Economics of
Industrial Organization

Lecture 9: Dynamic Games, Stackelburg, Cournot and Bertrand

Entry Game

Entry Game: An incumbent faces the possibility of entry by a challenger. The challenger may enter or not. If it enters, the incumbent may either accommodate or fight.

Payoff:

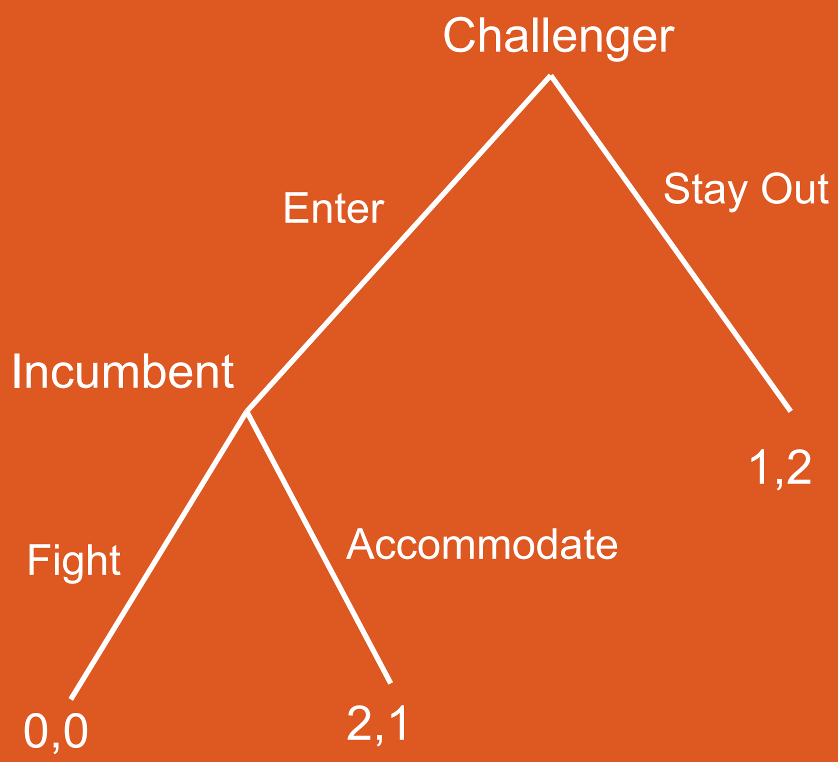
- Challenger: u_1(Enter, Accommodate)=2, u_1(Out)=1, u_1(Enter, Fight)=0
- Incumbent: u_2(Out)=2, u_2(E,A)=1, u_2(E,F)=0

Extensive Form Games with Perfect Information

Definition: an ***extensive form*** game consists of

- the players in the game
- when each player has to move
- what each player can do at each of her opportunities to move
- the payoff received by each player for each combination of moves that could be chosen by the players.

Extensive Form Game Tree



- Nash Equilibria?
- Solve via Backward Induction

Extensive Form Games with Perfect Information

Normal Form (Simultaneous Move).

Incumbent

Challenger		Accommodate	Fight
	Enter	2,1	0,0
	Stay Out	1,2	1,2

Extensive Form Games with Perfect Information

Normal Form (Simultaneous Move).

Incumbent

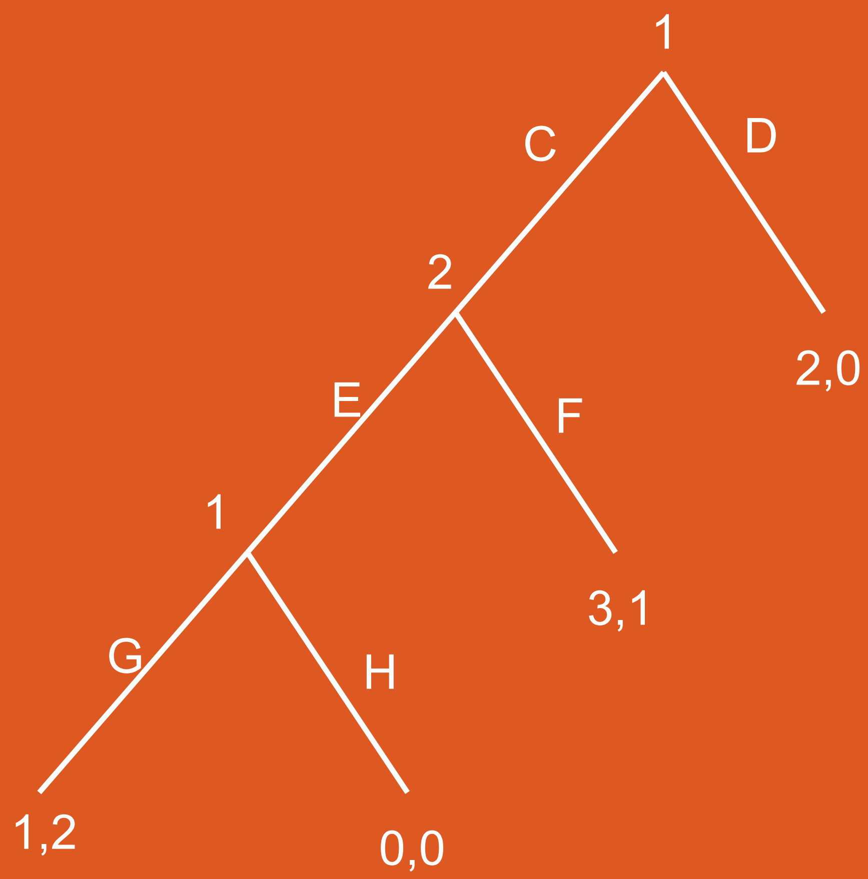
Challenger		Accommodate	Fight
	Enter	2,1	0,0
	Stay Out	1,2	1,2

So we have two pure strategy NE, (enter, accommodate) and (stay out, fight). How come in the extensive form we only have one equilibrium by backward induction ?

Extensive Form Games with Perfect Information

Definition: A ***strategy*** for a player is a complete plan of action for the player in every contingency in which the player might be called to act.

Extensive Form Games with Perfect Information



Strategies of Player 2: E, F
Strategies of Player 1:
CG,CH,DG,DH

"…a strategy of any player i specifies an action for EVERY history after which it is player i's turn to move, even for histories that, if that strategy is followed, do not occur."

Extensive Form Games with Perfect Information

Nash Equilibrium: each player must act optimally given the other players' strategies, i.e., play a best response to the others' strategies.

Problem: Optimality condition at the beginning of the game. Hence, some Nash equilibria of dynamic games involve non-credible threats.

Subgame and Subgame perfection

Definition: Consider a dynamic game of perfect information. A **subgame** of this game is a subset of the game starting at any node and continuing for the rest of the game.

A Nash equilibrium of is ***subgame perfect*** if it specifies Nash equilibrium strategies in every subgame. In other words, the players act optimally at every point during the game.

i.e., players play Nash Equilibrium strategies in EVERY subgame. This rules out non-credible threats.

Backward Induction

- A dynamic game of complete information can be solved by **backward induction**. Go to the end of the game, and work out what strategy last player to act should choose.
- Then, go back to the previous player's decision, and work out what strategy this previous player should choose, given that they now know the strategy that the final player will choose.
- Repeat this procedure iteratively back to the start of the game.

Stackelburg Cournot

2 firms, $i = 1,2$. $C(q_i) = cq_i$. $P(Q) = \alpha - Q$

Firm 1 chooses q_1. Firm 2 then observes this and chooses q_2.

Solve by backward induction:

Firm 2 solves $\max_{q2} (\alpha - q_1 - q_2 - c)q_2$

FOC: $\alpha - q_1 - 2q_2 - c = 0$

Solve for q_2 gives best response function.

BR_2: $q_2 = (\alpha - q_1 - c)/2$

Stackelburg Cournot 2

Now, solve for firm 1:

Max_{q1} $[\alpha - q_1 - (\alpha - q_1 - c)/2 - c]q_1$

Equivalently, this is Max_{q1} $q_1(\alpha - q_1 - c)/2$

FOC: $\alpha/2 - q_1 - c/2 = 0$

Solve for q_1; $q_1^* = (\alpha - c)/2$

Substitute into BR_2 to find q_2: $q_2^* = (\alpha - c)/4$

So: unique SPNE is $(q_1,q_2) = ((\alpha - c)/2, (\alpha - c)/4)$

Profits: Firm 1 gets $(\alpha - c)^2/8$.

Firm 2 gets $(\alpha - c)^2/16$

So game has first mover advantage.

Stackelburg Bertrand

Two firms, common marginal cost c.

$Q(P) = \alpha - \min[p_1, p_2]$

Firm 1 chooses p_1, then firm 2 observes this and chooses p_2.

Firm i captures entire market if $p_i < p_j$. Shares market equally if $p_i = p_j$.

Note that monopoly price (from solving $\max_p (\alpha - p)(p - c)$ is $p_M = (\alpha + c)/2$

Solve by backward induction.

Firm 2's best response:

If $c < p_1 < p_M$, then choose $p_2 = p_1 - \varepsilon$ (where ε is very small).

If $p_1 = c$, then any choose $p_2 \geq c$

If $p_2 < c$, then choose any $p_2 > p_1$.

If $p_1 > p_M$, then choose $p_2 = p_M$.

Firm 1's solution (knowing firm 2's best response): choose any $p_1 \geq c$ (and make zero profit).

This game has last mover advantage.

Stackelburg Bertrand with Differentiated Product

Suppose we have the differentiated product Bertrand model where $q_i = \alpha - p_i + bp_j$, but now we add dynamics.

Firm 1 chooses p_1 , then firm 2 observes this and chooses $p_{2.}$

Solve by backward induction: firm 2's best response function is the same as before,

BR_2: $p_2 = (\alpha + bp_1 + c)/2$

Now, player 1 solves:

$\text{Max}_{p1}\ [\alpha - p_1 + b(\alpha + bp_1 + c)/2][p_1 - c]$

Stackelburg Bertrand with Differentiated Product

This gives FOC:

$\alpha - 2p_1 + \alpha b/2 + b^2p_1 + bc/2 + c - b^2c = 0$

Solving for p_1 gives the equilibrium choice of p_1:

$p_1 = \{\alpha + c + b[\alpha + c(1\text{-}b)]/2\}/(2\text{-}b^2)$

Substituting into firm 2's best response function to find $p_{2.}$

$p_2 = (\alpha + c)/2 + (b/2)\{\alpha + c + b[\alpha + c(1\text{-}b)]/2\}/(2\text{-}b^2)$

This is the unique SPNE.

To find profits, substitute the prices into the original profit functions.

This shows that while prices are higher for both firms than in the simultaneous game, profits are higher for firm 2 than for firm 1; the game has last mover advantage, just like the undifferentiated version of Stackelburg Bertrand.