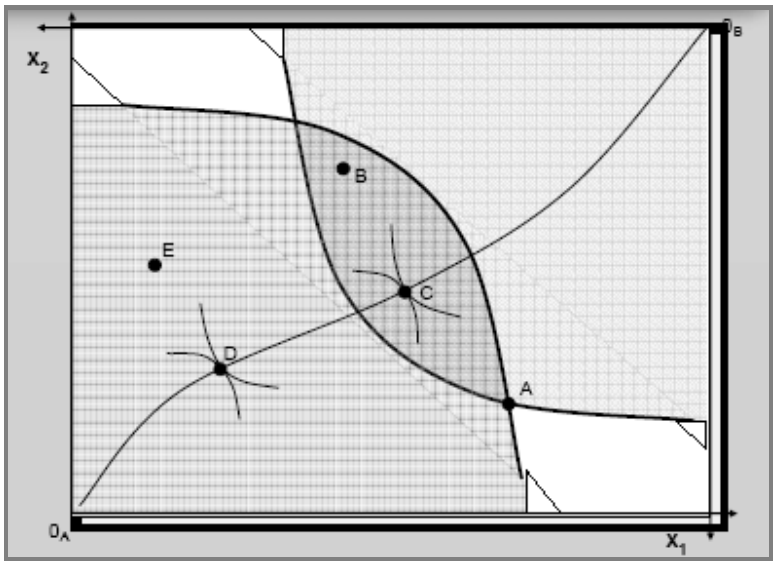
Lecture 15 – Economics of Welfare

1. Pareto Optimality

1) Illustrating Pareto Concepts

- Consider the Edgeworth Box of two individuals – Alice and Bob – trading two goods, x_1 and x_2.
- Starting at allocation A, the shaded area to the upper right represents the allocations that Alice weakly prefers to A.
- The shaded area to the lower left represents the allocations that Bob weakly prefers to A.
- The two sets overlap, and the lens-shaped area represents potential gains from trade – these are allocations that both Alice and Bob prefer to A.
- Since Alice and Bob individually prefer allocation B to allocation A, allocation B "Pareto dominates" allocation A – i.e. switching from A to B would be a Pareto improvement neither is worse off, and at least one is better off.



- Other allocations could Pareto dominate allocation B. Consider C, which Pareto dominates A and B, and is not Pareto dominated by any other alternative. From C, the only way to reach an allocation one or both prefer to C is to leave at least one worse off. For example, switching from allocation C to D would not be Pareto improving, because Bob prefers C to D.
- Allocation C is Pareto optimal, because no further unambiguous gains remain – all the gains from trade have been exhausted.
- There is generally not a single Pareto optimal allocation; Pareto optimal allocation form a set, the Pareto set – the upward-sloping curve in the Edgeworth box could be along one or two edges.

2) Defining Pareto Concepts

- A feasible allocation X Pareto dominates feasible allocation X' if and only if all individuals (weakly) prefer X to X' and at least one individual strictly prefers X to X'.
- A move from allocation X' to X is a Pareto improvement if X Pareto dominates X'.
- A feasible allocation X is Pareto optimal if there does not exist an allocation X' that would Pareto dominate X - Pareto optimal allocation is "optimal" in the sense that all the gains from trade have been exploited.

• The Pareto set is the set of Pareto optimal allocations.
- It is the locus of tangencies within the Edgeworth box, if such tangencies exist.
- If Alice's indifference curve crosses Bob's indifference curve at some allocation X', then X' would not be Pareto optimal: intersection of Alice's and Bob's preferred sets would not be empty (unless the allocation is along the edge of the box).

3) First Welfare Theorem

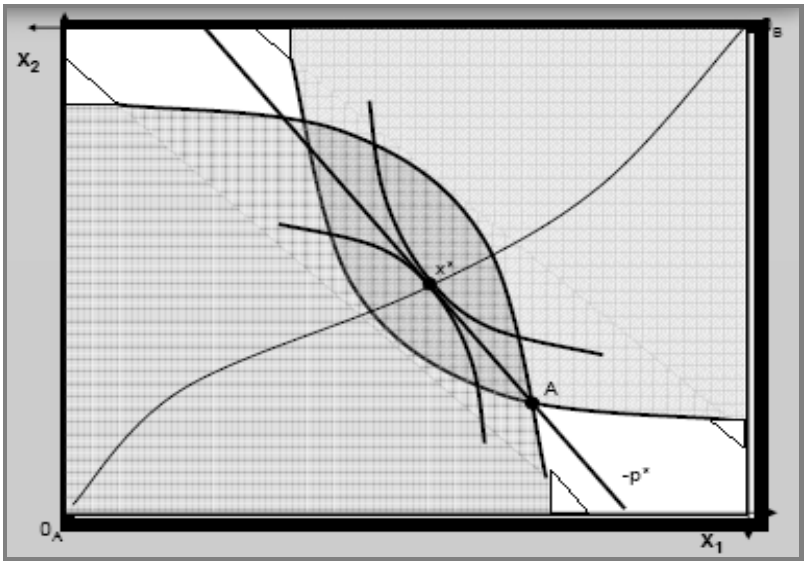
• In the *Wealth of Nations*, Adam Smith used the illusion of an invisible hand to present his famous case for a decentralized system of market transactions – capitalism.
• According to Smith (1776), each of us is "led by an invisible hand to promote a [social] end which was no part of his intention."
• Smith emphasizes the role of markets in directing resources to where they are valued most, so his standard for evaluating economic systems – his end – loosely fits the net benefit criterion (e.g. consumer's surplus).
• Smith's Invisible Hand translates into the modern era as the **First Welfare Theorem of Economics:** *If (x, p) is a competitive (i.e. Walrasian) equilibrium, then x is Pareto optimal.*

Proof: Suppose not, and let x' be a feasible allocation that all agents prefer to x. Then x' must not be affordable to each agent i. That is, $px_i' > p\omega_i$ for $i = 1, \cdots, I$. Sum these over i to find

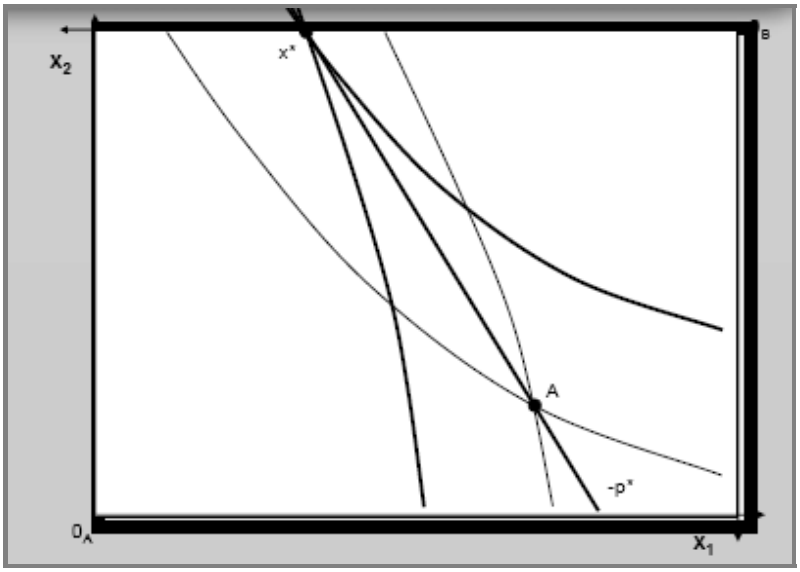
$$p\sum_{i=1}^{I} x_i' > p\sum_{i=1}^{I} \omega_i \equiv p\sum_{i=1}^{I} x_i'$$

which is a contradiction. Therefore, the theorem is true.

• If the equilibrium allocation is within the box, so choices are interior solutions, then proof is simple – Agents equate MRS (marginal rates of substitution) to price ratios, so MRSs are equated across agents, which means indifference curves are tangent (see the diagram below).



(First Welfare Theorem: Interior)



(First Welfare Theorem: Edge)

• The content of the First Welfare Theorem is that all the gains from trade are exhausted in a competitive equilibrium.

4) Individual Rationality and the Core

• The Pareto principle does not ask how we get to an allocation, only whether there could be gains to everyone from moving away from it. Allocation D (Diagram in page 82) is Pareto optimal, although it would not be individually rational for Bob to trade from A to D.

• The First Welfare Theorem guides us in our search for a competitive equilibrium: restrict our search to the Pareto set.

• The search can be further restricted by limiting our attention to individually rational allocations: an allocation is *individually rational* if each agent does at least as well as at this endowment: For each agent, construct the set of allocations that are weakly preferred to the endowment allocation. And intersection of these sets is the *region of mutual advantage*.

• Competitive equilibria must be individually rational, as well as Pareto optimal.

• In he two-agent economy, agents would bargain bilaterally rather than behave competitively, but the equilibrium will be individually rational and Pareto optimal. With two agents, this corresponds to the core.

• Feasible allocation x is an element of the *core* of an economy if no group (i.e. breaking coalition) can do better alone.

⇒ **Theorem: A competitive equilibrium allocation is a core allocation.**

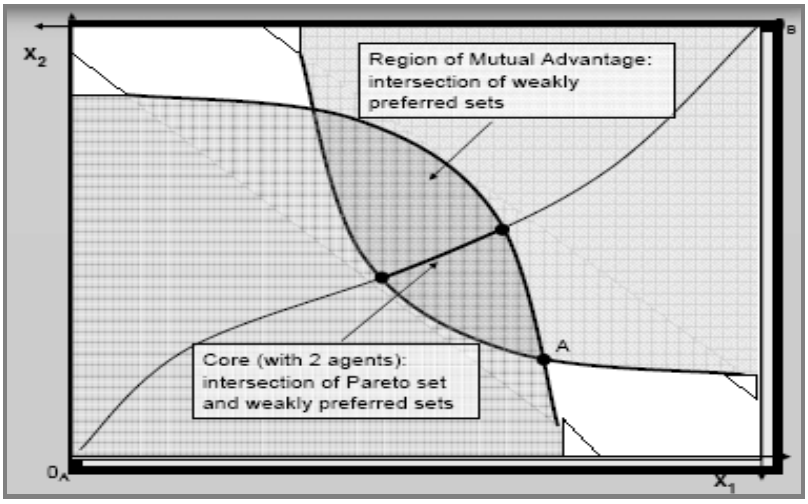
• Suppose we replicate the 2-agent (i.e. Alice and Bob) endowment economy, so there are r agents of each type: now, competition is more sensible.

⇒ **Replication Theorem:** The competitive equilibrium does not depend on r.

⇒ **Equal treatment Theorem:** Agents of he same type receive the same core allocation.

⇒ **Core Convergence Theorem:** As r grows, the core converges to the competitive equilibrium allocation.

• These results allow us to use the Edgeworth box to characterize the equilibrium with many agents of each type 9core is generally smaller than the set of individually rational Pareto optima).



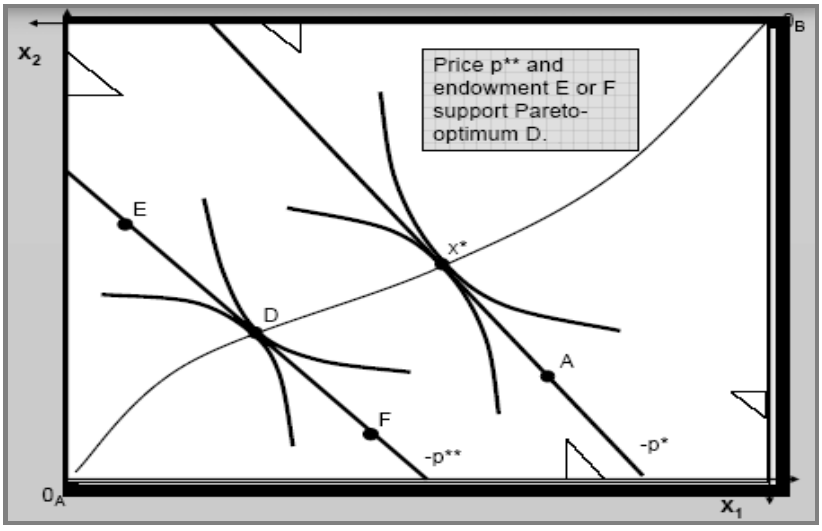
(Individual Rationality and the Core)

5) Second Welfare Theorem

• Any allocation within the Pareto set can be supported by a competitive equilibrium. Second Welfare Theorem of Economics: Suppose x is an element of the Pareto set such that each agent i receives a positive amount of each good. If preferences are convex (as well as continuous and monotonic), then x is a competitive (i.e. Walrasian) equilibrium allocation for any endowment w satisfying $pw_i = px_i$ for $i = 1, \cdots, I$.

• The impact of this theorem is that the issue of distribution can be separated from the issue of efficiency.

- Equity concerns are handled by moving along the Pareto set.
- Any egalitarian allocation can be achieved via competition by juggling the endowment, allocation in Diagram of page 82.
- First, society asks what alternative it likes best, second, it juggles endowments to get that as a competitive equilibrium.
- For example, the state might impose "land reform," which would switch the endowment point to E; from there, everyone would trade freely to alternative D, which is Pareto optimal.



(Second Welfare Theorem)

2. Social Welfare Function

1) Pareto and Complete Rankings

- Pareto principle ranks allocations but not completely.
- Many allocations are just not comparable on Pareto grounds
  – Pareto requires unanimous agreement for comparison.
- How should a society decide among all the alternatives within the Pareto set?
  – Pareto principle offers no guide for social choice.
- Perhaps we can construct a social welfare function that represents society's rankings of allocations.

2) Ranking Social Alternatives

- The problem is how to aggregate the preferences of individuals to determine the best alternative for the group.
- If there exists a solution to this problem in the form of complete and transitive social preferences, then social alternatives (e.g. the allocation of resources) can be ranked.
- Could social preferences be represented by a social welfare function, which assigns numbers to these alternatives such that better alternatives receive higher numbers?
  If complete, transitive, and continuous social preferences exist, then there exists a continuous social welfare function that represents those preferences.
  Social welfare function is ordinal.
- Generally, we have $\boxed{w = W(\mathrm{x}) = W(x_1, \cdots, x_I)}$. If we restrict attention to social welfare as a function of individual utilities,

$$w = W(u_1, \cdots, u_I) \qquad (1)$$

3) Maximizing Social Welfare

- The problem then is to allocate consumption goods across individuals to maximize social welfare w. That is,

$$\max_x W(u_1(x_1), \cdots, u_I(x_I)) \qquad (2)$$

$$\text{subject to } \sum_{i=1}^{I} x_i{}^j \le w^j, \quad j = 1, \cdots, n$$

- The following theorem establishes that this problem amounts to finding the best allocation within the Pareto set.
- **Theorem: If x^* maximizes a social welfare function, then x^* is Pareto optimal.**
- **Proof:** The FOCs are

$$W_i \frac{\partial u_i}{\partial x_i{}^j} = \lambda_j \qquad \begin{cases} i = 1, \cdots, I \\ j = 1, \cdots, n \end{cases} \qquad (3)$$

  Take the ratio of these for each agent I to find that marginal rates of substitution are equated across agents, i.e., x^* is a Pareto optimum.
- Solution depends on the form of the social welfare function, which in principle is supplied by social philosophers.
- A utilitarian simply sums the utilities.

$$w = u_1 + u_2 + \cdots + u_I \qquad (4)$$

  So, the utilitarian solution *equalizes marginal utilities of income across individuals.*
- More generally, the utilitarian could sum weighted utilities.

$$\boxed{w = \sum_{i=1}^{I} \alpha_i u_i(x_i)} \qquad (5)$$

  where the weights α_i are positive and sum to one.

• A Rawlsian weights only the least fortunate, so the social welfare function would be

$$w = \min(u_1, \cdots, u_I) \qquad (6)$$

So, the Rawlsian solution *equalizes utilities across individuals*.

• **Theorem: If x^* maximizes a social welfare function, then there exist weights α_i such that x^* maximizes the utilitarian social welfare function.**

• **We use the utilitarian social welfare function to derive the Pareto set simply by varying the weights.**

3. Arrow's Impossibility Theorem

1) Do social welfare functions exist?

• Social welfare functions go beyond the Pareto principle to complete social preferences.

• Candidate for a social welfare function is majority-rule voting. For example, each of us might vote for every pair of alternative allocations, and in each case the allocation with more votes would be deemed preferred. And the implied social preferences would be complete.

• There are other mechanisms for comparing all alternative allocations: would they deliver the same social preferences?

• Arrow's Impossibility Theorem establishes that social alternatives cannot be ranked. So, social welfare functions do not exist. For example, with majority-voting, social preferences would be complete but intransitive.

2) Conditions and Proof

• Do social preferences exist? – In special cases, yes. But generally, no.

• Arrow's conditions for social preferences are

(a) **Universal Domain**: Social preferences are complete and transitive for all individual preferences that are complete and transitive.

(b) **Pareto Principle**: If allocation x is preferred to allocation x' for all individuals, then x is preferred to x'.

(c) **Independence of Irrelevant Alternatives (IIA)**: For any two allocations x and x', whether x is preferred or vice versa does not depend on the rankings of other alternatives x''.

(d) **No Dictatorship**: No individual i is a dictator in the sense that social preferences are i's preferences no matter what the preferences of the other individuals.

• **Arrow's Impossibility Theorem:** If there are at least three alternative allocations (or social outcomes), then there does not exist social preferences satisfying Universal Domain, the Pareto principle, independence of Irrelevant Alternatives, and No dictatorship.

• This is a powerful blow to social engineers looking to implement "what's best" for society. It should also frustrate economists working with social welfare functions. So, the only hope for those requiring a social welfare function is to relax or to replace one of Arrow's assumptions (Is Arrow's Impossibility Theorem sensitive to its assumptions?).

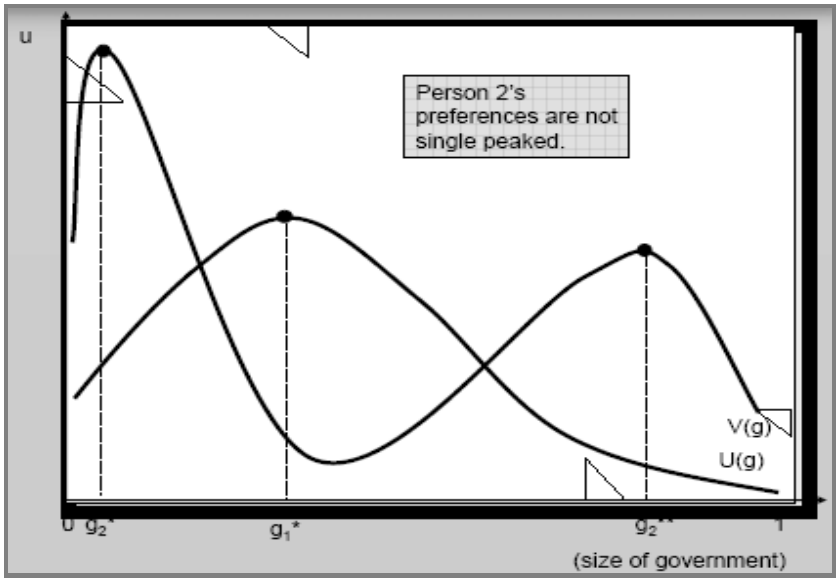
3) Single-Peaked Preferences

• By relaxing Universal Domain, might we to overturn Arrow's result for a narrower class of individual preferences?

• If allocations can be ordered such that each individual's ranking (or utility) is a single-peaked function of the allocations, then social preferences exist. For example, suppose the alternatives are summarized by the size of government from small to big. If each of us has

our own "most preferred" size of government and utility falls away from this peak, then a complete and transitive social ranking of the size of government exists. Indeed, majority rule would work as a social welfare function.

• There is no reason to believe it's sensible in terms of ethics.



(Single-Peaked Preference)

4) Benevolent Dictator

• Arrow's assumption of No Dictator is important. Without it, complete and transitive social preferences would exist as the preferences of any one individual – the dictator – and would thus be complete and transitive.

• This poses a problem for social engineering; all the weight of social policy would be directed to this one individual.

• However, if society's dictator were benevolent, caring for others like a parent cares for his or her children, then a social ranking of alternatives would exist.

• Suppose a benevolent dictator cares about how the alternative *x* impacts each individual *i* and he summarizes each individual *i*'s preferences by some utility function u_i.

• Even with ordinal utility, our dictator can represent his – the social – preferences with a social welfare function.*

• In terms of social choice, our dictator chooses from all the feasible alternatives *x* the one that maximizes the social welfare function.

• As we asserted above, varying the weights α_i - e.g. caring more for Bob and less for Alice – traces out the Pareto set.

• So, with explicit weights, the benevolent (non-paternalistic) dictator chooses one alternative from the Pareto set.

• Without explicit weights, solving the benevolent dictator's problem amounts to finding the Pareto set (his is how it is used in much of modern economics).

* The dictator's preferences are complete and transitive, so the existence of a social ordering of alternatives is not in doubt. But choice of a social welfare function and the utility function must be simultaneous. That is, if the dictator chose to represent my preferences using the *v* index rather than the *u* index then the form of this social welfare function would be affected.

- The empirically relevant case is the one without explicit weights, because the benevolent dictator does not exist.
- The benevolent dictator approach is equivalent – in terms of virtue, as well as incompleteness – to the Pareto principle.

5) Cardinal Utility

- A third attempt to resurrect social preferences allows for interpersonal comparisons of utility.
- Replace IIA with cardinal utility, and the utilities u_i in the social welfare function would carry cardinal significance.
- Given the premise of cardinal utility, complete and transitive social preferences exist, can be summarized by a social welfare function, and could guide social choices over social policies, economic systems, etc.
- These results build from a foundation of cardinal utility, which is no foundation at all.
- Nothing observable in behavior can provide any evidence for or against cardinal utility.[†]
- So it is rational to conclude that cardinal utility – like a magic elf – does not exist. Utility is ordinal.
- For matters of social engineering, one cannot escape from interpersonal comparison of value.
- But intensity of preference is not limited to cardinal utility; we compare valuations across agents without reference to utility. We use a surplus measure, such as the compensating or equivalent variation.
- Dollar-valuations are comparable, so perhaps summing valuations across agents would give a social welfare function.

6) Robustness of Arrow's Impossibility Theorem

- Going beyond the Pareto principle to a complete ranking of social alternatives is difficult if not impossible.
- Although the analysis does not constitute a proof that no relaxation of Arrow's assumptions could overturn the Impossibility Theorem, the theorem is quite robust.
- We are left a limited role for social welfare analysis – use the social welfare function to generate the Pareto set.

7) Robustness of the First Welfare Theorem

- Exploring robustness of the First Welfare Theorem remains.
- Can it handle externalities and public goods?

[†] Most economists seem to believe that utility is cardinal, but that analysis of the behavior of individuals requires only ordinal utility. Becker (1971, 54), however, stresses that the concept of cardinal utility is vacuous because "diminishing marginal utility of income or of commodities cannot be inferred from any evidence on consumer choices."