Lecture Note for Solid Mechanics

- Analysis of Strain -

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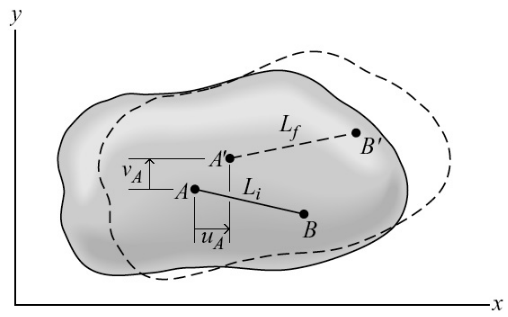
- Text book : Mechanics of Materials, 6th ed.,

  W.F. Riley, L.D. Sturges, and D.H. Morris, 2007.

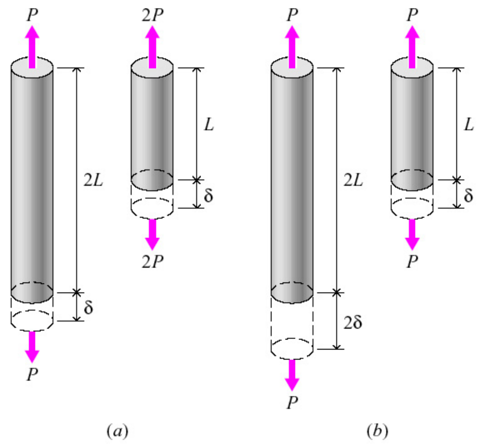
- Prerequisite : Knowledge of Statics, Basic Physics, Mathematics, etc.

Analysis of Strain

- Displacement
  - Movement of a point with respect to some convenient reference system of axes
  - Vector quantity



u_A : Scalar components of displacement in the x-direction
v_B : Scalar components of displacement in the y-direction



- Deformation (δ)
  - Change in dimension associated with relative displacements
  - Related to force or stress or to a change in temperature

- Strain
  - Quantity to measure the intensity of deformation (deformation per unit length)
  - Normal strain (ε) - change in size
  - Shear strain (γ) – change in shape (change in angle between two orthogonal lines)

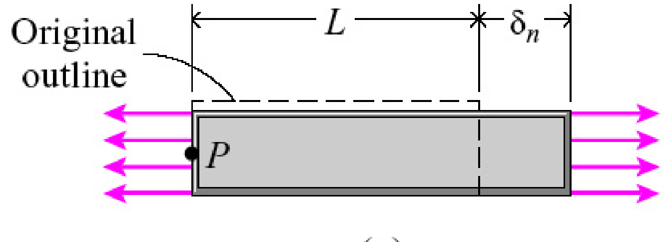
Analysis of Strain

- Average axial strain

$$\varepsilon_{avg} = \frac{\delta_n}{L}$$

- Axial strain at a point : non-uniform deformation

$$\varepsilon(P) = \lim_{\Delta L \to 0} \frac{\Delta \delta_n}{\Delta L} = \frac{d\delta_n}{dL}$$

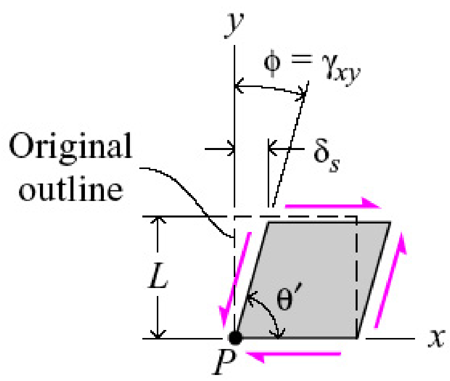


(a)

- Shear strain

$$\gamma_{avg} = \frac{\delta_s}{L} = \tan\phi \approx \phi \quad (if\ \delta_s / L < 0.001)$$

$$\gamma(P) = \lim_{\Delta L \to 0} \frac{\Delta \delta_s}{\Delta L} = \frac{d\delta_s}{dL} = \frac{\pi}{2} - \theta'$$



(b)

- Unit of strain (Dimensionless)
  - Normal strain : (in/in) or (μin/in)
  - Shear strain : rad or μrad

- Sign convention
  - tensile normal strain : elongation (+)
  - compressive normal strain : contraction (-)
  - shearing strain : (-) if angle increases and (+) if angle decreases

Analysis of Strain

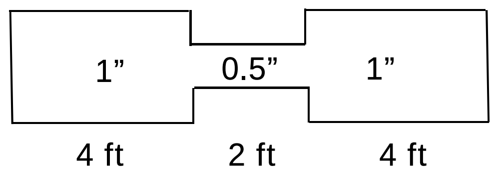
(Example 3-1) 1 in-diameter steel bar is 8 ft long. The diameter is reduced to 0.5 in in a 2 ft central portion of the bar. When an axial load is applied to the ends of the bar, the axial strain in the central portion of the bar is 960 μin/in and the total elongation of the bar is 0.04032 in. Determine (a) the elongation of the central portion of the bar, (b) the axial strain in the end portions of the bar.

(Sol)

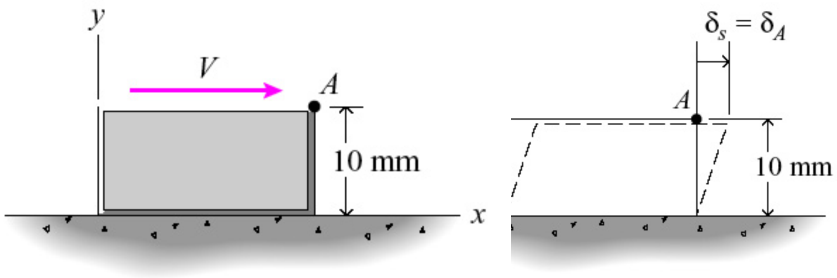
(a) $\delta_C = \varepsilon_{avg} L = 960\,(10^{-6})(2)(12) = 0.023\ in$

(b) $\delta_E = \delta_{total} - \delta_C = 0.04032 - 0.02304 = 0.01728\ in$

$$\varepsilon_E = \frac{\delta_E}{L} = \frac{0.01728}{6\,(12)} = 240\,(10^{-6}) = 240\ \mu in/in$$



(Example 3-2) Determine the horizontal movement of point A under shear force V causing average shear strain γ_{avg} of 1000 μm/m.



(a) (b)

(Sol)

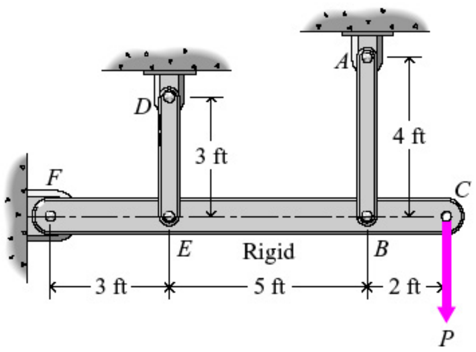
$$\gamma_{avg} = \frac{\delta_A}{L}$$

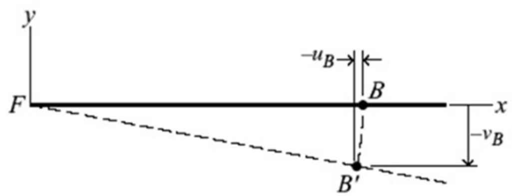
$$\delta_A = \gamma_{avg} L = 1000\,(10^{-6})(10)$$

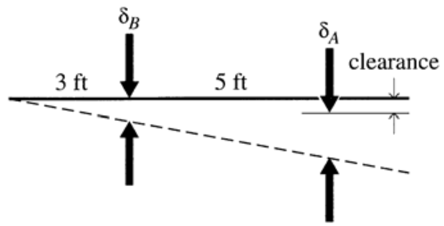
$$= 0.01\ mm = 10\ \mu m$$

Analysis of Strain

(Example 3-3) Rigid bar FEBC is supported by two steel rods. After load P is applied, the axial strain in the bar DE is 0.0006 in/in. Determine (a) axial strain in the bar AB and (b) axial strain in the bar AB if there is a 0.001 in clearance in the connection at B before the load is applied.







(Sol) $\delta_{AB} = \sqrt{u_B^2 + (L + v_B)^2} - L$

$$\delta_{AB}^2 + 2\delta_{AB}L + L^2 = L^2 + 2v_B L + u_B^2 + v_B^2$$

$$\delta_{AB} \cong v_B$$

B (0, L)
-> AB=L
B' ($-u_B$, $L+v_B$)
-> AB'=$[(-u_B)^2+(L+v_B)^2]^{1/2}$

$$\delta_{DE} = \varepsilon_{DE} L_{DE} = 0.0006(3)(12) = 0.0216\ in$$

(a) $\delta_{AB} = \frac{8}{3}\delta_{DE} = \frac{8}{3}(0.0216) = 0.0576\ in$

$$\varepsilon_{AB} = \frac{\delta_{AB}}{L_{AB}} = \frac{0.0576}{4(12)} = 0.0012\ in/in = 1200\ \mu in/in$$

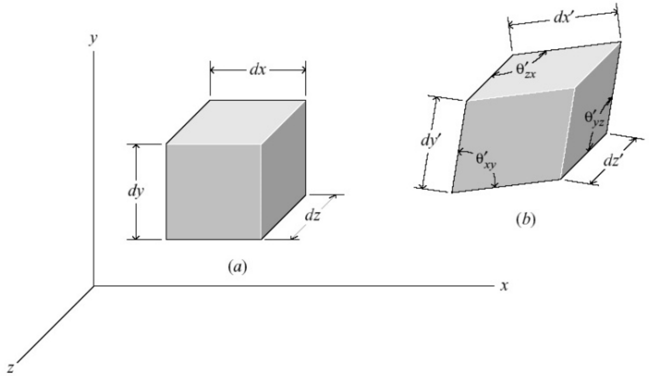
(b) $\delta_{AB} + clearance = \frac{8}{3}\delta_{DE}$

$$\delta_{AB} = \frac{8}{3}\delta_{DE} - clearance = \frac{8}{3}(0.0216) - 0.001 = 0.0566\ in$$

$$\varepsilon_{AB} = \frac{\delta_{AB}}{L_{AB}} = \frac{0.0566}{4(12)} = 0.001179\ in/in = 1179\ \mu in/in$$

Analysis of Strain

- State of strain at a point
  - Extension of the strain concept to biaxial loading
  - Necessary to experimental methods of stress evaluation



$$\varepsilon_x = \frac{dx' - dx}{dx} = \frac{d\delta_x}{dx} \qquad \gamma_{xy} = \frac{\pi}{2} - \theta'_{xy}$$

$$\varepsilon_y = \frac{dy' - dy}{dy} = \frac{d\delta_y}{dy} \qquad \gamma_{yz} = \frac{\pi}{2} - \theta'_{yz}$$

$$\varepsilon_z = \frac{dz' - dz}{dz} = \frac{d\delta_z}{dz} \qquad \gamma_{zx} = \frac{\pi}{2} - \theta'_{zx}$$

- Normal and shear strain components associated with a line oriented in n-direction and shear strain component associated with two orthogonal lines in n and t directions

$$\varepsilon_n = \frac{dn' - dn}{dn} = \frac{d\delta_n}{dn} \qquad \gamma_{nt} = \frac{\pi}{2} - \theta'_{nt}$$

- Change in length :

$$dx' = (1 + \varepsilon_x)dx \qquad \theta'_{xy} = \frac{\pi}{2} - \gamma_{xy}$$

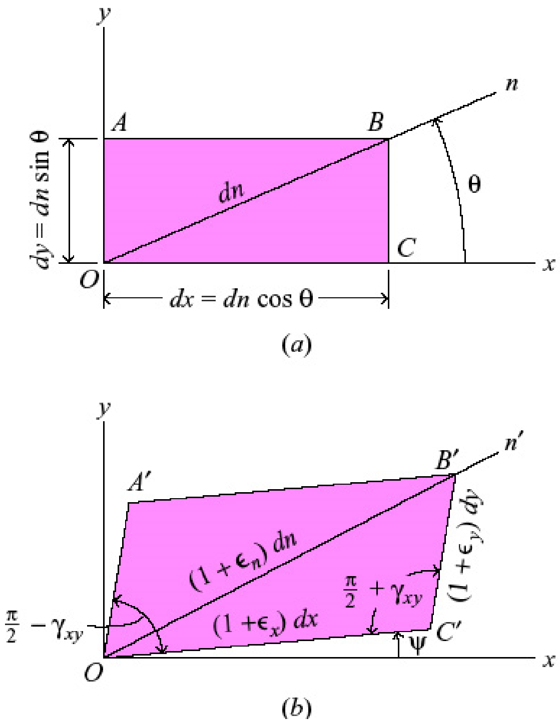
$$dy' = (1 + \varepsilon_y)dy \qquad \theta'_{yz} = \frac{\pi}{2} - \gamma_{yz}$$

$$dz' = (1 + \varepsilon_z)dz \qquad \theta'_{zx} = \frac{\pi}{2} - \gamma_{zx}$$

$$dn' = (1 + \varepsilon_n)dn \qquad \theta'_{nt} = \frac{\pi}{2} - \gamma_{nt}$$

Analysis of Strain

- Strain transformation equations for plane strain ($\varepsilon_z = \gamma_{yz} = \gamma_{zx} = 0$)



- Strains of the sides of the rectangle are known to $\varepsilon_x, \varepsilon_y, \gamma_{xy}$

  ⇨ Strain in n direction, ε_n , is to be determined

- Normal strain (ε_n)

$$(OB')^2 = (OC')^2 + (C'B')^2 - 2(OC')(C'B')\cos(\frac{\pi}{2} + \gamma_{xy})$$

$$[(1+\varepsilon_n)dn]^2 = [(1+\varepsilon_x)dx]^2 + [(1+\varepsilon_y)dy]^2 - 2[(1+\varepsilon_x)dx][(1+\varepsilon_y)dy][-\sin\gamma_{xy}]$$

$$[(1+\varepsilon_n)dn]^2 = (1+\varepsilon_x)^2(dn)^2(\cos^2\theta) + (1+\varepsilon_y)^2(dn)^2(\sin^2\theta) + 2(dn)^2(\sin\theta)(\cos\theta)(1+\varepsilon_x)(1+\varepsilon_y)(\sin\gamma_{xy})$$

Using $\varepsilon^2 << \varepsilon, \ \sin\gamma \cong \gamma, \ \gamma\varepsilon \approx 0$

$$1 + 2\varepsilon_n = (1+2\varepsilon_x)\cos^2\theta + (1+2\varepsilon_y)\sin^2\theta + 2\gamma_{xy}\sin\theta\cos\theta$$

$$1 + 2\varepsilon_n = (\cos^2\theta + \sin^2\theta) + 2(\varepsilon_x\cos^2\theta + \varepsilon_y\sin^2\theta + \gamma_{xy}\sin\theta\cos\theta)$$

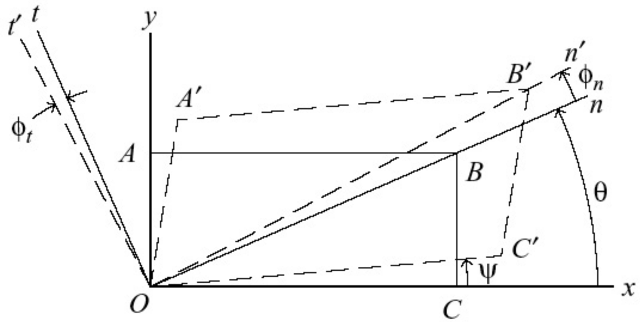
$$\varepsilon_n = \varepsilon_x\cos^2\theta + \varepsilon_y\sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$\varepsilon_n = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

(*refer to* $\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$)

Analysis of Strain

- shear strain (γ_{nt})



$$\frac{OB'}{\sin\angle OC'B'} = \frac{B'C'}{\sin\angle B'OC'}$$

$$B'C'\sin\angle OC'B' = OB'\sin\angle B'OC'$$

$$(1+\varepsilon_y)dy\sin\left(\frac{\pi}{2}+\gamma_{xy}\right) = (1+\varepsilon_n)dn\sin[\theta+(\phi_n-\psi)]$$

Using $\sin\left(\frac{\pi}{2}+\gamma_{xy}\right) = \cos\gamma_{xy} \cong 1$

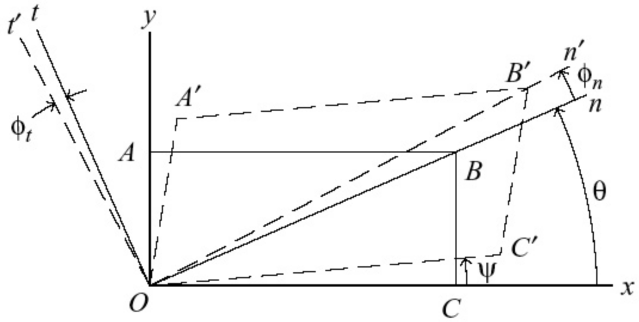
$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$$\sin[\theta+(\phi_n-\psi)] = \sin\theta\cos(\phi_n-\psi)+\cos\theta\sin(\phi_n-\psi)$$
$$\cong \sin\theta+(\phi_n-\psi)\cos\theta$$

$$(1+\varepsilon_y)dy \cong (1+\varepsilon_n)dn[\sin\theta+(\phi_n-\psi)\cos\theta]$$
$$(1+\varepsilon_y)dn\sin\theta \cong (1+\varepsilon_n)dn[\sin\theta+(\phi_n-\psi)\cos\theta]$$
$$(1+\varepsilon_y)\sin\theta \cong (1+\varepsilon_n)[\sin\theta+(\phi_n-\psi)\cos\theta]$$
$$\cong (1+\varepsilon_n)\sin\theta+(1+\varepsilon_n)(\phi_n-\psi)\cos\theta]$$
$$(\varepsilon_y-\varepsilon_n)\sin\theta \cong (\phi_n-\psi)\cos\theta+\varepsilon_n(\phi_n-\psi)\cos\theta$$
$$\cong (\phi_n-\psi)\cos\theta$$

Analysis of Strain

- shear strain (γ_{nt})



$$(\varepsilon_y - \varepsilon_n)\sin\theta \cong (\phi_n - \psi)\cos\theta$$

$$(\varepsilon_y - \varepsilon_x\cos^2\theta - \varepsilon_y\sin^2\theta - \gamma_{xy}\sin\theta\cos\theta)\sin\theta \cong (\phi_n - \psi)\cos\theta$$

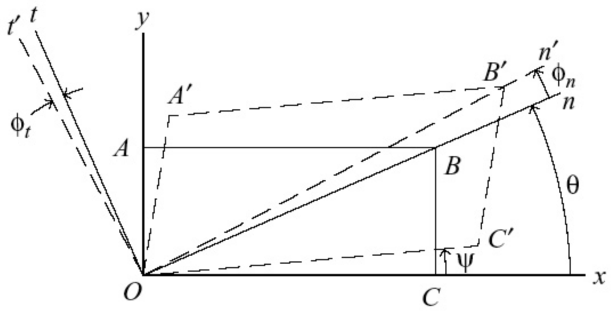
$$(\varepsilon_y - \varepsilon_x\cos^2\theta - \varepsilon_y\sin^2\theta - \gamma_{xy}\sin\theta\cos\theta)\frac{\sin\theta}{\cos\theta} \cong (\phi_n - \psi)$$

$$\varepsilon_n = \varepsilon_x\cos^2\theta + \varepsilon_y\sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$\begin{aligned}(\phi_n - \psi) &\cong \varepsilon_y\frac{\sin\theta}{\cos\theta} - \varepsilon_x\cos\theta\sin\theta - \varepsilon_y\frac{\sin^3\theta}{\cos\theta} - \gamma_{xy}\sin^2\theta \\ &\cong \varepsilon_y\frac{\sin\theta}{\cos\theta}(1-\sin^2\theta) - \varepsilon_x\cos\theta\sin\theta - \gamma_{xy}\sin^2\theta \\ &\cong \varepsilon_y\cos\theta\sin\theta - \varepsilon_x\cos\theta\sin\theta - \gamma_{xy}\sin^2\theta \\ &\cong -(\varepsilon_x - \varepsilon_y)\cos\theta\sin\theta - \gamma_{xy}\sin^2\theta\end{aligned}$$

$$\boxed{\phi_n = -(\varepsilon_x - \varepsilon_y)\sin\theta\cos\theta - \gamma_{xy}\sin^2\theta + \psi}$$

Analysis of Strain



$$\boxed{\phi_n = -(\varepsilon_x - \varepsilon_y)\sin\theta\cos\theta - \gamma_{xy}\sin^2\theta + \psi}$$

$$\begin{aligned}\phi_t &= \phi_n(\theta + \frac{\pi}{2})\\ &= -(\varepsilon_x - \varepsilon_y)\sin(\theta + \frac{\pi}{2})\cos(\theta + \frac{\pi}{2}) - \gamma_{xy}\sin^2(\theta + \frac{\pi}{2}) + \psi\\ &= (\varepsilon_x - \varepsilon_y)\sin\theta\cos\theta - \gamma_{xy}\cos^2\theta + \psi\end{aligned}$$

$$\sin(\theta + \frac{\pi}{2}) = \cos\theta$$

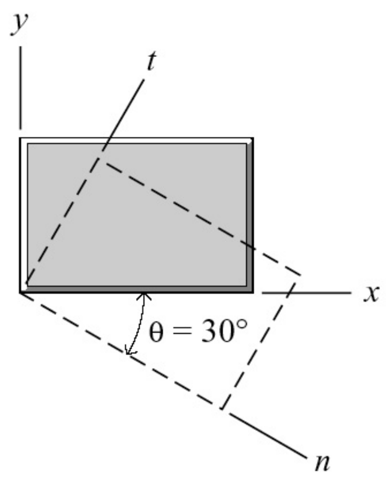
$$\cos(\theta + \frac{\pi}{2}) = -\sin\theta$$

$$\begin{aligned}\gamma_{nt} &= \phi_n - \phi_t\\ &= -2(\varepsilon_x - \varepsilon_y)\sin\theta\cos\theta + \gamma_{xy}(\cos^2\theta - \sin^2\theta)\\ &= -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta\end{aligned}$$

$$(\textit{refer to} \quad \tau_{nt} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta)$$

- Sign conventions
  - Tensile strains are positive and compressive strains are negative
  - Positive shear strains make the angle between two lines to decrease
  - Positive angle measures counterclockwise from x-axis
  - (x,y,z) and (n,t,z) coordinate systems are right-handed systems

(Example 3-4) Strain components at a point are ε_x=+800μ, ε_y=-1000μ, γ_{xy}=-600μ. Determine the strain components ε_n, ε_t, γ_{nt} , if xy- and nt-axes are oriented at followings.



(Sol)

$$\varepsilon_n = \varepsilon_x \cos^2\theta_n + \varepsilon_y \sin^2\theta_n + \gamma_{xy}\sin\theta_n\cos\theta_n$$
$$= (800)\cos^2(-30^\circ) + (-1000)\sin^2(-30^\circ) + (-600)\sin(-30^\circ)\cos(-30^\circ)$$
$$= 610\,\mu$$

$$\varepsilon_t = \varepsilon_x \cos^2\theta_t + \varepsilon_y \sin^2\theta_t + \gamma_{xy}\sin\theta_t\cos\theta_t$$
$$= (800)\cos^2(60^\circ) + (-1000)\sin^2(60^\circ) + (-600)\sin(60^\circ)\cos(60^\circ)$$
$$= -810\,\mu$$

$$\gamma_{nt} = -2(\varepsilon_x - \varepsilon_y)\sin\theta_n\cos\theta_n + \gamma_{xy}(\cos^2\theta_n - \sin^2\theta_n)$$
$$= -2[800 - (-1000)]\sin(-30^\circ)\cos(-30^\circ) + (-600)[\cos^2(-30^\circ) - \sin^2(-30^\circ)]$$
$$= 1259\,\mu$$

A Line element in the n-direction has increased in length, a line element in the t-direction has decreased in length, and the angle at the origin of nt-axes is less than 90°

Analysis of Strain

- Principal strains and maximum shear strain

  - Similarity between strain relation for plane strain and stress relation for plane stress indicates that all equations can be applied by substituting for ε_x for σ_x , ε_y for σ_y , and $\gamma_{xy}/2$ for τ_{xy}.

$$\sigma_x \Rightarrow \varepsilon_x$$
$$\sigma_y \Rightarrow \varepsilon_y$$
$$\tau_{xy} \Rightarrow \frac{\gamma_{xy}}{2}$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$
$$\varepsilon_{p1}, \varepsilon_{p2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$
$$\gamma_p = 2\sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$
$$\sigma_{p1}, \sigma_{p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$\tau_p = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

  - For the state of plane strain

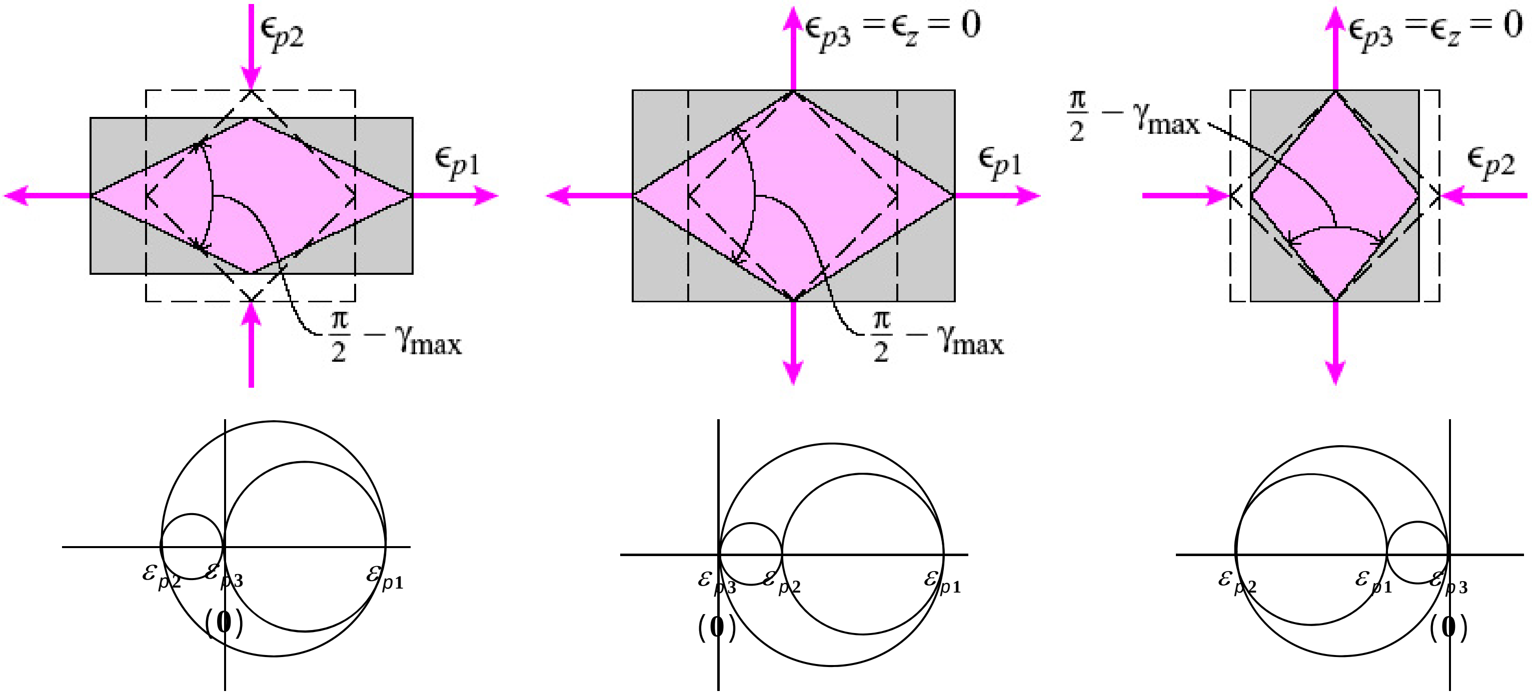
$$\varepsilon_{p1}, \varepsilon_{p2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$
$$\varepsilon_{p3} = \varepsilon_z = 0$$

Maximum in-plane shear strain is the difference between the in-plane strains, but this may not be the maximum shear strain at the point

- Principal strains and maximum shear strain

  - Maximum shear strain at the point is determined by depending on the relative magnitudes and the signs of the principal strains

$$\boxed{(\varepsilon_{p1}-\varepsilon_{p2})\ \ or\ \ (\varepsilon_{p1}-0)\ \ or\ \ (0-\varepsilon_{p2})}$$



(Example 3-5) Strain components at a point are ε_x=+1200μ, ε_y =-600μ, γ_{xy}=+900μ. Determine the principal strains and maximum shear strain at the point. Show principal strain deformations and maximum shear strain distortion on a sketch.

(Sol) The in-plane principal strains are

$$\varepsilon_{p1}, \varepsilon_{p2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$(\varepsilon_{p1}, \varepsilon_{p2})(10^6) = \frac{1200 + (-600)}{2} \pm \sqrt{\left(\frac{1200 - (-600)}{2}\right)^2 + \left(\frac{900}{2}\right)^2}$$

$$= 300 \pm 1006.2$$

$$\varepsilon_{p1} = (300 + 1006.2)(10^{-6}) = 1306\ \mu$$

$$\varepsilon_{p2} = (300 - 1006.2)(10^{-6}) = -706.2\ \mu$$

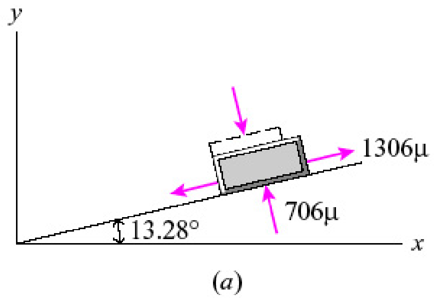
$$\varepsilon_{p3} = \varepsilon_z = 0$$

The principal directions are

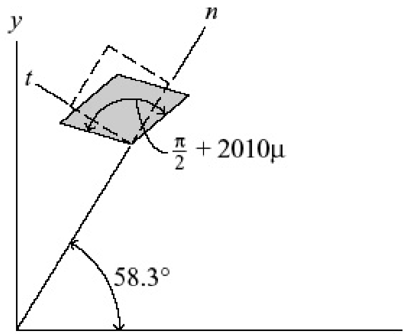
$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{900}{1200 - (-600)}$$

$$2\theta_p = 26.57^o \quad and \quad 206.57^o$$

$$\theta_p = 13.28^o \quad and \quad 103.28^o$$



(a)



The in-plane principal strains is found by substituting the values of the principal strain directions into the strain transformation equation

$$\begin{aligned}\varepsilon_n &= \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta \\ &= (1200\,\mu)\varepsilon_x \cos^2(13.28) + (-600\,\mu)\sin^2(13.28) + (900\,\mu)\sin(13.28)\cos(13.28) \\ &= 1306\,\mu = \varepsilon_{p1}\end{aligned}$$

The principal strains are perpendicular and there is no shear strain

The maximum in-plane shear strain is given as

$$\gamma_p = 2\sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_p(10^6) = 2\sqrt{\left(\frac{1200 - (-600)}{2}\right)^2 + \left(\frac{900}{2}\right)^2} \quad \Rightarrow \quad \gamma_{\max} = \gamma_p = 2010\,\mu$$

$$\gamma_p = 2010(10^{-6}) = 2010\,\mu$$

The angle associated with maximum shear strain is found as

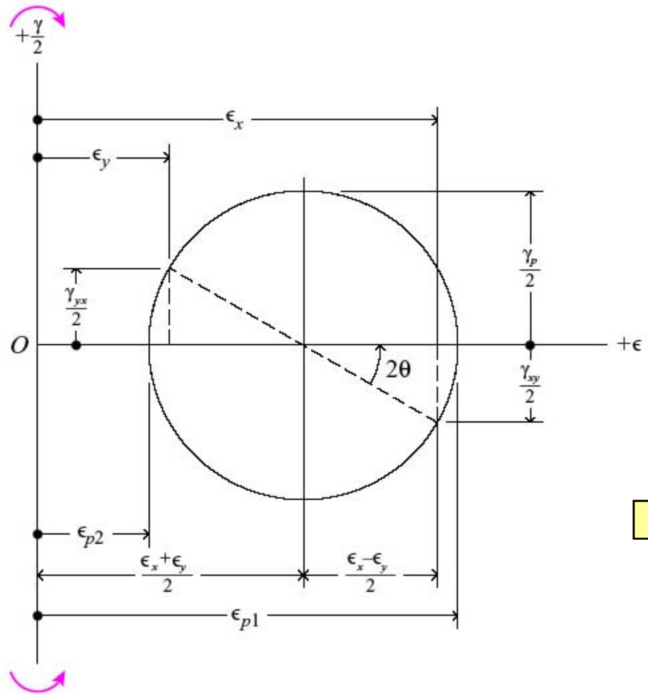
$$\tan 2\theta_\gamma = \frac{-(\varepsilon_x - \varepsilon_y)}{\gamma_{xy}} = \frac{-(1200 - (-600))}{900} \quad \Rightarrow \quad \theta_\gamma = -31.7^o \text{ and } 58.3^o$$

Substituting the values of the principal strain directions of -31.7 or 58.3 into the strain transformation equation gives

$$\begin{aligned}\gamma_{nt} &= -2(\varepsilon_x - \varepsilon_y)\sin\theta\cos\theta + \gamma_{xy}(\cos^2\theta - \sin^2\theta) \\ &= -2[(1200) - (-600)]\sin(58.3)\cos(58.3) + \gamma_{xy}[\cos^2(58.3) - \sin^2(58.3)] \\ &= -2010\,\mu = \gamma_{\max}\end{aligned}$$

Analysis of Strain

- Mohr circle for plane strain ($\varepsilon_z = \gamma_{yz} = \gamma_{zx} = 0$)



$${}_n - \frac{\varepsilon_x + \varepsilon_y}{2} = \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\Leftarrow \quad \varepsilon_n = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\frac{\gamma_{nt}}{2} = -\frac{(\varepsilon_x - \varepsilon_y)}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\Leftarrow \quad \gamma_{nt} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$\left(\varepsilon_n - \frac{\varepsilon_x + \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{nt}}{2}\right)^2 = \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2$$

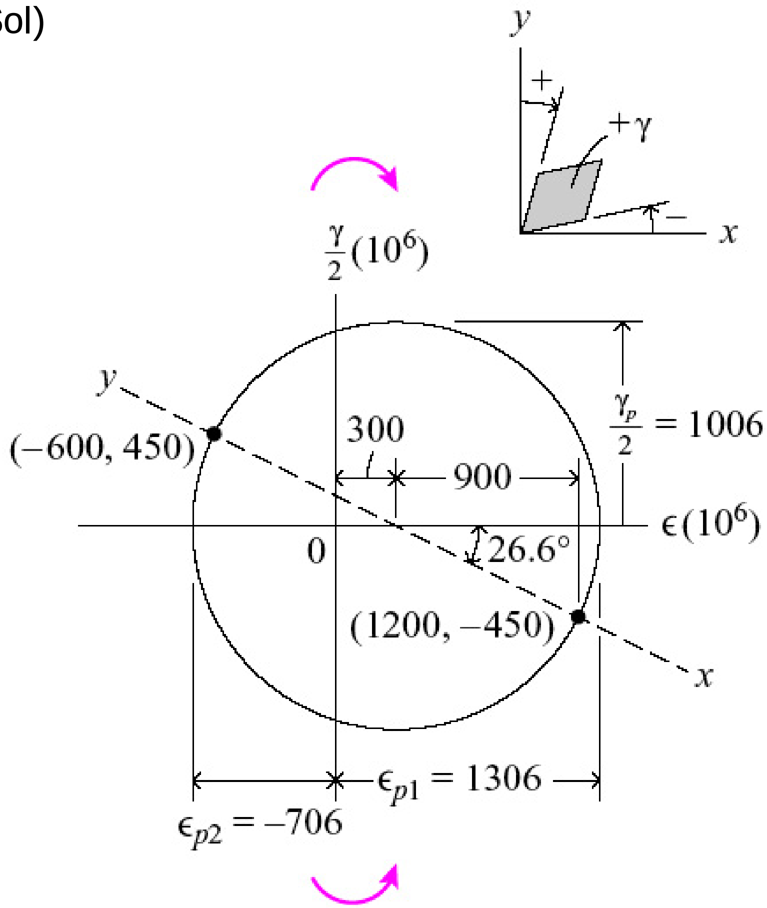
$$C\left(\frac{\varepsilon_x + \varepsilon_y}{2}, 0\right) \quad and \quad R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

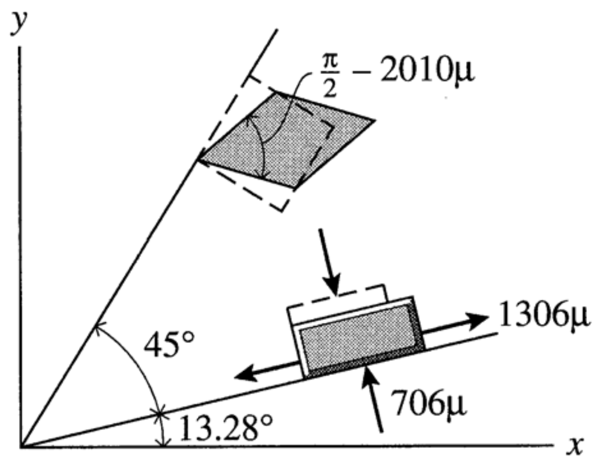
For a positive shear strain, the edge of the element parallel to the x-axis tends to rotate counterclockwise and the edge parallel to the y-axis tends to rotate clockwise.

Analysis of Strain

(Example 3-5) Strain components at a point are $\varepsilon_x = +1200\mu$, $\varepsilon_y = -600\mu$, $\gamma_{xy} = +900\mu$.
Determine the principal strains and maximum shear strain at the point.

(Sol)

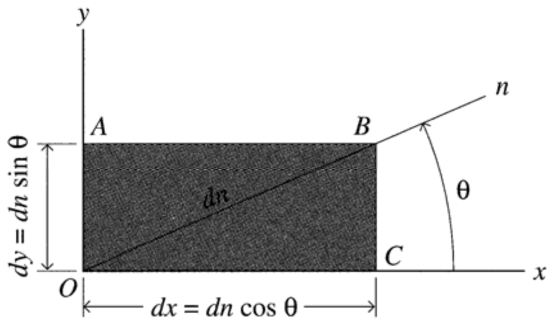




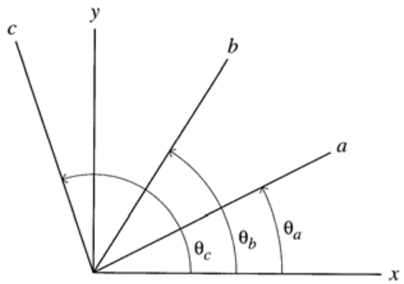
Analysis of Strain

- Strain measurement and rosette analysis

  ✓ In experimental work involving strain measurement, the strains are measured on a free surface of a member where a state of stress exist.



$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$$



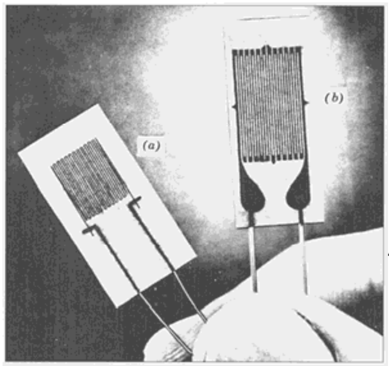
$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy} \sin\theta_a \cos\theta_a$$
$$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy} \sin\theta_b \cos\theta_b$$
$$\varepsilon_c = \varepsilon_x \cos^2\theta_c + \varepsilon_y \sin^2\theta_c + \gamma_{xy} \sin\theta_c \cos\theta_c$$

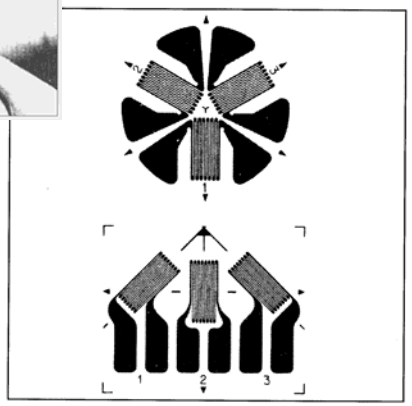
✓ If the measured values of the strains (ε_a , ε_b , ε_c) and the orientations (θ_a , θ_b , θ_c), the values of the strains (ε_x , ε_y , γ_{xy}) can be determined from the three equations

✓ Once the ε_x , ε_y , γ_{xy} have been determined, the in-plane principal strains (ε_{p1} , ε_{p2}), their orientations (θ_p), and the maximum in-plane strains (τ_p) at the point can be obtained

Analysis of Strain

- Strain gages and rosettes
  - ✓ Strain gages provide accurate measurements of the strain
  - ✓ As the material is strained, the wires are lengthened or shortened; this changes the electrical resistance of the gage
  - ✓ Strain gages are sensitive only to normal strains and respond to shear strains
  - ✓ Shear strains are often obtained by measuring normal strains in two or three different directions





Since the plane stress offers no restraint to out-of-plane deformation, ε_z develops in addition to in-plane strains (ε_x , ε_y , γ_{xy})

$$\varepsilon_z = \varepsilon_{p3} = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y)$$

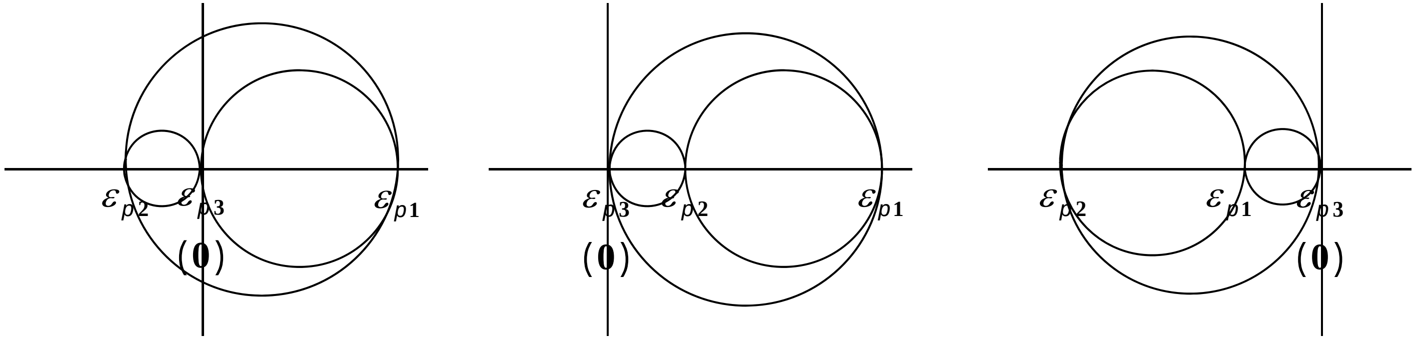
$$where \;\; \nu \; is \; Poisson's \; ratio)$$

This out-of-plane principal strain is important because the maximum shear strain may be (ε_{p1}- ε_{p2}), (ε_{p1}- ε_{p3}), or (ε_{p3}- ε_{p2}), depending on the relative magnitudes and the signs of the principal strains at the point

Analysis of Strain

- Maximum shear strain for plane strain

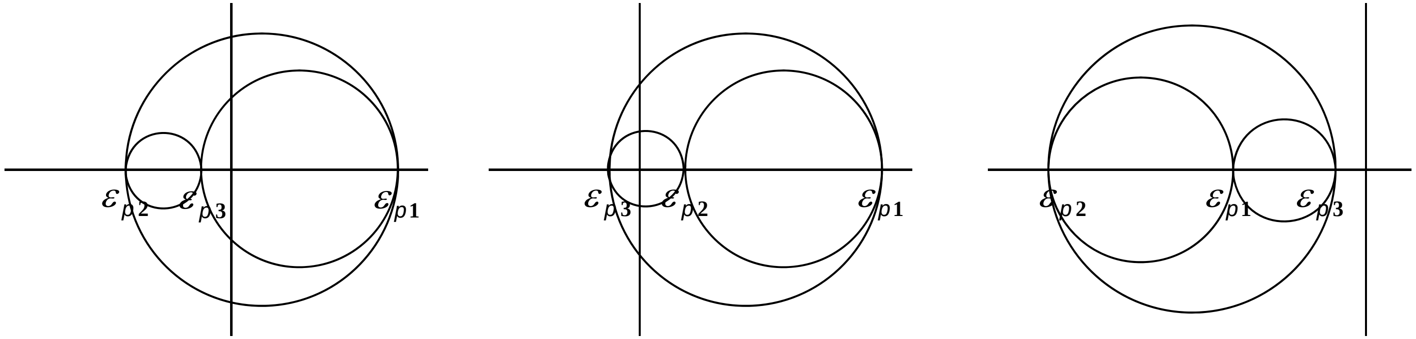
$(\varepsilon_{p1} - \varepsilon_{p2})$ $(\varepsilon_{p1} - 0)$ $(0 - \varepsilon_{p2})$



- Maximum shear strain for plane stress

$$\varepsilon_z = \varepsilon_{p3} = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y)$$

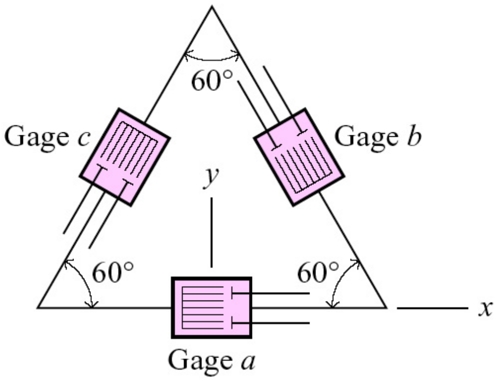
$(\varepsilon_{p1} - \varepsilon_{p2})$ $(\varepsilon_{p1} - \varepsilon_{p3})$ $(\varepsilon_{p3} - \varepsilon_{p2})$



Analysis of Strain

(Example 3-7) The following strains are measured as $\varepsilon_a=+1000\mu$, $\varepsilon_b=750\mu$, $\varepsilon_c=-650\mu$. Determine the principal strains and maximum shear strain. Show the directions of the in-plane principal strains on a sketch. Assume that Poisson's ration is 1/3.

(Sol)



$$\varepsilon_a = \varepsilon_o = +1000\ \mu = \varepsilon_x \cos^2(0) + \varepsilon_y \sin^2(0) + \gamma_{xy}\sin(0)\cos(0)$$

$$\varepsilon_c = \varepsilon_{60} = -650\ \mu = \varepsilon_x \cos^2(60) + \varepsilon_y \sin^2(60) + \gamma_{xy}\sin(60)\cos(60)$$

$$\varepsilon_b = \varepsilon_{120} = 750\ \mu = \varepsilon_x \cos^2(120) + \varepsilon_y \sin^2(120) + \gamma_{xy}\sin(120)\cos(120)$$

$$\Rightarrow \varepsilon_x = +1000\ \mu \quad \varepsilon_y = -266.7\ \mu, \quad \gamma_{xy} = -1616.7\ \mu$$

$$\varepsilon_{p1}, \varepsilon_{p2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$(\varepsilon_{p1}, \varepsilon_{p2})(10^6) = \frac{1000 + (-266.7)}{2} \pm \sqrt{\left(\frac{1000 - (-266.7)}{2}\right)^2 + \left(\frac{-1616.6}{2}\right)^2}$$

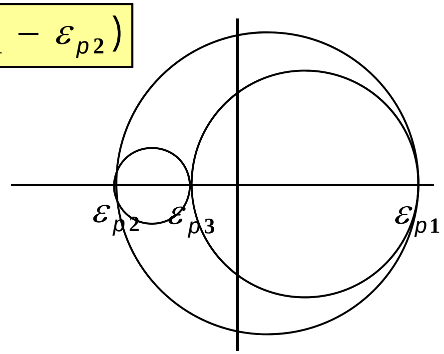
$$\varepsilon_{p1} = 1394\ \mu$$

$$\varepsilon_{p2} = -660\ \mu$$

$$\varepsilon_z = \varepsilon_{p3} = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = -\frac{1/3}{1-(1/3)}(1000\ \mu - 266.7\ \mu) = -367\ \mu$$

Analysis of Strain

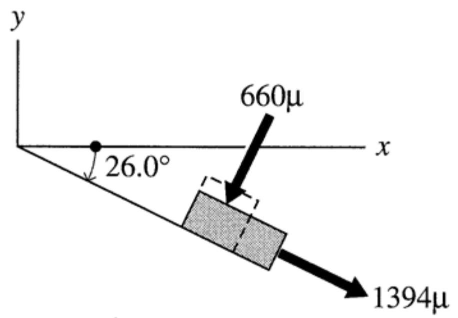
$$\varepsilon_{p1} = 1394\ \mu$$
$$\varepsilon_{p2} = -660\ \mu$$
$$\varepsilon_{p3} = -367\ \mu$$



$$\gamma_{\max} = \gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 1393.6\ \mu - 660.2\ \mu = 2050\ \mu$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-1616.6}{1000 - (-266.7)} = 1.2763$$

$$2\theta_p = -51.92^o \qquad \theta_p = -26.0^o$$



(Homework)

(3-12), (3-20), (3-33), (3-53), (3-64)