Economics of Industrial Organization

Lecture 6: Cournot Oligopoly

꾸르노 복점 모형

- Single good produced by n firms
- Cost to firm *i* of producing q_i units: $C_i(q_i)$, where C_i is nonnegative and increasing
- If firms' total output is Q then market price is P(Q), where P is nonincreasing
- Profit of firm *i*, as a function of all the firms' outputs:

$$\pi_i(q_1, q_2, \ldots, q_n) = q_i P\left(\sum_{j=1}^{n} q_j\right) - C_i(q_i)$$

꾸르노 복점 모형

Strategic game:

- players: firms
- each firm's set of actions: set of all possible outputs
- each firm's preferences are represented by its profit

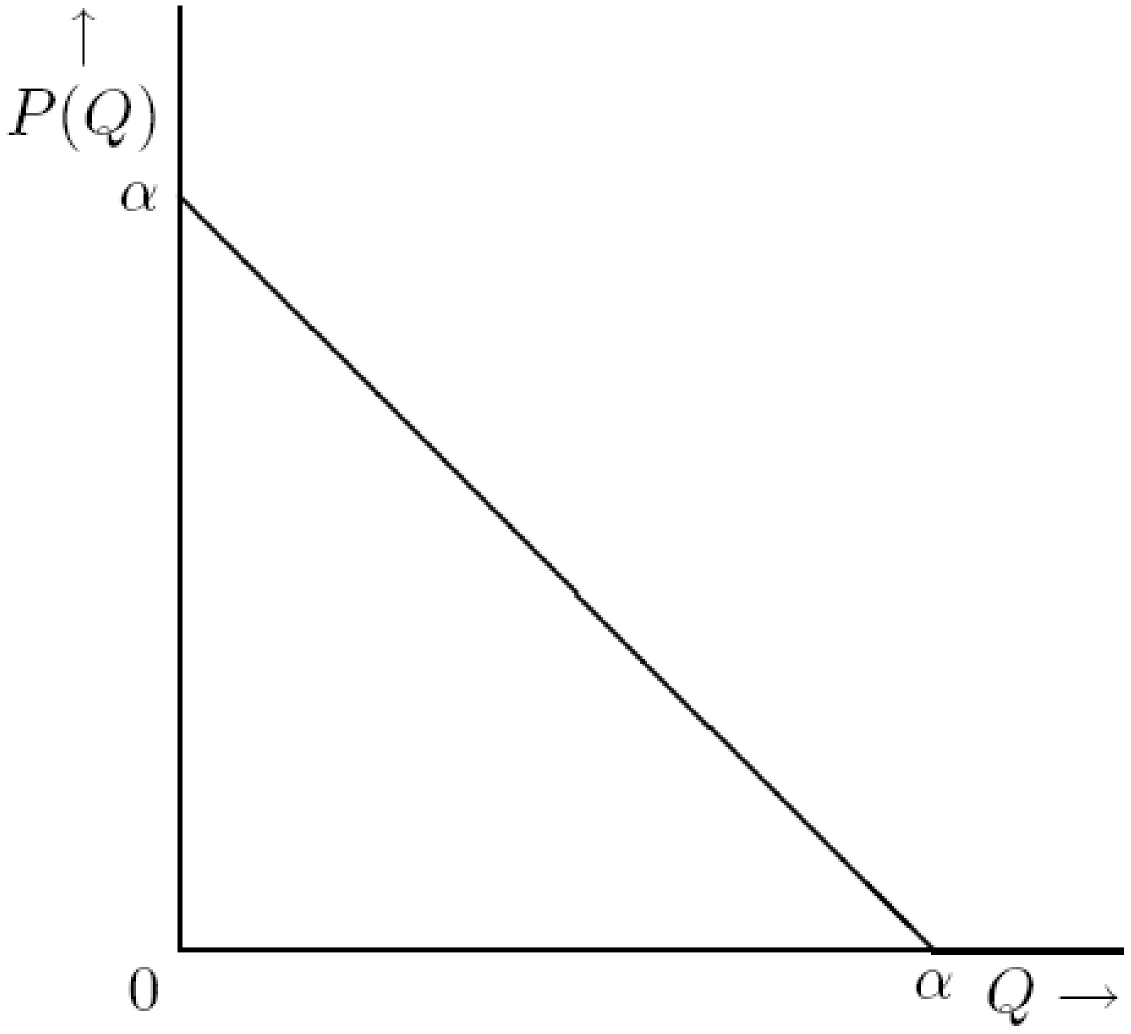
Duopoly

- two firms
- Inverse demand:

$$P(Q) = \max\{0, \alpha - Q\} = \begin{cases} \alpha - Q & \text{if } Q \leq \alpha \\ 0 & \text{if } Q > \alpha \end{cases}$$

- constant unit cost: $C_i(q_i) = cq_i$, where $c < \alpha$

Duopoly



Duopoly

Recall for a perfectly competitive firm, P=MC, so

$\alpha - Q = c$, or $Q = \alpha - c$.

Recall for a monopolist, MR=MC, so

$\alpha - 2Q = c$, or $Q = (\alpha - c)/2$.

[We could verify this by using calculus to solve the profit maximization problem.]

Duopoly

Payoff functions

Firm 1's profit is

$$\pi_1(q_1, q_2) = q_1(P(q_1 + q_2) - c)$$
$$= \begin{cases} q_1(\alpha - c - q_2 - q_1) & \text{if } q_1 \leq \alpha - q_2 \\ -cq_1 & \text{if } q_1 > \alpha - q_2 \end{cases}$$

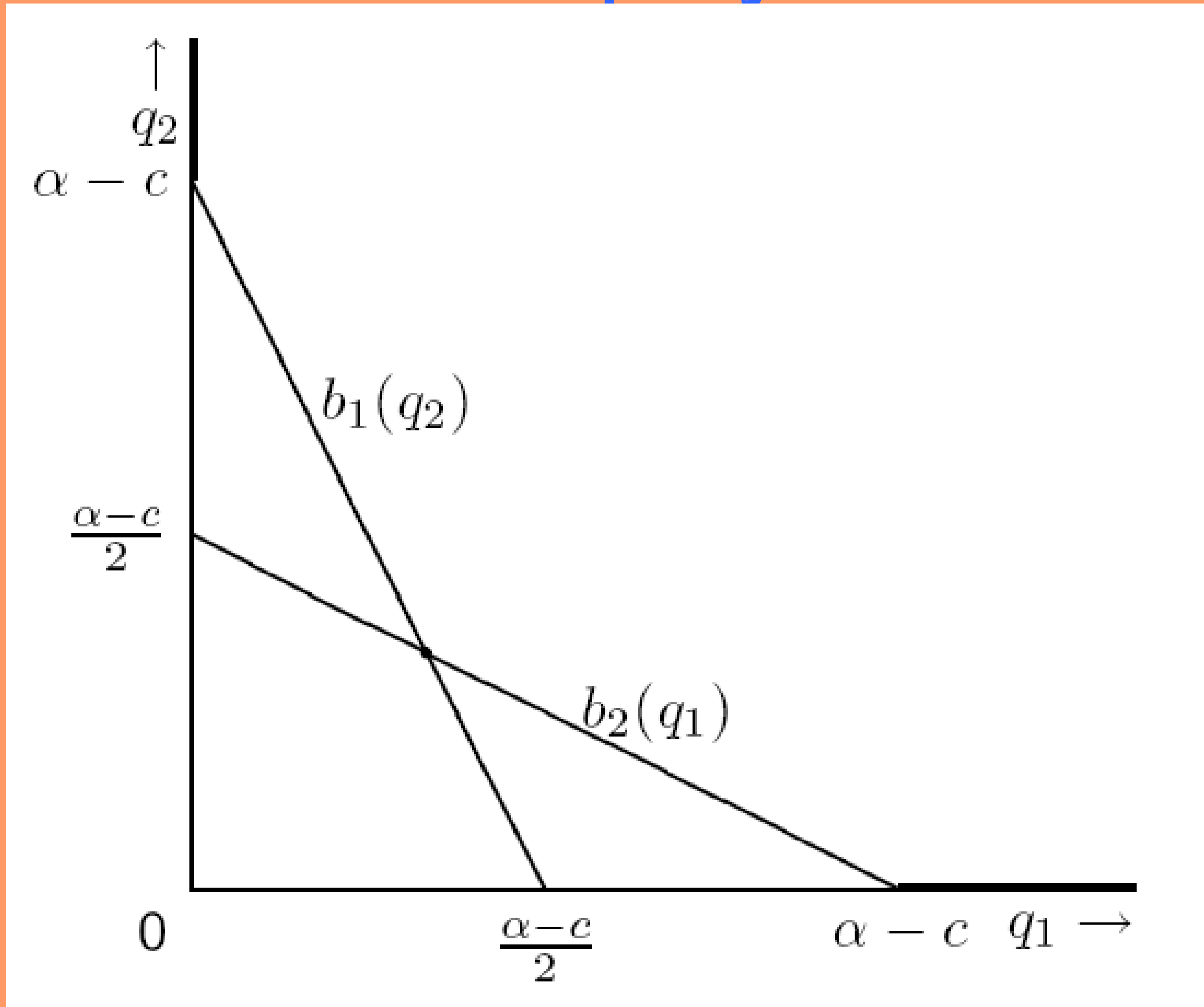
Duopoly

- Best response function is:

$$b_1(q_2) = \begin{cases} (\alpha - c - q_2)/2 & \text{if } q_2 \le \alpha - c \\ 0 & \text{if } q_2 > \alpha - c. \end{cases}$$

Same for firm 2: $b_2(q) = b_1(q)$ for all q.

Duopoly



Duopoly

Nash equilibrium:

Pair (q^*_1, q^*_2) of outputs such that each firm's action is a best response to the other firm's action

or

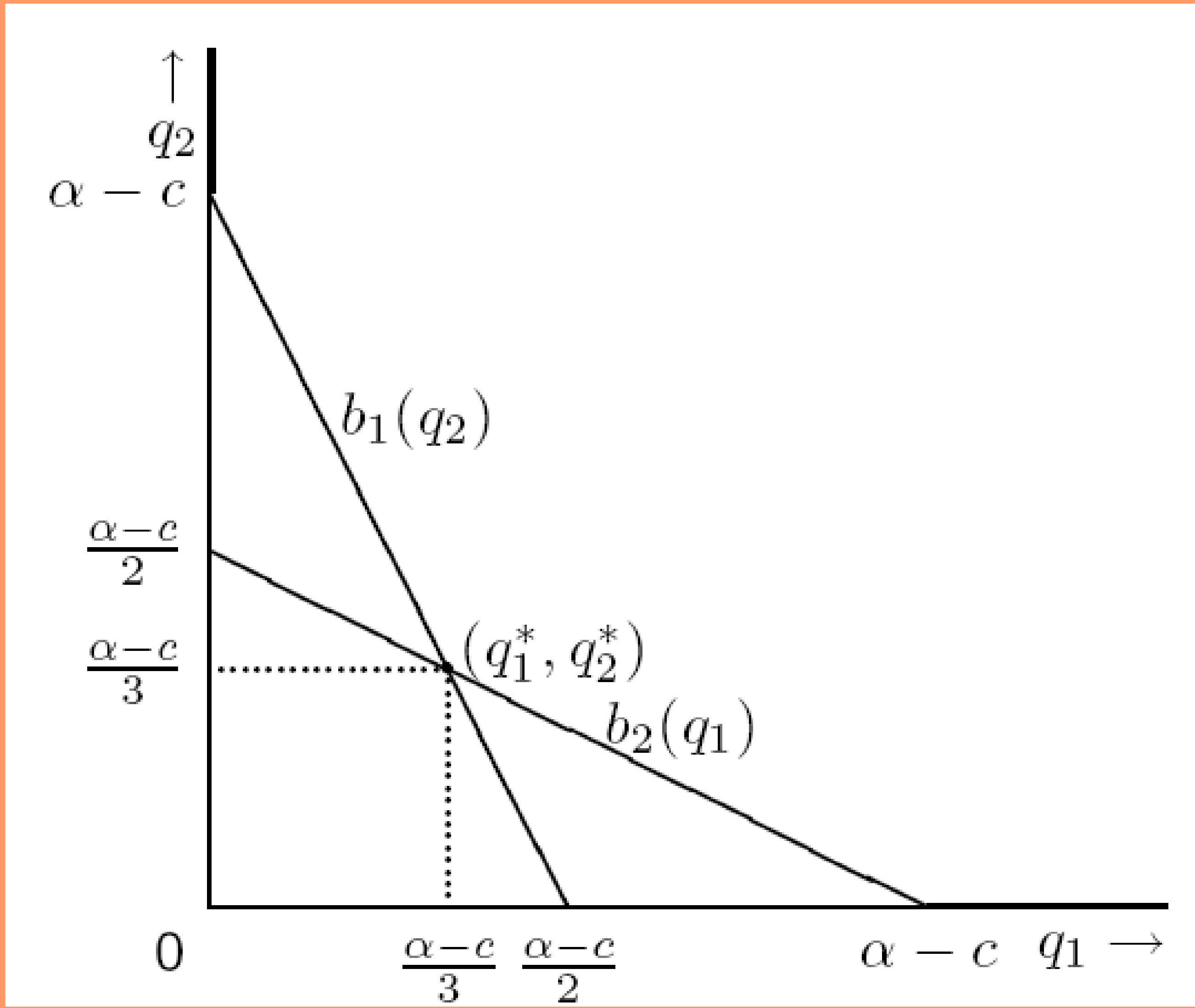
$$q^*_1 = b_1(q^*_2) \text{ and } q^*_2 = b_2(q^*_1)$$

Solution:

$$q_1 = (\alpha - c - q_2)/2 \text{ and } q_2 = (\alpha - c - q_1)/2$$

$$q^*_1 = q^*_2 = (\alpha - c)/3$$

Duopoly



↑
q_2
$\alpha - c$
$b_1(q_2)$
$\frac{\alpha - c}{2}$
$\frac{\alpha - c}{3}$
(q_1^*, q_2^*)
$b_2(q_1)$
0
$\frac{\alpha - c}{3}$
$\frac{\alpha - c}{2}$
$\alpha - c$
q_1 →

Duopoly

Conclusion:

- Game has unique Nash equilibrium:

$$(q^*_1, q^*_2) = ((\alpha - c)/3, (\alpha - c)/3)$$

- At equilibrium, each firm's profit is

$$\pi = ((\alpha - c)^2)/9$$

- Total output $2(\alpha - c)/3$ lies between monopoly output $(\alpha - c)/2$ and competitive output $\alpha - c$.

N-player Cournot Model (1)

- Demand function $P = \alpha - Q$
- Cost function $C_i(q_i) = cq_i$
- n firms, i = 1,2,…,n. So $Q = q_1 + q_2 + \ldots + q_n$.
- Solve using a representative firm i.

$\pi_i = q_i(\alpha - Q - c)$

$= q_i(\alpha - q_i - \sum q_{-i} - c)$

FOC: $(\alpha - 2q_i - \sum q_{-i} - c) = 0$

Solve for firm i's best response function:

$q_i = (\alpha - \sum q_{-i} - c)/2$

This gives n linear equations which we could solve simultaneously (for i = 1,2,….,n)

N-player Cournot Model (2)

- Instead, we will impose symmetry.
- It should be clear from the symmetric nature of the problem and the best response functions that the solution will be symmetric – i.e. $q_1 = q_2 = \ldots = q_n$. We could see for example that simultaneously solving the best response functions for q_1 and q_2 will imply that $q_1 = q_2$, and we could repeat this for all other pairs of equation.
- Thus, we can impose $q_i = q^*$ for all i on our representative firm best response function.
- This implies

$q^* = (\alpha - (n-1)q^* - c)/2$
$2q^* = \alpha - (n-1)q^* - c$
$(n+1)q^* = \alpha - c$
$q^* = (\alpha - c)/(n+1)$.So this is our unique Nash equilibrium.

N-player Cournot Model (3)

- To find prices and profits, we can substitute this solution for q^* into our original demand function and profit function.
- Industry output $Q = nq^* = n(\alpha - c)/(n+1)$
- Market price $P = \alpha - n(\alpha - c)/(n+1)$
  $= (\alpha + nc)/n+1$
- Firm profit
  $\pi_i = [(\alpha - c)/(n+1)][\alpha - c - n(\alpha - c)/(n+1)]$
  $= [(\alpha - c)/(n+1)]\,[(\alpha - c)/(n+1)]$
  $= [(\alpha - c)/(n+1)]^2$

N-player Cournot Model (4)

- This then is a generalization of our duopoly case, where n = 2. Substitute n = 2 into the solutions before, and see that we get our duopoly outcomes.
  $q_1 = q_2 = (\alpha - c)/3$
  $\pi_i = [(\alpha - c)/3]^2$
- Look also at how the model converges to perfectly competitive outcomes as $n \rightarrow \infty$.
  $q_i \rightarrow 0$
  $Q \rightarrow \alpha - c$
  $P \rightarrow c$
  $\pi_i \rightarrow 0$.