Lecture Note for Solid Mechanics

- Essential Concept and Statics Review -

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- **Text book : Mechanics of Materials, 6th ed.,**

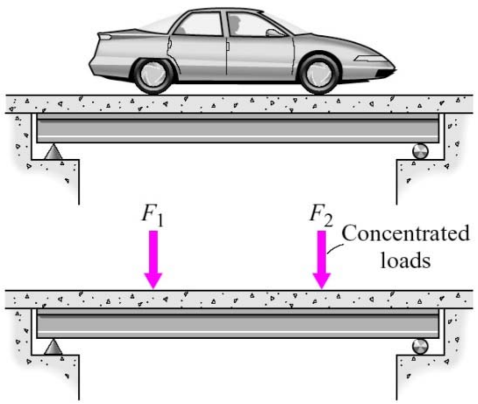
  W.F. Riley, L.D. Sturges, and D.H. Morris, 2007.

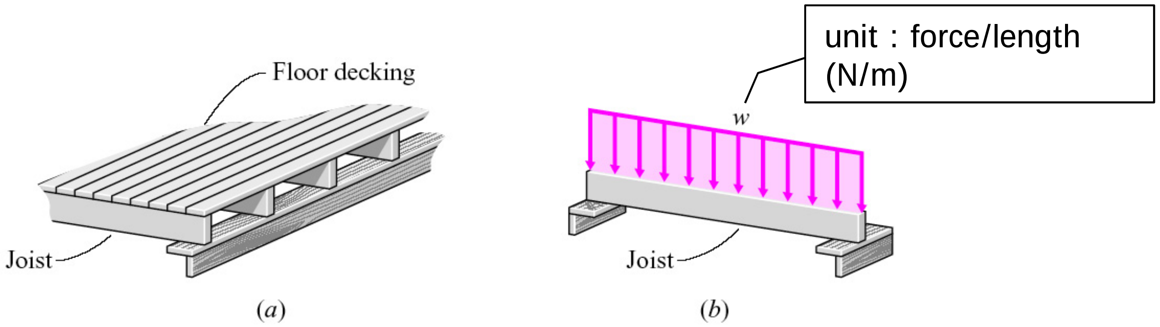
- **Prerequisite : Knowledge of Statics, Basic Physics, Mathematics, etc.**

Essential Concepts and Statics Review

- Objective of mechanics of materials
  - Relationships between loads applied to a nonrigid body and internal forces and deformations induced in the body
    - Determination of load carrying capability of structural members
    - Development of mathematical and experimental methods of analysis
- Subject matter of this course
  - Given a certain function to perform, of what materials should be constructed, and what sizes and proportions of various elements? (Designer's task)
  - Given the completed design, does it perform the function economically and without excessive deformation? (Checkers' problem)
  - Given a completed structure, what is its actual load carrying capability? (Rating problem)

- Classification of forces
  - Forces : action of one body on another
    (equal magnitude, opposite direction pairs)
- Direct and indirect contact between two bodies
  - Surface forces (direct contact)
    - Concentrated force and distributed force (uniform or non-uniform)
  - Body forces (indirect contact)
    - Gravitational attraction force and electromagnetic force

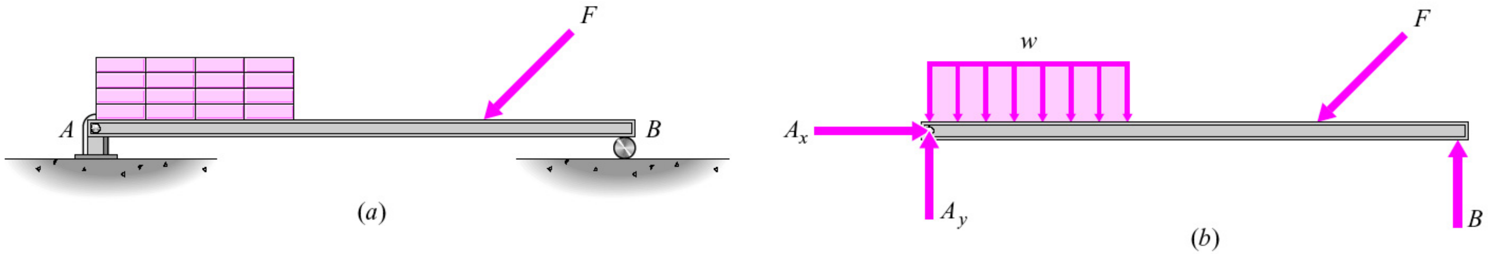




Essential Concepts and Statics Review

- Applied forces and reactions
  - Applied forces
    - Concentrated and/or distributed forces designed to carry
  - Reactions
    - Forces at supports to prevent movements

- External forces and internal forces



- Static forces and dynamic forces
  - Static forces
    - Forces which do not vary with time
  - Dynamic forces
    - Forces which vary with time such as impact load and repeated load

- Equilibrium of a rigid body
  - Rigid body
    - Body that does not deform under the action of applied forces
  - Equilibrium condition
    - Resultant of the system of forces acting on the body is zero
    - $\sum F = \mathbf{0}$ *and* $\sum M_o = \mathbf{0}$ (o : *on or off the body*)

$$\sum \vec{F} = 0 \quad and \quad \sum \vec{M}_o = 0$$

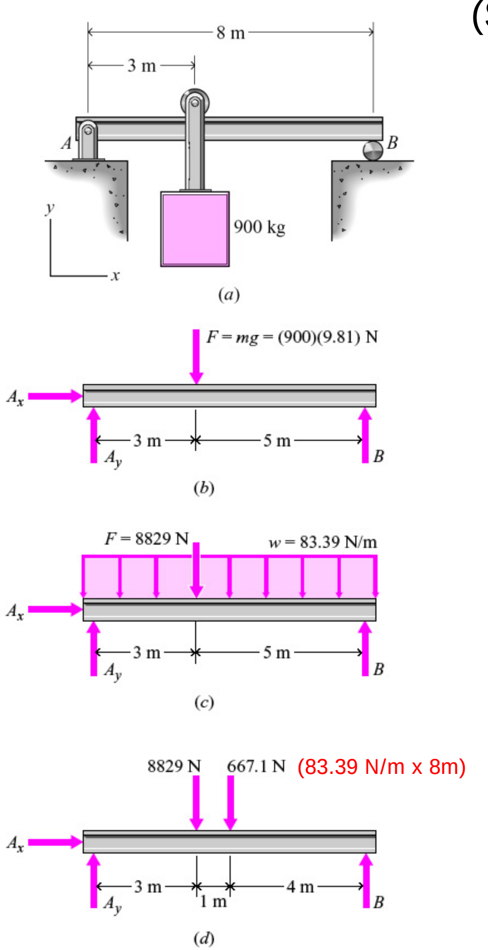
$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$
$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$
(3-Dim)

$$\sum F_x = \mathbf{0} \quad \sum F_y = \mathbf{0} \quad \sum M_z = \mathbf{0}$$
(2-Dim)

$$\sum F_x = \mathbf{0} \quad \sum F_y = \mathbf{0}$$
(2-Dim)

- Free body diagram
  - Free body : Body of interest separated from all other interacting bodies
  - Free body diagram : Free body with all external forces
  - Use for determining the unknown forces acting on rigid bodies

(Example) 900kg mass is supported by a roller that can move along the beam.
Determine (a) reactions at A and B neglecting the mass of beam and (b) reactions at A and B if the mass of the beam is 8.5kg/m.



(Sol)

(a) Using $\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_z = 0$

$$\sum F_x = 0 \Rightarrow A_x = 0$$
$$\sum F_y = 0 \Rightarrow A_y + B - 8829 = 0$$
$$\sum M_A = 0 \Rightarrow B(8) - 8829(3) = 0$$

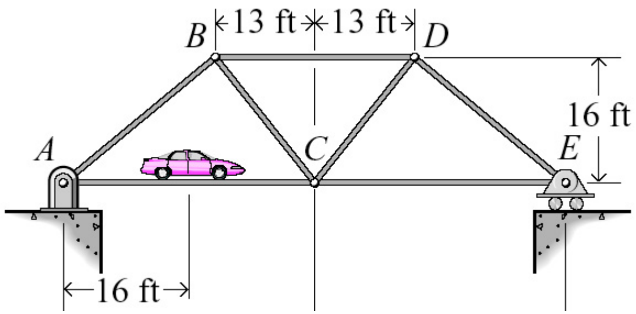
$$\Rightarrow \quad A_x = 0 \quad A_y = 5520\ N \quad B = 3310\ N$$

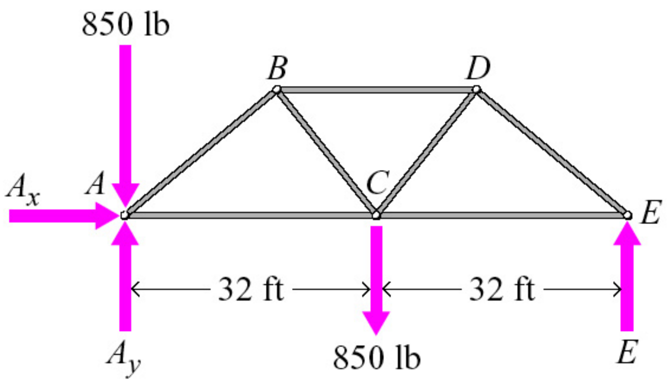
(b) Using $\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_z = 0$

$$\sum F_x = 0 \Rightarrow A_x = 0$$
$$\sum F_y = 0 \Rightarrow A_y + B - 8829 - 8(83.39) = 0$$
$$\sum M_A = 0 \Rightarrow B(8) - 8829(3) - 8(83.39)(4) = 0$$

$$\Rightarrow \quad A_x = 0 \quad A_y = 5850\ N \quad B = 3640\ N$$

(Example) Calculate the support reactions and the forces in the BD, DE, and CE. Assume that the truss supports one side of bridge and floor beams carry vehicle loads (W=3400 lb) to the truss joints.





(Sol)

Since the truss is a two-force member, it is subjected to either tension or compression.

Half of the car's weight (W=1700 lb) is carried by the truss and the other half is carried by the truss on the other side of the bridge.

Using $\sum \vec{F} = 0 \;\; and \;\; \sum \vec{M} = 0$

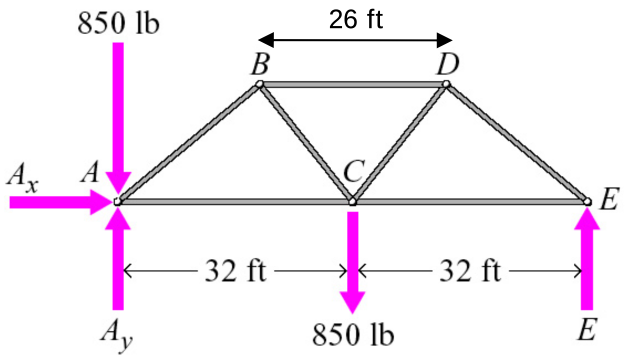
$$\sum F_x = 0 \;\Rightarrow\; A_x = 0$$

$$\sum F_y = 0 \;\Rightarrow\; A_y + E - 850 - 850 = 0$$

$$\sum M_A = 0 \;\Rightarrow\; E(64) - 850(32) = 0$$

$$\Rightarrow \quad A_x = 0 \quad A_y = 1275\ lb \quad E = 425\ lb$$

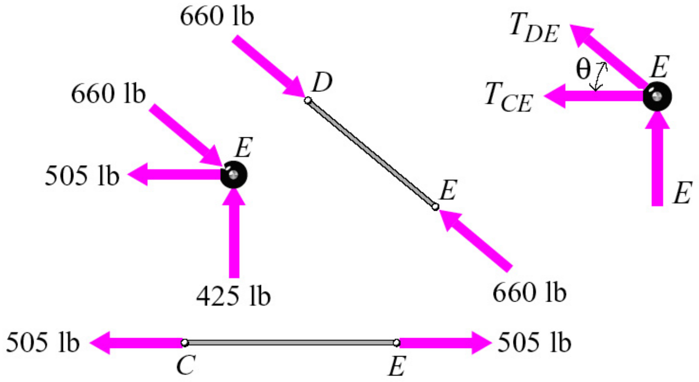
Essential Concepts and Statics Review



Free body diagram of pin E

$$\sum F_x = 0 \Rightarrow -T_{CE} - T_{DE}\cos\theta = 0$$

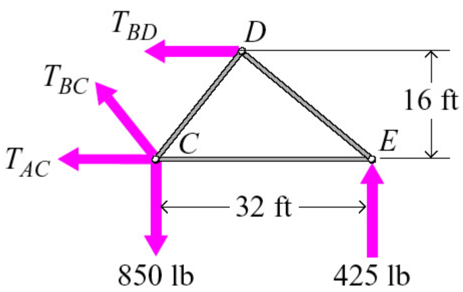
$$\sum F_y = 0 \Rightarrow E + T_{DE}\sin\theta = 425 + T_{DE}\sin\theta = 0$$



Since $\theta = \tan^{-1}(16/19) = 40.10^{\circ}$

$$\Rightarrow \quad T_{DE} = -659.8\ lb \quad T_{CE} = 504.7\ lb$$

Summing moments about point C

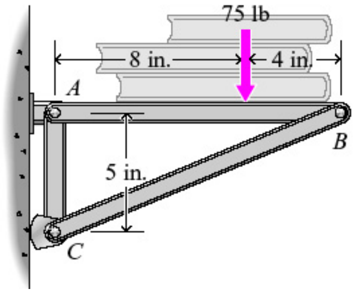


$$\sum M_C = 0 \Rightarrow T_{BD}(16) + 425(32) = 0$$

$$T_{BD} = -850\ lb = 850\ lb\ (C)$$

Essential Concepts and Statics Review

(Example) The weight of books on a shelf bracket is 75 lb. Determine all forces on all three members of this frame.



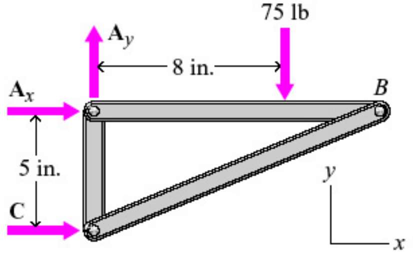
(Sol) Using $\sum \vec{F} = 0 \;\; and \;\; \sum \vec{M} = 0$

$$\sum M_A = 0 \;\Rightarrow\; 5C - 8(75) = 0$$

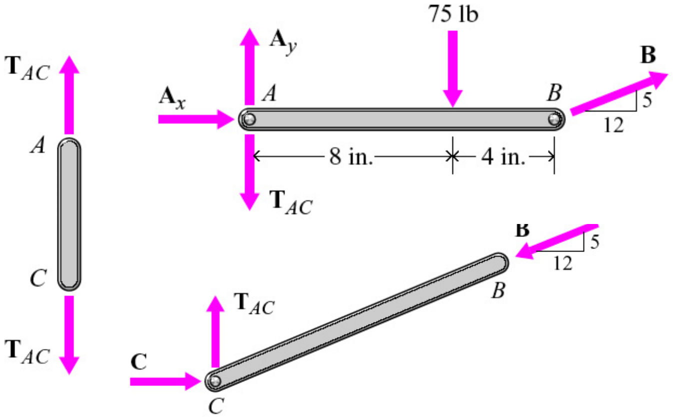
$$\sum F_x = 0 \;\Rightarrow\; A_x + C = 0$$

$$\sum F_y = 0 \;\Rightarrow\; A_y - 75 = 0$$

$$\Rightarrow \quad A_x = -120.0\,lb \quad A_y = 75.0\,lb \quad C = 120.0\,lb$$



Dismember the bracket and draw free body diagrams of each member

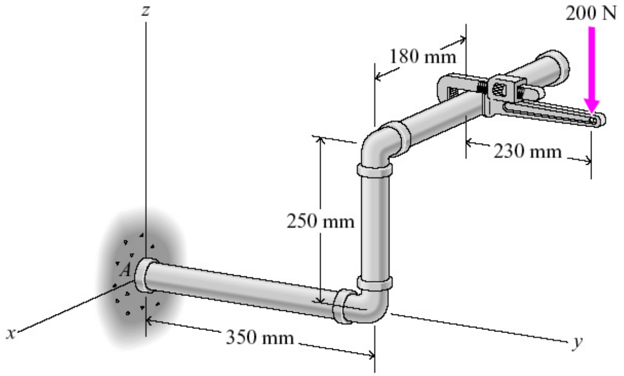


$$\sum M_B = 0 \;\Rightarrow\; 4(75) + 12T_{AC} - 12(75) = 0$$

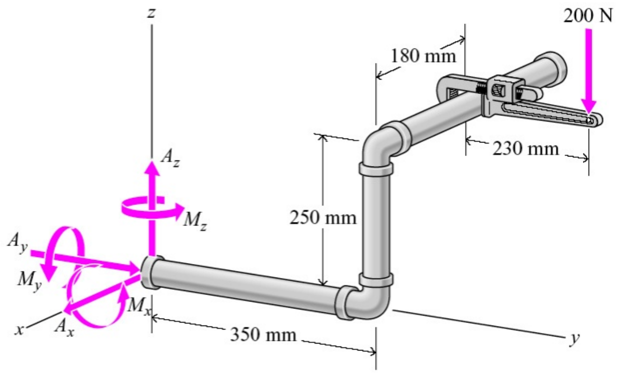
$$\sum M_A = 0 \;\Rightarrow\; 12\left[\left(5/13\right)B\right] - 8(75) = 0$$

$$\Rightarrow \quad T_{AC} = 50.0\,lb \quad B = 130.0\,lb$$

(Example) Determine reactions at support A neglecting the weights of the pipe and the wrench.



(a)



(b)

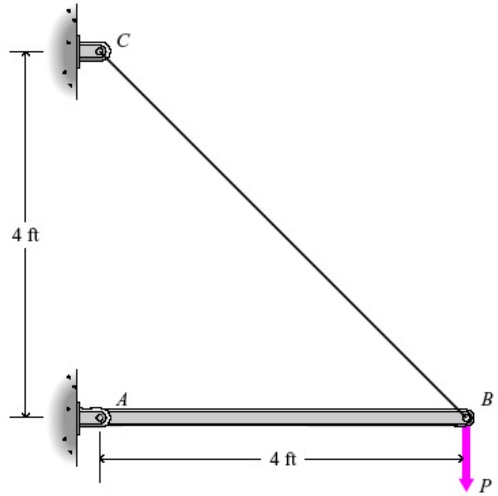
(Sol)

(a) Using $\sum F = 0$ and $\sum M = 0$

$$\sum F_x = 0 \Rightarrow A_x = 0$$
$$\sum F_y = 0 \Rightarrow A_y = 0$$
$$\sum F_z = 0 \Rightarrow A_z - 200 = 0$$
$$\sum M_x = 0 \Rightarrow M_x - 200\,(0.350 + 0.230) = 0$$
$$\sum M_y = 0 \Rightarrow M_y - 200\,(0.18) = 0$$
$$\sum M_z = 0 \Rightarrow M_z = 0$$

$$A_x = 0\,N \quad A_y = 0\,N \quad A_z = 200\,N$$
$$M_x = 116.0\,N \cdot m \quad M_y = 36.0\,N \cdot m \quad M_z = 0\,N \cdot m$$

(Example) Determine the forces at A and B on rigid structure AB subjected to a 5000-lb force.



(Sol)

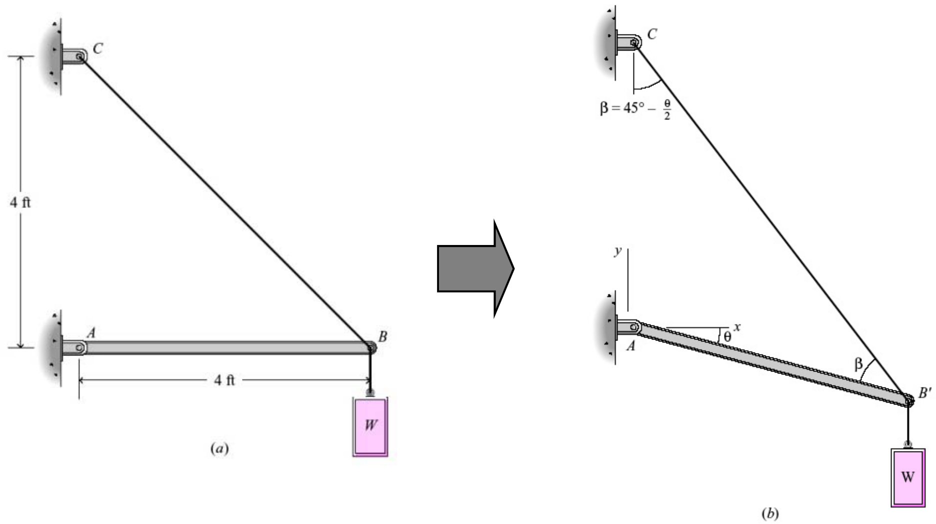
Using $\sum \vec{F} = 0 \ \ and \ \ \sum \vec{M} = 0$

$$\sum F_x = 0 \Rightarrow A_x - T\cos 45 = 0$$
$$\sum F_y = 0 \Rightarrow A_y + T\sin 45 - 5000 = 0$$
$$\sum M_z = 0 \Rightarrow (T\sin 45)(4) - 5000(4) = 0$$

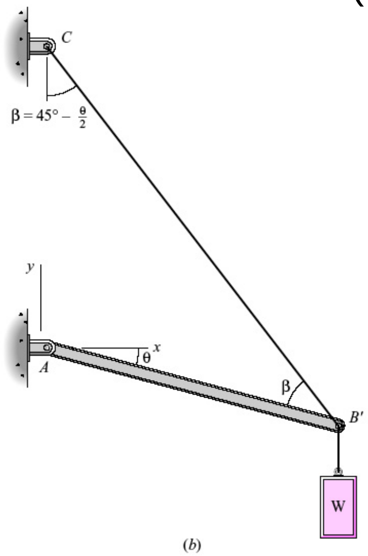
$$\Rightarrow \quad A_x = 5000 \ lb \quad A_y = 0.04795 \ lb \approx 0 \ lb \quad T = 7071 \ lb$$

- Equilibrium of a deformable body
  - Interaction between loads acting on a body, deformations of the body, and the geometry of the free body diagram makes the solution of the deformable body problems more complex
  - Solution requires either a trial-and-error solution or a numerical solution or an iterative solution method

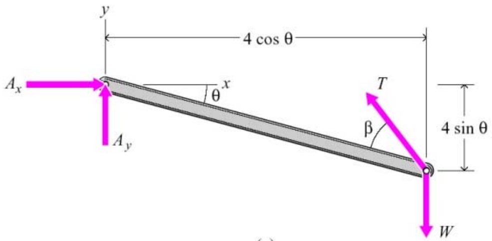
(Example) 5000 lb weight is to be supported by AB (rigid bar) and BC (nonrigid wire). Using $T_{BC}=k\delta$ and $\delta=L_f-L_i$, determine the tension in the wire after the load is applied for k=5000 lb/in.



(Example) Determine the tension in the wire if k=5000 lb/in.



(b)



(c)

(Sol)

$$\sum F_x = 0 \Rightarrow A_x - T\cos(45+\theta/2) = 0$$
$$\sum F_y = 0 \Rightarrow A_y + T\sin(45+\theta/2) - W = 0$$
$$\sum M_A = 0 \Rightarrow [T\sin(45+\theta/2)](4\cos\theta)$$
$$-[T\cos(45+\theta/2)](4\sin\theta) - W(4\cos\theta) = 0$$
$$\Rightarrow 4T\sin(45-\theta/2) - 4W\cos\theta = 0$$

Using the trigonometric relation, $\sin(A-B) = \sin A\cos B - \cos A\sin B$

Force-deformation relation is introduced

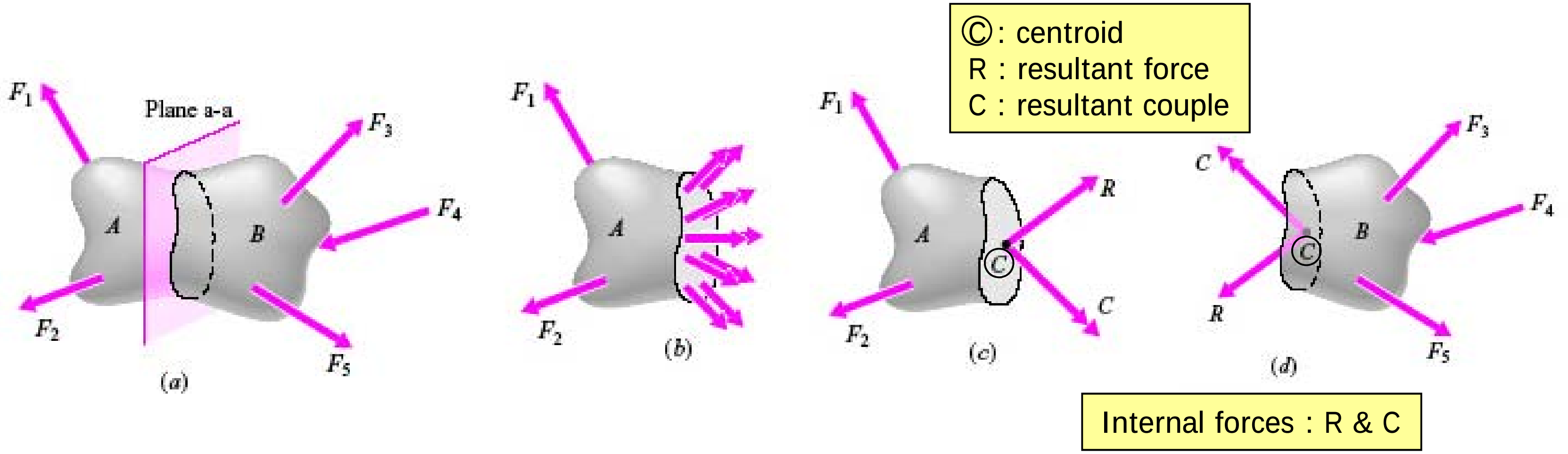
$$L_i = \sqrt{(4)^2 + (4)^2} = 5.657\ ft = 67.88\ in$$
$$L_f^2 = (48)^2 + (48)^2 - (2)(48)(48)\cos(90+\theta)$$
$$\Rightarrow L_f = 67.88\sqrt{1+\sin\theta}\ in$$
$$\delta = L_f - L_i = 67.88\sqrt{1+\sin\theta} - 67.88$$
$$= 67.88\left[\sqrt{1+\sin\theta} - 1\right] in$$

Thus, $$67.88k\left(\sqrt{1+\sin\theta} - 1\right)\sin(45-\theta/2) = W\cos\theta$$

If k is 5000 lb/in, θ=2.465deg, T=7221 lb, A_x= 4995 lb, A_y = -215 lb

- Internal forces

  - Forces that exist throughout the interior of a body



$$\sum \vec{F} = 0 \; : \; \vec{F}_1 + \vec{F}_2 + \vec{R} = 0$$

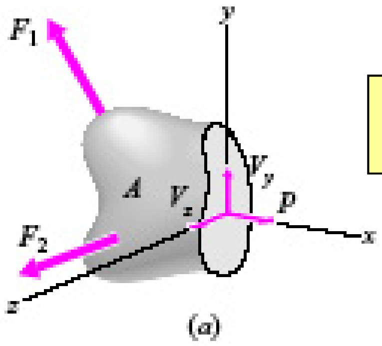
$$\sum \vec{M}_C = 0 \; : \; \vec{M}_1 + \vec{M}_2 + \vec{C} = 0$$

where M_1 and M_2 are the moments of F_1 and F_2 about the centroid.

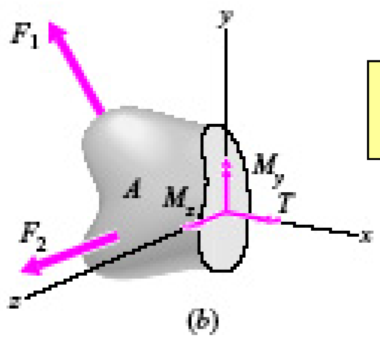
We find the resultant of the internal force system using the equilibrium equations. However, we can not find the exact distribution of the internal forces without information of the deformation of the body.

Essential Concepts and Statics Review

- Resolve the resultants R and C into components along and perpendicular to the section



P : Normal force
V : Shear force

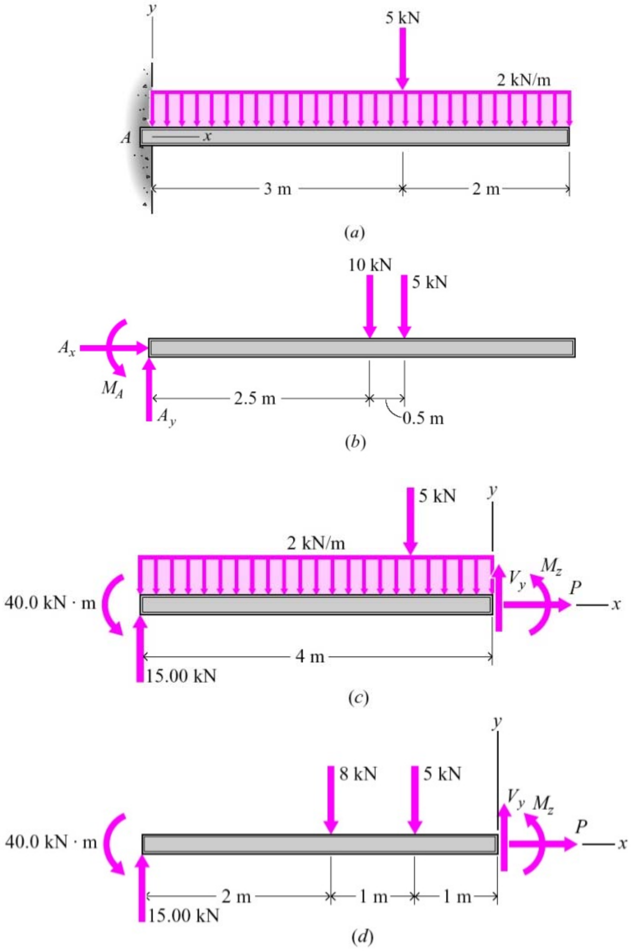


T : Twisting moment
M : Bending moment

- Components of the internal force system can be found using the equilibrium equations

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$
$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

(Example) Determine (a) the support reactions and (b) the internal forces on a section 4m to the right of the support A.



(Sol)

(a) Using $\sum \vec{F} = 0 \;\; and \;\; \sum \vec{M} = 0$

$$\sum F_x = 0 \Rightarrow A_x = 0$$

$$\sum F_y = 0 \Rightarrow A_y - 10 - 5 = 0$$

$$\sum M_A = 0 \Rightarrow M_A - 10(2.5) - 5(3) = 0$$

$$\Rightarrow \quad A_x = 0\,kN \quad A_y = 15\,kN \quad M_A = 40\,kN \cdot m$$

(b)

$$\sum F_x = 0 \Rightarrow P = 0$$

$$\sum F_y = 0 \Rightarrow 15 - 8 - 5 + V_y = 0$$

$$\sum M_z = 0 \Rightarrow 40 - 15(4) + 8(2) + 5(1) + M_z = 0$$

$$\Rightarrow \quad P = 0\,kN \quad V_y = -2\,kN \quad M_z = -1\,kN \cdot m$$

(Homework)

(1-8), (1-9), (1-22), (1-34), (1-56), (1-61)