Lecture 16 – Investment, Time, and Risk (Basic issues in Finance)

1. Intertemporal Investment Decisions: The Importance of Time and Discounting

1) Time as one of the most important factors affecting firm's investment decisions: A firm's decision to purchase a capital good, such as a new plant or a new piece of heavy machinery, differs from its decision to hire more labor or purchase additional raw materials, in large part because of very different time horizons for these decisions. Once a firm purchases a new capital good, the firm is stuck with that decision for years to come, while labor can be laid off on a short notice, and raw material orders can be quickly cancelled.

2) Present Value and Discounting

- X dollars invested today at an interest rate r would increase in value to $X(1+r)$ dollars in one year, $X(1+r)(1+r) = X(1+r)^2$ dollars in two years, and $X(1+r)^t$ dollars in t years. The present value of $X(1+r)^t$ dollars received t years from today equals $X(1+r)^t/(1+r)^t = X$ dollars. By similar reasoning, the promise to pay X dollars t years from today has a present value of:

$$PV = \frac{X}{(1+r)^t}$$

- **Present Value of Profit**: Suppose π_t represents the firm's profit in time period t, and $(1+r)$ represents the rate at which the firm is willing to trade future for present income just as it represented the rate at which an individual would trade future for present consumption.

$$\pi_{PV} = \frac{\pi_1}{(1+r)} + \frac{\pi_2}{(1+r)^2} + \frac{\pi_3}{(1+r)^3} + \cdots + \frac{\pi_n}{(1+r)^n} \quad (1)$$

The maximization of long-run profit implies the maximization of π_{PV}. Equation (1) shows that the maximization of long-run profit depends not only one the annual flow of economic profit, π_t, but also critically depends on the discount rate, r. Firms with short time horizons place a low value on future profits and are willing to trade current profit for future profit only a at high interest rate, and therefore have high discount rates. These firms want to earn high profits in early periods even if it means sacrificing large future profits. Firms with longer time horizons are willing to trade current profits for future profits at a low interest rate, and therefore have lower discount rates. These firms are willing to sacrifice current profits in order to earn higher profits in the future.

Assuming that n goes to infinity and π_t is constant and equal to π for all values of t, Equation (1) can be rewritten as:

$$\pi_{PV} = \frac{\pi}{(1+r)} + \frac{\pi}{(1+r)^2} + \frac{\pi}{(1+r)^3} + \cdots = \frac{1}{(1+r)}\left(\pi + \frac{\pi}{(1+r)} + \frac{\pi}{(1+r)^2} + \cdots\right) \quad (2)$$

$$\pi_{PV} = \frac{1}{(1+r)}(\pi + \pi_{PV}), \quad \pi_{PV}(1+r) = \pi + \pi_{PV},$$

$$\boxed{\therefore \pi_{PV} = \frac{\pi}{r}} \qquad (3)$$

• **Generalized Present Value Calculations**

Equation (2) can be generalized for any sum beginning at any time *t*, not just for $t = 1$.

$$\pi_{PV} = \sum_{t=c}^{\infty} \frac{\pi}{(1+r)^t} = \frac{\pi}{(1+r)^c} + \frac{\pi}{(1+r)^{c+1}} + \frac{\pi}{(1+r)^{c+2}} + \cdots \qquad (4)$$

where c is any number less than, equal to, or greater than zero. In Equation (4), *c* represents the time when the flow of profit begins. If the profit flow begins in the current time period, $c = 0$. Rewriting Equation (4) will yield:

$$\boxed{\pi_{PV} = \frac{1}{(1+r)^{c-1}}\left[\frac{\pi}{(1+r)} + \frac{\pi}{(1+r)^2} + \frac{\pi}{(1+r)^3} + \cdots\right] = \frac{1}{(1+r)^{c-1}}\left[\frac{\pi}{r}\right] = \frac{\pi}{r(1+r)^{c-1}}} \qquad (5)$$

Ex) Suppose that profit flow is \$100. And discount rate is 0.10 with $c = 4$ and -4. Then use Equation (5) to calculate present value of respective cases.

$$\pi_{PV} = \frac{\$100}{0.1(1+0.1)^{4-1}} = \$751.88. \text{ And } \pi_{PV} = \frac{\$100}{0.1(1+0.1)^{-5}} = \$1{,}610$$

3) The Value of a Bond or Perpetuity

One important use of discounting is to determine the value of *bonds* and *perpetuities*.

* **Bond:** A debt security issued by a government or a firm where the purchaser lends the issuer a lump sum amount of money today in return for a promise from the issuer to pay a finite stream of future payments.

* **Coupon**: The annual interest payment on a bond.

* **Perpetuity**: A debt security where the purchaser lends the issuer a lump sum amount of money today in return for a promise from the issuer to pay a fixed amount of income to the purchaser forever.

• To determine the value of a bond to its buyer, we calculate the present value of the payments stream. Suppose, for example, a bond is sold for \$1,000 and pays a coupon of \$100 per year for each of the next 10 years and a principal repayment in the tenth year of \$1,000.

$$PV = \frac{\$100}{(1+r)} + \frac{\$100}{(1+r)^2} + \frac{\$100}{(1+r)^3} + \cdots + \frac{\$100}{(1+r)^{10}} + \frac{\$1{,}000}{(1+r)^{10}}$$

At a discount rate of zero, the present value of the bond will be equal to $PV = 10(\$100) + \$1{,}000 = \$2{,}000$. As the discount rate increases above zero, the present value will continuously decline. Using Equations (3) and (5), we can simplify the calculation of the present value of the bond as follows:

$$PV = \frac{\$100}{r} - \frac{\$100}{r(1+r)^{10}} + \frac{\$1{,}000}{(1+r)^{10}}$$

$\boxed{\frac{\$100}{r}}$: PV of an annual payment of \$100 forever with the first payment being made in one year (from Eq.(3))

$\boxed{\frac{\$100}{r(1+r)^{10}}}$: PV of the payments that would be made from $t = 11$ to $t = \infty$ on a bond that pays a \$100 coupon forever.

So the first two terms on the *r.h.s.* of the above calculation measure the present value of the 10 years of coupon payments on the bond. And the third term of the above calculation is the PV of the principal repayment that is received in 10 years.

Now, if the purchaser of the bond has a 5% discount rate, the present value of the bond is:

$$PV = \frac{\$100}{.05} - \frac{\$100}{(.05)(1.05)^{10}} + \frac{\$1{,}000}{(1.05)^{10}} = \$2{,}000 - \$1{,}227.83 + \$613.91 = \$1{,}386.08$$

• To find the present value of a perpetuity, which, beginning in year 1, pays \$1,000 per year forever, use Equation (3) to calculate:

$$PV = \frac{\$1{,}000}{(1+r)} + \frac{\$1{,}000}{(1+r)^2} + \frac{\$1{,}000}{(1+r)^3} + \cdots = \frac{\$1{,}000}{r}$$

• Government and corporate bonds are sold in the bond market in the same way that shares of stock are sold in the stock market. Buyers and sellers agree to exchange a bond at an equilibrium price *p*. The yield on a bond is simply the rate of return on the market price (market value) of the bond. The face value of the bond equals the principal payment to the owner when the bond reaches maturity. To determine the yield or rate of return on a bond, we rewrite Equation (1) in the following form:

$$\boxed{p = \frac{x_1}{(1+rr)} + \frac{x_2}{(1+rr)^2} + \cdots + \frac{x_n}{(1+rr)^n} + \frac{principal}{(1+rr)^n}} \qquad (6)$$

Buyer knows bond price (*p*), the coupon payments (the values of x_i for all *i*), and the principal payment; therefore, the only unknown is the yield or rate of return of the bond.

2. The Firm's Investment Decision

1) Net present Value Criterion

• If the PV of the expected cash flow generated by an investment is greater than the PV of the cost of the investment, then a firm should make the investment.

$$NPV = -C + \frac{X_1}{(1+r)} + \frac{X_2}{(1+r)^2} + \frac{X_3}{(1+r)^3} + \cdots + \frac{X_n}{(1+r)^n} \qquad (7)$$

• If NPV > 0, the firm undertakes the investment. If NPV < 0, the firm does not.

2) Firm's Discount Rate

• The discount rate on a particular project is the rate of return on the firm's next best alternative investment project with the same risk. In other words, the discount rate is the opportunity cost of capital invested in projects with the same risk. It is necessary for the firm to put projects in different risk categories, because investors insist on earning higher rates of return on riskier investments.

• For "safe" or low-risk investments, a common measure of the opportunity cost of capital is the rate of return on a 10-year U.S. Treasury Bond. Because there is essentially no risk of the U.S. government defaulting on its bonds, the return on a government bond is considered a risk-free return. Firms and individuals always have the opportunity to invest in risk-free U.S. government bonds.

Ex) Suppose that prior to August 1, 2008, Disney is considering building a Phantasmic theme restaurant at its MGM Studios them park at Disney World in Orlando, FL. The restaurant will cost $6million to build and Disney expects it to generate a cash flow of $1 million per year for each of the next 10 years. After 10 years, Disney plans to completely change the theme of the restaurant and expects to sell the salvageable equipment for $1.2 million. Based on the estimates, Disney needs to determine whether it should build the new restaurant. Now, using different discount rates of 5%, 10%, and 15%, calculate the respective NPVs. And if the Disney's expected cash flow from the restaurant decreases by 25% due to unexpected financial turmoil, how do the NPVs change?

3) Internal Rate of Return Criterion

• An alternative to the net present value criterion for evaluating investments is the internal rate of return (IRR) of an investment, which is the discount rate for which the net present value of the investment equals zero. He IRR criterion for an investment states that if the IRR is greater than the firm's required rate of return on investments, the firm should undertake the investments; if the IRR is less than the firm's required rate of return, the firm should not undertake the investment. What is the firm's required rate of return? If the firm is borrowing to pay for the investments, the required rate of return will be the cost of borrowing. The firm then undertakes the investment if and only if the IRR is greater than the firm's cost of borrowing.

- For any investment, there always exists at least one IRR. Consider the Disney example (page 93). The IRR is calculated by setting the NPV equal to zero as follows:

$$NPV = -\$6 + \frac{\$1}{1+IRR} + \frac{\$1}{(1+IRR)^2} + \cdots + \frac{\$1}{(1+IRR)^{10}} + \frac{\$1.2}{(1+IRR)^{10}} = 0$$

$$NPV = -\$6 + \frac{\$1}{IRR} - \frac{\$1}{(IRR)(1+IRR)^{10}} + \frac{\$1.2}{(1+IRR)^{10}} = 0$$

Solving by trial and error, IRR=0.121:

$$-\$6 + \frac{\$1}{0.121} - \frac{\$1}{(0.121)(1.121)^{10}} + \frac{\$1.2}{(1.121)^{10}} = -\$6 + \$8.26 - 2.63 + 0.38 \cong 0.01 = 0$$

If Disney's cost of borrowing is less than 12.1 percent, Disney will build the restaurant.

4) Real Versus Nominal Discount Rates and Cash Flows

- Real values adjust for inflation; nominal values do not. The real interest rate on a bond measures the increase in real purchasing power that the purchaser receives each year. The difference between the real and nominal interest rate can be closely approximated by:

$$\boxed{r \cong i - \pi} \qquad (8)$$

- For example, if a government bond pays a nominal 7% each year and the inflation rate is 2%, then the real interest rate is approximately 5%. The use of the nominal interest rate overstates the increase in purchasing power to the lender each year. Suppose an individual purchases a \$10,000 one-year bond paying a nominal 7% interest rate. The bond holder receives a \$700 interest payment plus a \$10,000 principal repayment next year. In nominal terms, the bond holder sacrificed \$10,000 of purchasing power this year in return for \$10,700 of purchasing power next year. In real terms, however, if the price index is 1.00 today and inflation is 2%, then the price index next year is 1.02. In return for sacrificing \$10,000 in purchasing power today, the lender receives \$10,700/\$1.02=\$10,490.20 in purchasing power next year, or slightly under a 5% real return.

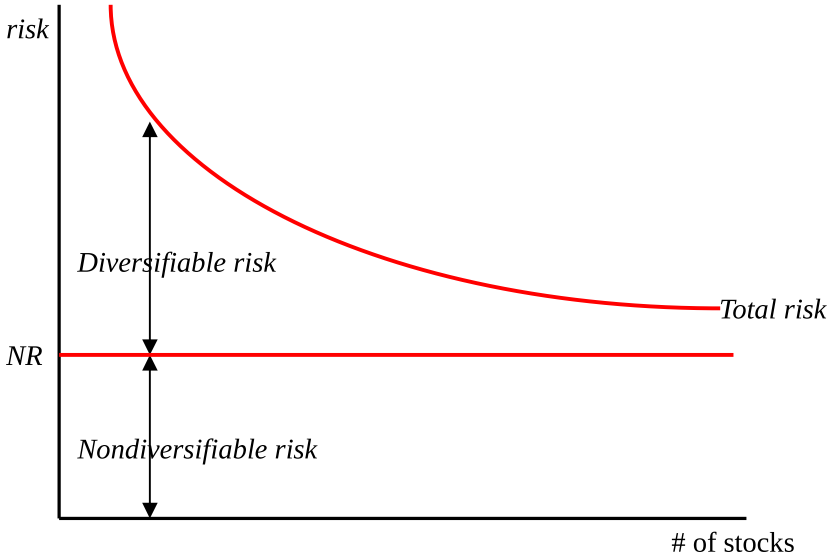
3. The Impact of Risk on Investment Decisions

1) Diversifiable and Non-diversifiable Risk

- Diversification requires a firm to reduce its risk by choosing an appropriate collection of investments, some of which are high-risk gambles and some low-risk gambles. Risk that can be eliminated through diversification is called **diversifiable risk**. Risk that cannot be eliminated through diversification is called **nondiversifiable risk** or **market risk**. It is sometimes useful to think of diversifiable risk as risk that affects only a small number of assets, whereas nondiversifiable risk affects a large number of assets. The risk of investing in just a few stocks is primarily diversifiable risk, because purchasing a broader base of stocks can reduce it. Similarly, the risk undertaken by life and automobile insurance companies is also diversifiable risk, because by selling insurance to many different individuals, the companies can reduce their risk.

• What constitutes nondiversifiable risk? One example is risk associated with fluctuations in general economic conditions (ex. Economic downturn resulting from 9.11 terrorist attack, unanticipated oil price shock, a financial crisis in Southeast Asia, unexpected increase in interest rate, etc.)

• Diversifiable risk is a function of the number of stocks owned. And nondiversifiable risk is independent of the number of stocks owned.



2) Capital Asset Pricing Model (CAPM)

• Need to examine how firms include risk in their net present value calculations. The most commonly used method is called the **capital asset pricing model**, or the **CAPM**.

• The CAPM measures the risk on a particular capital investment by comparing that risk with the risk of investing in the entire stock market. Suppose an individual invested in the entire stock market; that is, bought every single stock offered for sale. The investor would bear no diversifiable risk, because she would be completely diversified; she would, however, bear some nondiversifiable risk. Because investment in the entire market includes some nondiversifiable risk, she would demand a risk premium for investing in the market instead of investing in risk-free government bonds. If the return on the entire market is r_m and the return on risk-free government bonds is r_f, then the risk premium is $(r_m - r_f)$.

- **An Investment's Asset Beta (β)**: If an investor is considering investing in the stock of just one company, he can use the capital asset pricing model to calculate the risk premium he would demand to invest in that stock. According to the CAPM, he would demand a risk premium on that stock that is proportional to the risk premium on the entire market, so that:

$$\boxed{\text{risk premium} = r_i - r_f = \beta(r_m - r_f)} \qquad (9)$$

or

$$\boxed{r_i = r_f + \beta(r_m - r_f)} \qquad (10)$$

  where r_i is the expected return on the stock, and β, called the asset beta, is a measure of the nondiversifiable risk associated with the investment. The asset beta β is related only to nondiversifiable risk. Because borrowers know that lenders can eliminate all of their diversifiable risk, they will not pay a risk premium to lenders to cover diversifiable risk. The asset beta measures the nondiversifiable risk associated with a particular investment compared to an investment in the entire market.* For an investment in the entire market, the risk premium is ($r_m - r_f$), and therefore $\beta = 1$.
- Investments that are riskier (less risky) than investing in the entire market have asset betas greater (smaller) than 1. Some investments are more highly correlated with movements in the stock market than others. If a 1 percent decrease in the value of the market, r_m, is associated with a 2 percent decrease in the return on a particular investment, then $\beta = 2$ for that investment. This investment has a very high positive correlation with the entire market and is quite risky.
- Profits in the airline and steel industries are very sensitive to macroeconomic shocks, and therefore have relatively high asset betas. Profits in the ready-to-eat cereal, soaps and detergents, and food-processing industries are much less sensitive to macroeconomic shocks, and therefore have relatively low asset betas.
- If the β associated with a particular stock investment is known, it is possible to calculate the correct discount rate to use in calculating the NPV of that investment. The discount rate is:

---

* In statistics, β is technically defined as:

$$\beta = \frac{\text{cov}(r_i, r_m)}{\text{var}(r_m)},$$

where $\text{cov}(r_i, r_m)$ is the covariance of r_i and r_m, which is a measure of the linear association between them; and $\text{var}(r_m)$ is the variance of r_m, which measures the dispersion of r_m around its mean value. This ratio, and therefore, β, measures the volatility of the investment i divided by the volatility of the entire market.

$$\boxed{\text{discount rate} = r_f + \beta(r_m - r_f)} \qquad (11)$$

- For a stock investment, calculating Equation (11) is straightforward. Over many years, the risk premium on the entire stock market, $r_m - r_f$, has averaged approximately 8 percent. If the current risk-free return on government bonds is 5 percent, then:

  $$\text{discount rate} = 0.05 + \beta(0.08)$$

  By following past trends in a stock's value, it is possible to calculate the stock's β.
- Firms use the CAPM to determine the discount rate to use when calculating the net present value of an investment. According to the CAPM, the larger the firm's asset beta, the larger the firm's discount rate, and the less likely the firm is to undertake an investment.