

5.6 Brittle fracture of polymer

Rectangular bar of width W under uniaxial tension (See Fig 5.17):

Brittle fracture

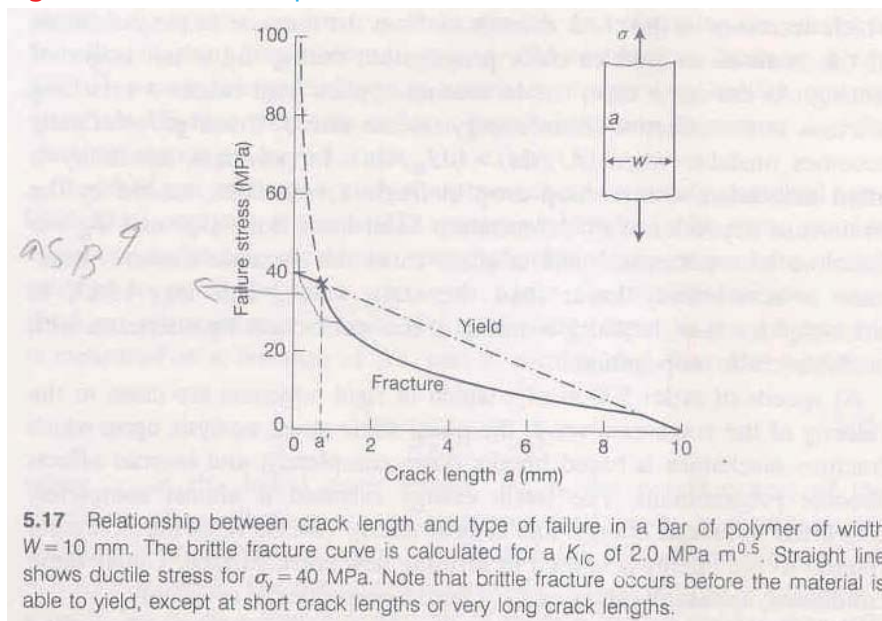
$$K_{IC} = Y\sigma_F(\pi a)^{\frac{1}{2}} \quad (5.30)$$

Y = Geometry factor (See 5.31)

Ductile fracture (yield)

$$\sigma_{\max} = \sigma_y(W - a)/W \quad (5.36)$$

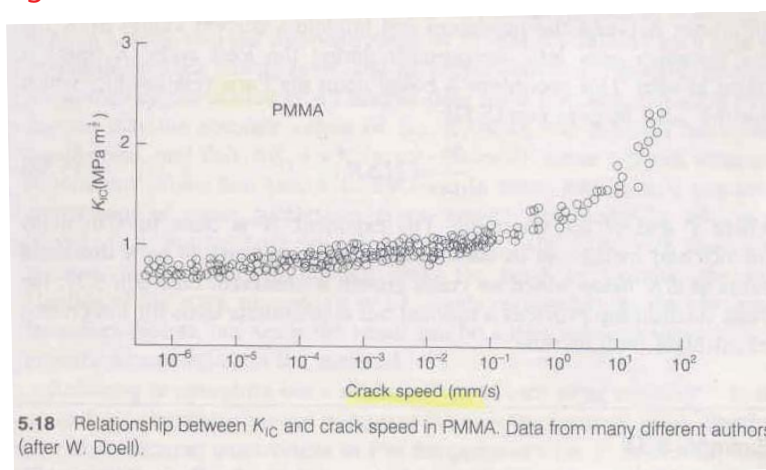
Fig 5.17 here (Pls replace)



For $a < a' \Rightarrow$ Yield (ductile fracture) (See the caption in the figure)

$a > a' \Rightarrow$ Brittle fracture, a' : critical crack length (see Fig 5.17)

Fig 5.18 here



As crack speed $\uparrow \Rightarrow K_{IC} \uparrow \Rightarrow a' \downarrow \Rightarrow$ Impact and other forms of rapid loading tend to cause brittle fracture. (The increase of K_{IC} is smaller than the increase in yield stress, so a' decreases)

Static fatigue : Slow crack growth under long term steady loading \rightarrow Threshold value for K_I below which no crack grows. Above this, a sub-critical crack growth is related to growth rate, da/dt as

$$\frac{da}{dt} = \beta K_I^m \quad (5.37)$$

$\beta, m = \text{constant}$

Time to failure can be calculated from (37) by integration or iteration. Final fracture occurs when $K_I \Rightarrow K_{IC}$

Dynamic fatigue

$$\frac{da}{dn} = \zeta (\Delta K)^N \quad (5.38)$$

$\zeta, N = \text{const}$ ($N \approx 4$ for plastic)

$n = \#$ of fatigue cycle

$\Delta K = K_{\max} - K_{\min}$ (Threshold ΔK for fracture)

\uparrow For tension-compression, $K(\text{compression})=0$

$$K = Y\bar{\sigma}(\pi a)^{\frac{1}{2}}$$

A threshold value exists for ΔK below which no crack growth is observed.

EX 5.11 [here](#)

Example 5.11

Rigid PVC is a polymer that conforms well to the Paris relationship.

Fatigue crack growth data at 20°C for PVC can be represented by the equation

$$\frac{da}{dn} = 0.035\Delta K^{2.4},$$

where da/dn is in μm per cycle and ΔK is in $\text{MPa m}^{0.5}$. A compact tension specimen has $B = 6 \text{ mm}$, $W = 50 \text{ mm}$, and $a = 20 \text{ mm}$. Calculate da/dn when the specimen is cycled between $F_{\text{max}} = 100 \text{ N}$ and $F_{\text{min}} = 50 \text{ N}$.

Solution

Using the expression for Y given in eqn 5.31, at $a/W = 0.4$,

$$Y = 16.70 - 41.88 + 59.18 - 36.72 + 9.23 = 6.51;$$

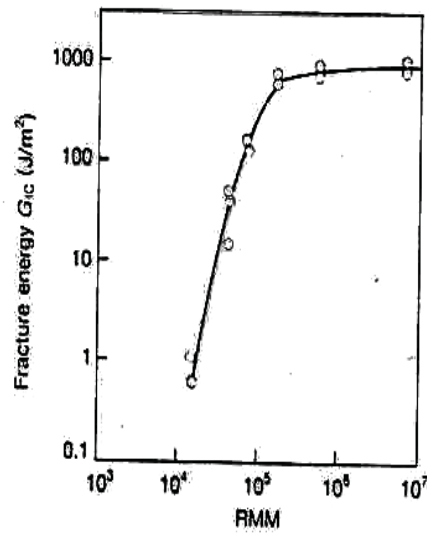
$$\Delta K = Y(\pi a)^{\frac{1}{2}}(\sigma_{\text{max}} - \sigma_{\text{min}}) = Y(\pi a)^{\frac{1}{2}} \frac{(F_{\text{max}} - F_{\text{min}})}{BW},$$

$$= 6.51 \times (\pi \times 0.02)^{\frac{1}{2}} \frac{(100 - 50)}{0.006 \times 0.05}$$

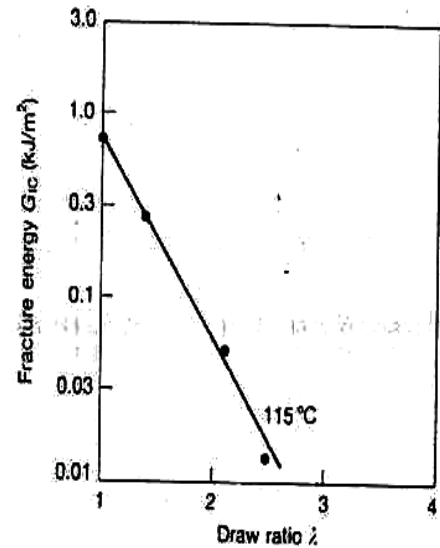
$$= 0.272 \text{ MPa m}^{0.5};$$

$$\frac{da}{dn} = 0.035\Delta K^{2.4}$$

$$= 1.54 \text{ nm per cycle.}$$



5.19 Relationship between G_{IC} and relative molar mass in PMMA (© D. T. Turner).



5.20 Fracture energy of polystyrene at 23°C after drawing above the glass transition at 10^{-2} s^{-1} , showing effect of molecular orientation. The crack plane is parallel to the draw direction (after L. J. Broutman and F. J. McGary).

5.7 Rubber toughening

Fig 5.21 here

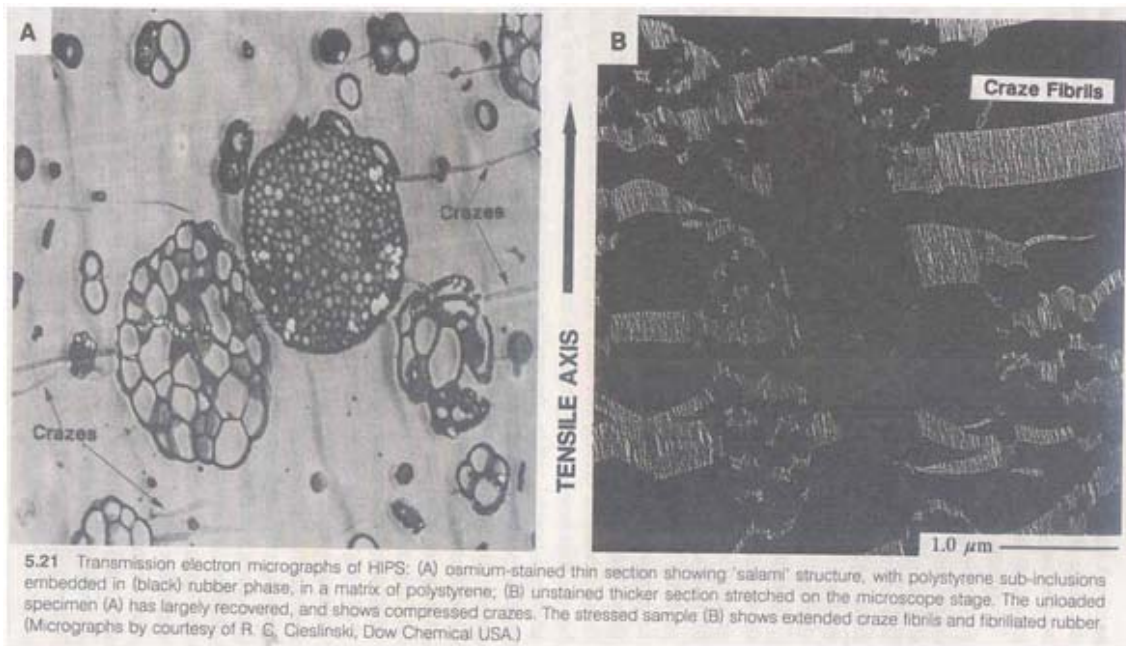


Fig 5.22 here

