제 2장 열전도 방정식

2-1. 서 론

2-3. 1차원 열전도 방정식

ⅰ) 직각 좌표계

$$\frac{\partial}{\partial x}(k\frac{\partial T}{\partial x}) + \dot{g} = \rho C\frac{\partial T}{\partial t}$$

a) 정상 상태 $\frac{\partial T}{\partial t} = 0$

b) 열발생이 없는 경우 $\dot{g} = 0$

ⅱ) 원통 좌표계

$$\frac{1}{r}\frac{\partial}{\partial r}(rk\frac{\partial T}{\partial r}) + \dot{g} = \rho C\frac{\partial T}{\partial t}$$

ⅲ) 구 좌표계

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2k\frac{\partial T}{\partial r}) + \dot{g} = \rho C\frac{\partial T}{\partial t}$$

2-4. 일반적인 열전도 방정식

ⅰ) 직각 좌표계

$$\frac{\partial}{\partial x}(k\frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(k\frac{\partial T}{\partial y}) + \frac{\partial}{\partial z}(k\frac{\partial T}{\partial z}) + \dot{g} = \rho C\frac{\partial T}{\partial t}$$

ⅱ) 원통 좌표계

$$\frac{1}{r}\frac{\partial}{\partial r}(kr\frac{\partial T}{\partial r}) + \frac{1}{r^2}\frac{\partial}{\partial \phi}(k\frac{\partial T}{\partial \phi}) + \frac{\partial}{\partial z}(k\frac{\partial T}{\partial z}) + \dot{g} = \rho C\frac{\partial T}{\partial t}$$

ⅲ) 구 좌표계

$$\frac{1}{r^2}\frac{\partial}{\partial r}(kr^2\frac{\partial T}{\partial r}) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}(k\frac{\partial T}{\partial \phi}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}(ksin\theta\frac{\partial T}{\partial \theta}) + \dot{g} = \rho C\frac{\partial T}{\partial t}$$

2-5. 경계조건과 초기조건

1. 주어진 온도 경계 조건

$$T(0,t) = T_1$$
$$T(L,t) = T_2$$

2. 주어진 열 유속 경계조건

$$\dot{q} = -k\frac{\partial T}{\partial x} = \dot{q}$$

i) 단열경계 $\dot{q} = 0$ $\quad \frac{\partial T}{\partial t} = 0$

3. 대류 경계조건

$$-k\frac{\partial T(0,t)}{\partial x} = h_1[T_\infty - T(0,t)] \text{ 또는}$$
$$-k\frac{\partial T(L,t)}{\partial x} = h_2[T(L,t) - T_\infty]$$

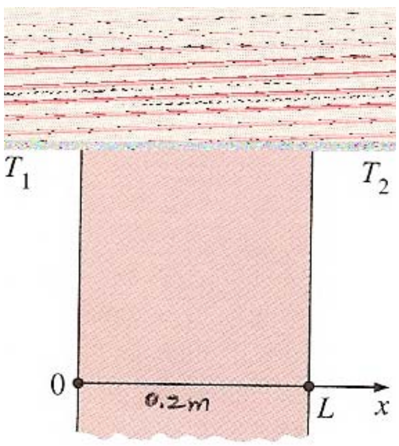
4. 복사 경계조건

$$-k\frac{\partial T(0,t)}{\partial x} = \epsilon_1\sigma[T_s^{\ 4} - T(0,t)^4] \text{ 또는}$$
$$-k\frac{\partial T(L,t)}{\partial x} = \epsilon_2\sigma[T(L,t)^4 - T_s^{\ 4}]$$

5. 접촉면 경계조건

접촉면에서 $T_A = T_B$ 와
$\dot{q}_A = \dot{q}_B$

2-6. 1차원 정상 열전도 문제 해



(Ex 2-11) 평면 벽에서의 열전도

$k = 1.2\ W/m \cdot {}^\circ C$

$A = 15m^2$

$T_1 = 120\,{}^\circ C$

$T_2 = 50\,{}^\circ C$

$a)$ $x = 0.1m$의 온도

$b)$ 열전도율

(해)

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) = 0 \quad \Rightarrow \quad \frac{d^2 T}{dx^2} = 0$$

$T(0) = 120^\circ C \qquad T(L) = 50^\circ C$

$T(x) = C_1 x + C_2$

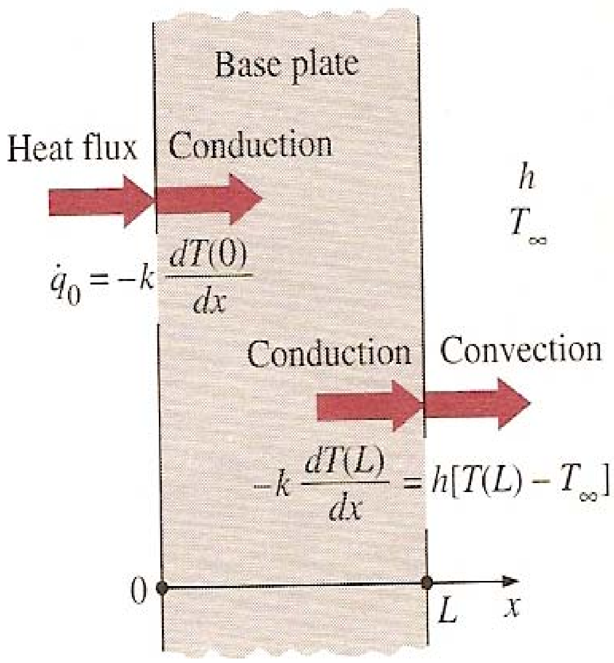
경계조건을 대입하면

$$T(x) = \frac{T_2 - T_1}{L} x + T_1$$

$$T(0.1) = \frac{120 - 50}{0.2} \bullet 0.1 + 50 = 85^\circ C$$

$$\dot{Q} = -kA\frac{dT}{dx} = -kA\frac{T_2 - T_1}{L} = 6300\,(W)$$

(Ex 2-13) 다리미 바닥판의 열전도



$L = 0.5cm = 0.005m$

$A = 300cm = 0.03m^2$

$k = 15\ W/m^\circ C$

$\dot{Q}_0 = 1200\ W$

$T_\infty = 20^\circ C$

$h = 80\ W/m^2 \bullet {}^\circ C$

(해) $$\frac{d^2T}{dx^2} = 0$$

$$\dot{q}_0 = -k\frac{dT(0)}{dx} = \frac{Q}{A} = 40000\ W/m^2$$

$$\dot{q}_1 = -k\frac{dT(L)}{dx} = h[T(L) - T_\infty]$$

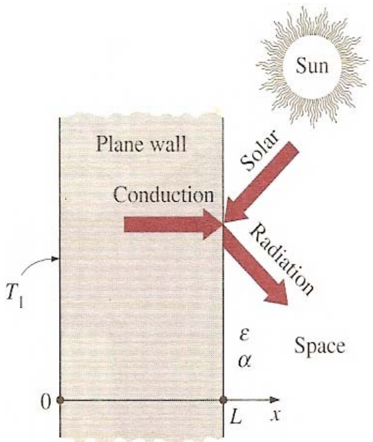
$$T = C_1x + C_2$$

$$\dot{q}_0 = -kC_1 \quad \Rightarrow \quad C_1 = -\frac{\dot{q}_0}{k}$$

$$-kC_1 = h[C_1L + C_2 - T_\infty] \quad C_2 = T_\infty + \frac{\dot{q}_0}{h} + \frac{\dot{q}_0}{k}L$$

$$\therefore\ T(x) = T_\infty + \dot{q}_0(\frac{L - x}{k} + \frac{1}{h})$$

(Ex 2-14) 태양열로 가열된 벽에서의 열전도



$$L = 0.06m,\ k = 1.2\ W/m \cdot {}^\circ C$$

$$\epsilon = 0.85\ ,\ \alpha = 0.26$$

$$\dot{q}_{solar} = 800\ W/m^2$$

$$T_1 = 300K,\ T_{space} = 0\,K$$

(해) $$\frac{d^2T}{dx^2} = 0$$

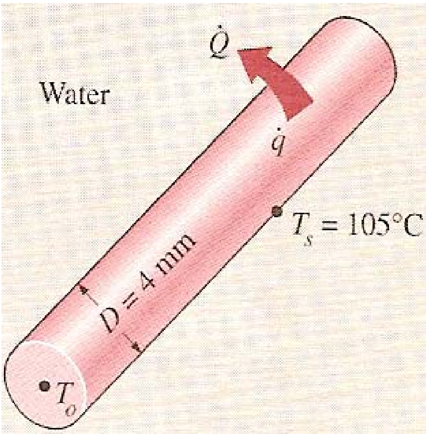
$$T(0) = T_1$$

$$-k\frac{dT(L)}{dx} = \epsilon\sigma[T(L)^4 - T_{space}^4] - \alpha\dot{q}_{solar}$$

C_1은 수치적으로 구한다.

2-7. 고체 내부의 열발생

(Ex 2-17) 전열기의 중심선에서의 온도



$$k = 15\ W/m \bullet {}^\circ C$$
$$D = 4m$$
$$L = 0.5m$$
$$\dot{Q}_{\geq n} = 2000\ W$$
$$T_s = 105\ {}^\circ C$$
$$T_0 = ?$$

(해) $\dot{g} = \dfrac{\dot{Q}_{gn}}{V} = \dfrac{\dot{Q}_{gn}}{\dfrac{\pi D^2}{4} \bullet L} = 0.318 \times 10^9$

$$\frac{1}{r}\frac{\partial}{\partial r}(rk\frac{\partial T}{\partial r}) + \dot{g} = 0$$

$$T(r_0) = T_s$$

$$\frac{dT(0)}{dr} = 0$$

$$rk\frac{dT}{dr} = -\dot{g}\frac{r^2}{2} + C_1$$

$$\frac{dT}{dr} = -\frac{\dot{g}r}{2k} + C_1$$

$\dfrac{dT(0)}{dr} = 0$ 이므로 $C_1 = 0$

$$\frac{dT}{dr} = -\frac{\dot{g}r}{2k} \Rightarrow T = -\frac{\dot{g}r^2}{4k} + C_2$$

$$T(r_0 = \frac{D}{2}) = -\frac{\dot{g}r_0^2}{4k} + C_2 = T_s$$

$$\therefore C_2 = T_s + \frac{\dot{g}}{4k}r_0^2$$

$$\therefore T(r) = T_s + \frac{\dot{g}}{4k}(r_0^2 - r^2)$$

(Ex 2-19) 2중 매체 내에서의 열전도

$k_1 = 15\ W/m \cdot {}^\circ C$

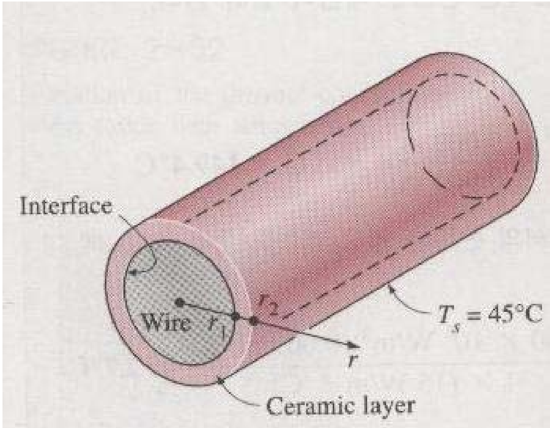
$k_2 = 1.2\ W/m \cdot {}^\circ C$

$r_1 = 0.002\,m$

$r_2 = 0.007\,m$

$\dot{g} = 50\ W/cm^3 = 50 \times 10^6\ W/m^3$

$T_s = 45\ {}^\circ C$



(해) $\dfrac{1}{r}\dfrac{d}{dr}(k\dfrac{dT}{dr}) + \dot{g} = 0$ in wire

$T(r_1) = T_1$

$\dfrac{dT(0)}{dr} = 0$

$\therefore T_{wire}(r) = T_1 + \dfrac{\dot{g}}{4k_1}(r_1^2 - r^2)$ ——(1)

$\dfrac{1}{r}\dfrac{d}{dr}(kr\dfrac{dT}{dr}) = 0$ in ceramic

$T(r_1) = T_1$

$T(r_2) = T_s$

$\dfrac{d}{dr}(r\dfrac{dT}{dr}) = 0 \Rightarrow r\dfrac{dT}{dr} = C_1$

$\dfrac{dT}{dr} = \dfrac{C_1}{r}$

$T = C_1 \ln r + C_2$

$T(r_1) = C_1 \ln r_1 + C_2 = T_1$

$T(r_2) = C_1 \ln r_2 + C_2 = T_s$

$\therefore T_{ceramic}(r) = \dfrac{\ln(r/r_1)}{\ln(r_2/r_1)}(T_s - T_1) + T_1$ ——(2)

경계면에서의 열유속은 같다. 즉,

$$-k\frac{dT}{dr} = -k_2\frac{dT}{dr} \Rightarrow T_1 = \frac{\dot{g}r_1^2}{2k_2}ln\frac{r_2}{r_1} + T_s$$

$$\therefore\ T_{wire}(0) = T_1 + \frac{gr_1^2}{4k_1}$$

2-8. 가변열전도도

물질의 열전도도는 온도에 따라 변한다 → 평균값 사용.

$$k_{ave} = \frac{\int_{T_1}^{T_2} k(T)dT}{T_2 - T_1}$$

(Ex 2-20) k(T)인 벽에서의 온도 변화

$k(T) = k_0(1 + \beta T)$에서의 열전달률과 벽의 온도분포

i) $\dot{Q} = k_{ave}A\frac{T_2 - T_1}{L}$

$$k_{ave} = \frac{\int_{T_1}^{T_2} k_0(1+\beta T)dT}{T_2 - T_1} = k_0(1+\beta\frac{T_2+T_1}{2})$$

ii) $\frac{d}{dx}(k\frac{dT}{dx}) = 0$

$T(0) = T_1,\ \ T(L) = T_2$

$k\frac{dT}{dx} = C_1$

$k_0(1+\beta T)dT = C_1 dx \qquad 1+\beta T \Rightarrow \theta$

$\beta dT \Rightarrow d\theta$

$$\frac{1}{\beta}\theta d\theta = \frac{C_1}{k_0}dx \Rightarrow \theta \bullet d\theta = \frac{\beta C_1}{k_0}dx$$

$$\frac{\theta^2}{2} = \frac{\beta C_1}{k_0}x + C_2 \Rightarrow \theta^2 = \frac{2\beta C_1}{k_0}x + C_2^{'}$$

$$\theta = \pm\sqrt{\frac{2\beta C_1}{k_0}x + C_2^{'}}$$

$$1+\beta T = \pm\sqrt{\frac{2\beta C_1}{k_0}x + C_2^{'}}$$

$$\beta T = -1 \pm\sqrt{\frac{2\beta C_1}{k_0}x + C_2}$$

$$\therefore\ T = -\frac{1}{\beta} \pm\sqrt{\frac{2C_1}{k_0\beta}x + \frac{C_2}{\beta^2}}$$

$T(0) = T_1$과 $T(L) = T_2$를 代入하면

C_1과 C_2 계산

결과적으로

$$T(x) = -\frac{1}{\beta} \pm\sqrt{\frac{1}{\beta^2} - \frac{2k_{ave}}{\beta k_0}\frac{x}{L}(T_1 - T_2) + T_1^2 + \frac{2}{\beta}T_1}$$