity of substitution equals 1.



(σ is relatively small)
"Little" opportunity to substitute between inputs

(σ is relatively big)
"Big" opportunity to substitute between inputs

8. Special Production Functions

a) Linear Production Function (Perfect Substitutes, $\boxed{\sigma = \infty}$)

$\boxed{Q = cL + dK}$, *c* and *d* are positive constants.

Ex) natural gas or fuel oil in manufacturing process.
company data storage b/w high-capacity and low-capacity computers.

b) Fixed-proportions Production Function (Perfect Complements, $\boxed{\sigma = 0}$, *Leontief Function*)

$\boxed{Q = \min(aL, bK)}$, *a* and *b* are positive constants.

Ex) fixed portions of oxygen and hydrogen atoms to make water molecules.
One frame with two tires for bicycle. One chassis with four tires for a car.

c) Cobb-Douglas Production Function

$\boxed{Q = AL^{\alpha}K^{\beta}}$, $A,\ \alpha,\ \beta$ are positive constants.

Homogeneous Function of degree r

$y = f(x,\ z)$

If we *k-fold* all the independent variables *x* and *z*, $\boxed{f(kx,\ kz) \equiv k^r f(x,\ z) = k^r y}$

If $r = 1$, the function is also known as *linear* homogeneous function.

Ex) Identify the following functions.

a) $y = 3x^2 + xz - 2z^2$ b) $y = \dfrac{x}{3z} + 5$ c) $\boxed{y = x^a z^{1-a}}$

Returns to scale for a Cobb-Douglas Production Function

Let L_1 and K_1 denote the initial quantities of labor and capital, and let Q_1 denote the initial output, so $Q_1 = AL_1^{\alpha}K_1^{\beta}$. Now let's increase all input quantities by the same proportional amount λ, where $\lambda > 1$, and let Q_2 denote the resulting volume of output:

$Q_2 = A(\lambda L_1)^{\alpha}(\lambda K_1)^{\beta} = \lambda^{\alpha+\beta}AL_1^{\alpha}K_1^{\beta} = \lambda^{\alpha+\beta}Q_1$. From this, we can see that if:

a) $\alpha + \beta > 1$, then $\lambda^{\alpha+\beta} > \lambda$, and so $Q_2 > \lambda Q_1$ (increasing returns to scale, *IRS*)

b) $\alpha + \beta = 1$, then $\lambda^{\alpha+\beta} = \lambda$, and so $Q_2 = \lambda Q_1$ (constant returns to scale, *CRS*)

c) $\alpha + \beta < 1$, then $\lambda^{\alpha+\beta} < \lambda$, and so $Q_2 < \lambda Q_1$ (decreasing returns to scale, *DRS*)

d) Constant Elasticity of Substitution (CES) Production Function

σ is independent of $MRTS_{L,K}$ or input ratio (K/L) or even output Q.

$\boxed{Q = A\left[aL^{-\rho} + (1-a)K^{-\rho}\right]^{-\frac{r}{\rho}}}$, where $A > 0,\ 0 < a < 1,\ \rho \geq -1$.

r is the degree of homogeneity. $\boxed{\sigma = \dfrac{1}{1+\rho}}$

a) If $\rho = -1\ (\sigma = \infty)$, $Q = A[aL + (1-a)K]^r$ (isoquant is a straight-line).

b) If $\rho = 0\ (\sigma = 1)$, we need a trick because we can't define 1^{∞}.

Taking log on both sides, we can get $\log Q = \log A - \frac{r}{\rho}\log[aL^{-\rho} + (1-a)K^{-\rho}]$.

But the second term of *r.h.s.* is indeterminate because of $\frac{0}{0}$. The best way to solve this problem is to use *L'Hospital rule*.

Generally, when we have $\lim_{\rho\to 0} f(\rho) = 0,\ \lim_{\rho\to 0} g(\rho) = 0$, $\boxed{\lim_{\rho\to 0}\frac{f(\rho)}{g(\rho)} = \lim_{\rho\to 0}\frac{f'(\rho)}{g'(\rho)}}$

Let's define $f(\rho) = r\log[aL^{-\rho} + (1-a)K^{-\rho}]$ and $g(\rho) = \rho$.

$$\lim_{\rho\to 0} f'(\rho) = \lim_{\rho\to 0}\frac{-r}{aL^{-\rho} + (1-a)K^{-\rho}}(aL^{-\rho}\log L + (1-a)K^{-\rho}\log K)$$

$$= \frac{-r}{1}(a\log L + (1-a)\log K) = -r\log(L^a K^{1-a})$$

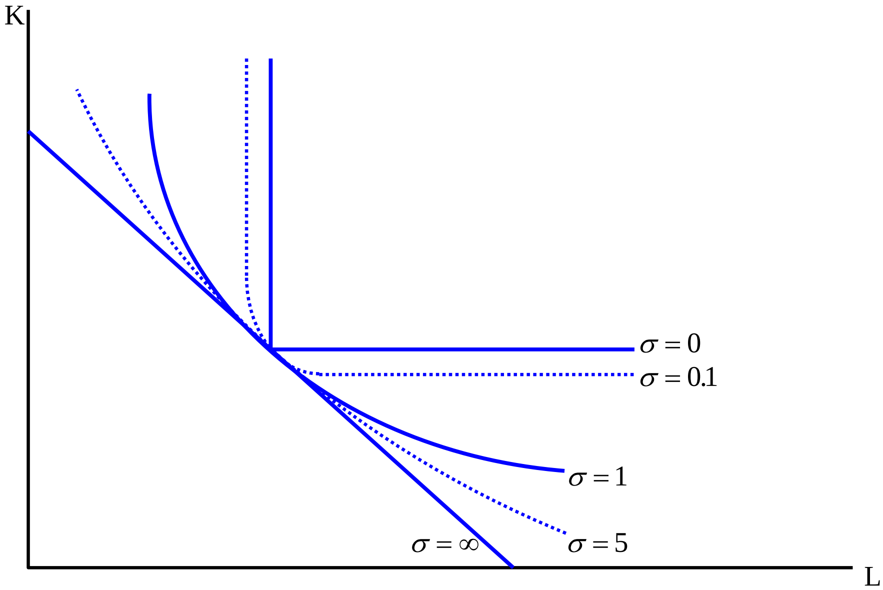
$$\lim_{\rho\to 0} g'(\rho) = 1$$

Finally, we know $\lim_{\rho\to 0}\log Q = \log A - \lim_{\rho\to 0}\frac{f(\rho)}{g(\rho)} = \log A + r\log(L^a K^{1-a})$

$\boxed{\lim_{\rho\to 0} Q = A(L^a K^{1-a})^r}$ (Cobb-Douglas Production Function)

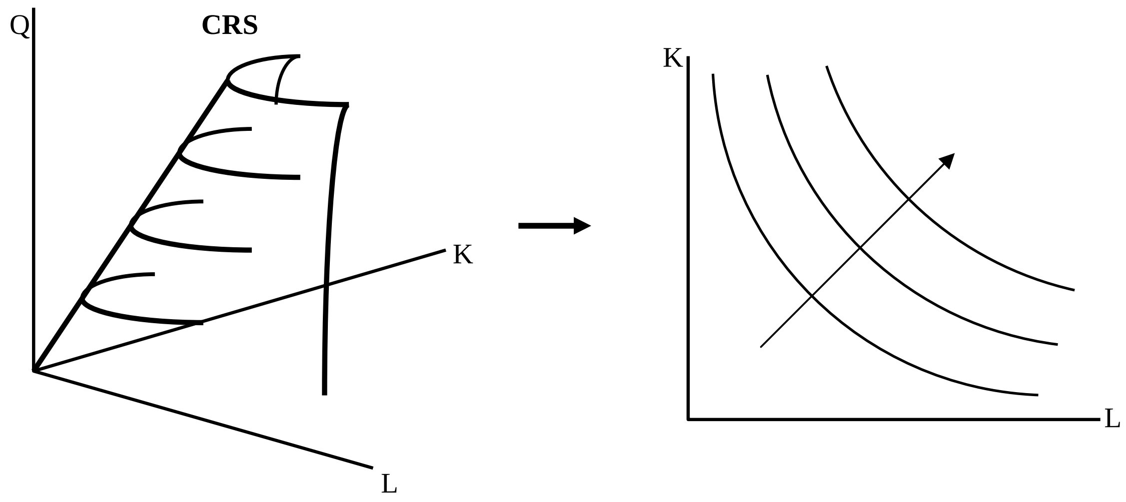
(You can check out why Cobb-Douglas has $\sigma = 1$)

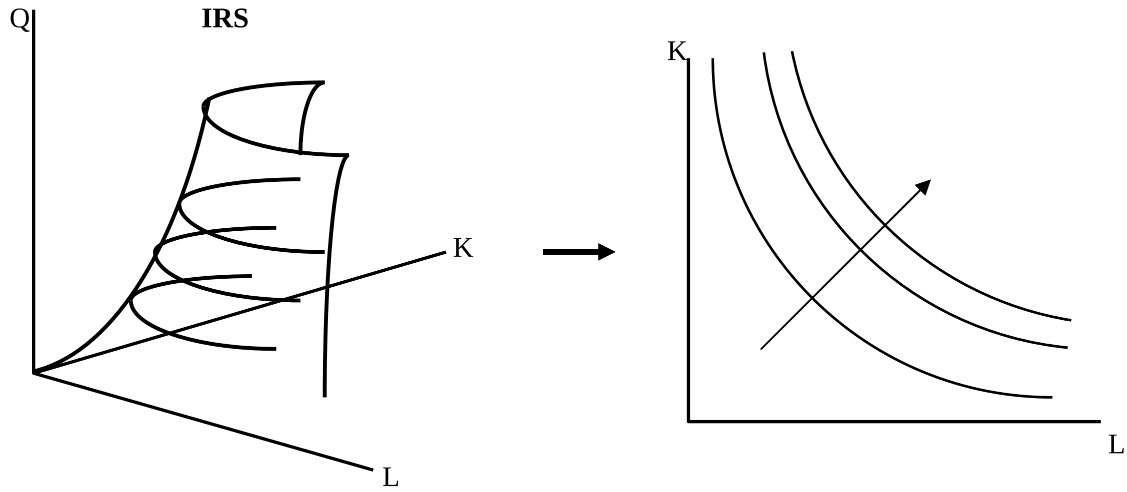
c) If $\rho \to \infty$ ($\sigma = 0$). Leontief Production Function $Q = \min[aL,\ bK]$

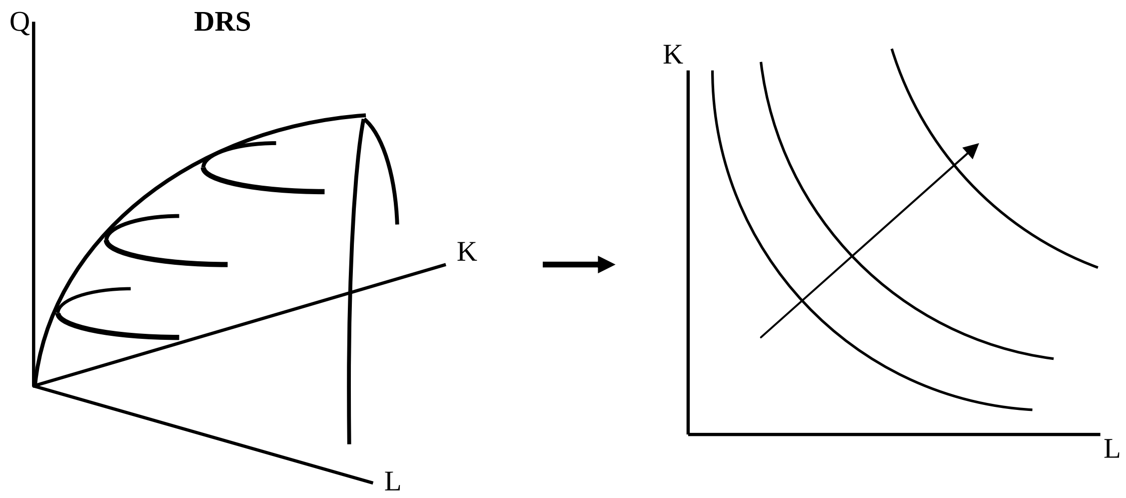


9. Returns to Scale: revisited

$$\begin{cases} f(kL,\ kK) = kf(L,\ K) : \text{CRS} \\ f(kL,\ kK) > kf(L,\ K) : \text{IRS} \\ f(kL,\ kK) < kf(L,\ K) : \text{DRS} \end{cases}$$


CRS
Q
K
L
K
L


IRS
Q
K
L
K
L


DRS
Q
K
L
K
L