#### LECTURE 5: Cost Theory 비용이론

- Meaning and Measurements of Cost:
  - Which Costs Matter?
- Cost Minimization
- Costs in the Short Run and in the Long Run: Long-Run versus Short-Run Cost Curves
- Production with Two Outputs—Economies of Scope
- Dynamic Changes in Costs—The Learning Curve
- Estimating and Predicting Cost

#### Cost of production: The long run total cost function

- Given production technology, that is, input-output relation, and markets for these inputs
- How to combine factor inputs determines the firm's cost of production
- The long run total cost function relates minimized total cost to output, Q, and to the factor prices (w and r).
  - $\Box TC(Q,w,r) = wL^*(Q,w,r) + rK^*(Q,w,r)$
  - Where: L\* and K\* are the long run input demand functions

# Meaning of costs & Measuring Cost

Meaning of costs, and how to measure them

- For example, if a firm has to rent equipment or buildings, is the rent they pay a cost?
- What if a firm owns its own equipment or building?
- How are costs calculated here?

# Measuring Cost: Which Costs Matter?

- Accountants tend to take a retrospective view of firms costs, where as economists tend to take a forward-looking view
- □ Accounting Cost 회계적 비용
  - Actual expenses plus depreciation charges for capital equipment
- □ Economic Cost 경제(학)적 비용
  - Cost to a firm of utilizing economic resources in production, including opportunity cost

# Measuring Cost: Which Costs Matter?

- Economic costs distinguish between costs the firm can control and those it cannot
  - Concept of opportunity cost plays an important role
- □ Opportunity cost 기회비용
  - Cost associated with opportunities that are foregone when a firm's resources are not put to their highestvalue use.

# **Opportunity Cost**

#### An Example

- A firm owns its own building and pays no rent for office space
- Does this mean the cost of office space is zero?
- The building could have been rented instead
- Foregone rent is the opportunity cost of using the building for production and should be included in economic costs of doing business

**Opportunity Cost** 

- A person starting their own business must take into account the opportunity cost of their time
  - Could have worked elsewhere making a competitive salary
- Accountants and economists often treat depreciation differently as well

# Measuring Cost: Which Costs Matter?

- Although opportunity costs are hidden and should be taken into account, sunk costs should not
- Sunk Cost
  - Expenditure that has been made and cannot be recovered
  - Should not influence a firm's future economic decisions.

# Sunk Cost 매몰비용

- Firm buys a piece of equipment that cannot be converted to another use
- Expenditure on the equipment is a sunk cost
  - Has no alternative use so cost cannot be recovered opportunity cost is zero
  - Decision to buy the equipment might have been good or bad, but now does not matter

### Short Run costs

- Some costs vary with output, while some remain the same no matter amount of output
- Total cost can be divided into TC & VC
- □ total cost (TC or C) 총비용
  - Total economic cost of production, consisting of fixed and variable costs.

#### fixed cost (FC) 고정비용

 Cost that does not vary with the level of output and that can be eliminated only by shutting down.

#### variable cost (VC) 변동비용

Cost that varies as output varies

#### Fixed and Variable Costs

- Total output is a function of variable inputs and fixed inputs.
- Therefore, the total cost of production equals the fixed cost (the cost of the fixed inputs) plus the variable cost (the cost of the variable inputs), or...

# TC = FC + VC

#### Fixed and Variable Costs

- Which costs are variable and which are fixed depends on the time horizon
- Short time horizon most costs are fixed
- Long time horizon many costs become variable
- In determining how changes in production will affect costs, must consider if affects fixed or variable costs

#### Fixed Cost Versus Sunk Cost

- Fixed cost and sunk cost are often confused
  - Sunk costs are costs that have been incurred and cannot be recovered.
  - An example is the cost of R&D to a pharmaceutical company to develop and test a new drug and then, if the drug has been proven to be safe and effective, the cost of marketing it.
  - Whether the drug is a success or a failure, these costs cannot be recovered and thus are sunk.
  - Amortizing Sunk Costs매몰비용의 이연상각
    - amortization 이연상각 Policy of treating a one-time expenditure as an annual cost spread out over some number of years.

# Measuring Cost: Which Costs Matter?

- Personal Computers
  - Most costs are variable
  - Largest component: labor
- Software
  - Most costs are sunk
  - Initial cost of developing the software

### Marginal and Average Cost

- In completing a discussion of costs, must also distinguish between
  - Average Cost (ATC or AC) 총평균비용
  - Marginal Cost (MC) 한계비용
- After definition of costs is complete, one can consider the analysis between short-run and longrun costs

- Marginal Cost (MC):
  - The cost of expanding output by one unit.
  - Fixed cost have no impact on marginal cost, so it can be written as:

$$MC = \frac{\Delta VC}{\Delta q} = \frac{\Delta TC}{\Delta q}$$

#### Average Total Cost (ATC)

Cost per unit of output, consisting of

- average fixed cost (AFC) 평균고정비용
- Fixed cost divided by the level of output.
- average variable cost (AVC) 평균변동비용
- Variable cost divided by the level of output.

#### □ ATC, AFC, AVC

$$ATC = \frac{TC}{q} = AFC + AVC$$
$$ATC = \frac{TC}{q} = \frac{TFC}{q} + \frac{TVC}{q}$$

- All the types of costs relevant to production have now been discussed
- Can now discuss how they differ in the long and short run
- Costs that are fixed in the short run may not be fixed in the long run
- Typically in the long run, most if not all costs are variable

### Determinants of Short-run Costs

- The rate at which these costs increase depends on the nature of the production process
  - The extent to which production involves diminishing returns to variable factors
- Diminishing returns to labor
  - When marginal product of labor is decreasing

### Determinants of Short-run Costs

- If marginal product of labor decreases significantly as more labor is hired
  - Costs of production increase rapidly
  - Greater and greater expenditures must be made to produce more output
- If marginal product of labor decreases only slightly as increase labor
  - Costs will not rise very fast when output is increased

# Determinants of Short-run Costs – An Example

- Assume the wage rate (w) is fixed relative to the number of workers hired.
- Variable costs is the per unit cost of extra labor times the amount of extra labor: wL

$$MC = \frac{\Delta VC}{\Delta q} = \frac{w\Delta L}{\Delta q}$$

# Determinants of Short-run Costs – An Example

Remembering that



And rearranging

 $\Delta L$  for a 1 unit  $\Delta Q = \frac{\Delta L}{\Lambda O} = \frac{1}{\Delta M P_L}$ 

# Determinants of Short-run Costs – An Example

We can conclude:



 ...and a low marginal product (MP) leads to a high marginal cost (MC) and vise versa.

#### Cost Curves for a Firm



#### **Cost Curves**

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Output (units/yr)

#### Cost Curves

- When MC is below AVC, AVC is falling
- □ When MC is above AVC, AVC is rising
- When MC is below ATC, ATC is falling
- □ When MC is above ATC, ATC is rising
- Therefore, MC crosses AVC and ATC at the minimums
  - The Average Marginal relationship

#### Cost Curves for a Firm



# Cost in the Long Run

- □ In the long run a firm can change all of its inputs
- The Cost-Minimizing Input Choice 비용최소화 (하는) 생산
  - In making cost minimizing choices, must look at the cost of using capital and labor in production decisions

### Cost in the Long Run

#### The Price of Capital 자본의 가격

- Capital is either rented/leased or purchased
  - We will consider capital rented as if it were purchased
  - The price of capital is its user cost, given by r = Depreciation rate + Interest rate.
  - The Rental Rate of Capital 자본의 임대료
    rental rate
    - Cost per year of renting one unit of capital.

#### User Cost of Capital 자본의 사용자비용

□ The user cost of capital must be considered

- The annual cost of owning and using the airplane instead of selling or never buying it
- Sum of the economic depreciation and the interest (the financial return) that could have been earned had the money been invested elsewhere

### Cost in the Long Run

- Example: Assume Delta is considering purchasing an airplane for 150 million
  - Plane lasts for 30 years
  - 5 per year economic depreciation for the plane
- User Cost of Capital = Economic Depreciation + (Interest Rate)\*(Value of Capital)
  - = 5 mil + (.10)(150 mil depreciation)
  - Year 1 = 5 million + (.10)(150 million) = 20 million
  - Year 10 = 5 million +(.10)(100 million) = 15 million

### Cost in the Long Run

- User cost can also be described as;
  - Rate per dollar of capital, r
  - r = Depreciation Rate + Interest Rate
- In our example, depreciation rate was 3.33% and interest was 10% so

□ r = 3.33% + 10% = 13.33%

# Cost Minimizing Input Choice

- How do we put all this together to select inputs to produce a given output at minimum cost?
- Assumptions
  - Two Inputs: Labor (L) & capital (K)
  - Price of labor: wage rate (w)
  - The price of capital
    - r = depreciation rate + interest rate
    - Or rental rate if not purchasing
    - These are equal in a competitive capital market

### Cost in the Long Run

- □ The Isocost Line 등비용선
  - A line showing all combinations of L & K that can be purchased for the same cost
  - Total cost of production is sum of firm's labor cost, wL and its capital cost rK

$$C = wL + rK$$

For each different level of cost, the equation shows another isocost line

### Cost in the Long Run

Rewriting C as an equation for a straight line:

$$\square \mathsf{K} = \mathsf{C}/\mathsf{r} - (\mathsf{w}/\mathsf{r})\mathsf{L}$$

- Slope of the isocost:
  - -w/r is the ratio of the wage rate to rental cost of capital.
  - This shows the rate at which capital can be substituted for labor with no change in cost.
## **Choosing Inputs**

- We will address how to minimize cost for a given level of output by combining isocosts with isoquants
- We choose the output we wish to produce and then determine how to do that at minimum cost
  - Isoquant is the quantity we wish to produce
  - Isocost is the combination of K and L that gives a set cost

## Producing a Given Output at Minimum Cost



# Input Substitution When an Input Price Change

- If the price of labor changes, then the slope of the isocost line change, w/r
- It now takes a new quantity of labor and capital to produce the output
- If price of labor increases relative to price of capital, and capital is substituted for labor

## Input Substitution When an Input Price Change



How does the isocost line relate to the firm's production process?

$$MRTS = -\frac{\Delta K}{\Delta L} = \frac{MP_{L}}{MP_{K}}$$

Slope of isocost line =  $\frac{\Delta K}{\Delta L} = -\frac{W}{r}$ 

$$\frac{MP_L}{MP_K} = \frac{W}{r}$$
 when firm minimizes cost

The minimum cost combination can then be written as:

$$\frac{MP_L}{W} = \frac{MP_K}{r}$$

Minimum cost for a given output will occur when each dollar of input added to the production process will add an equivalent amount of output.

- □ If w = 10, r = 2, and  $MP_L = MP_K$ , which input would the producer use more of?
  - Labor because it is cheaper
  - Increasing labor lowers MP<sub>L</sub>
  - Decreasing capital raises MP<sub>K</sub>
  - Substitute labor for capital until

$$\frac{MP_L}{w} = \frac{MP_K}{r}$$

- Cost minimization with Varying Output Levels
  - For each level of output, there is an isocost curve showing minimum cost for that output level
  - A firm's expansion path shows the minimum cost combinations of labor and capital at each level of output.
  - $\blacksquare$  Slope equals  $\Delta K/\Delta L$

## A Firm's Expansion Path 확장경로



## Expansion Path & Long-run Costs

- Firms expansion path has same information as longrun total cost curve
- □ To move from expansion path to LR cost curve
  - Find tangency with isoquant and isocost
  - Determine min cost of producing the output level selected
  - Graph output-cost combination

## A Firm's Long-Run Total Cost Curve



#### **LECTURE 6: Cost-2**

- Long-Run versus Short-Run Cost Curves
- Production with Two Outputs—Economies of Scope
- Dynamic Changes in Costs—The Learning Curve
- Estimating and Predicting Cost

- In the short run some costs are fixed
- In the long run firm can change anything including plant size
  - Can produce at a lower average cost in long run than in short run
  - Capital and labor are both flexible
- We can show this by holding capital fixed in the short run and flexible in long run

## The Inflexibility of Short-Run Production



- □ Long-Run Average Cost (LAC) 장기평균비용
  - Most important determinant of the shape of the LR AC and MC curves is relationship between scale of the firm's operation and inputs required to min cost
- 1. Constant Returns to Scale
  - If input is doubled, output will double
  - AC cost is constant at all levels of output.

#### Long-Run Average Cost

- Iong-run average cost curve (LAC) 장기평균비용 곡선 Curve relating average cost of production to output when all inputs, including capital, are variable.
  - eshort-run average cost curve (SAC) 단기평균비용 곡선 Curve relating average cost of production to output when level of capital is fixed.
  - long-run marginal cost curve (LMC) 장기한계비용 곡선 Curve showing the change in long-run total cost as output is increased incrementally by 1 unit.

- 2. Increasing Returns to Scale
  - If input is doubled, output will more than double
  - AC decreases at all levels of output.
- 3. Decreasing Returns to Scale
  - If input is doubled, output will less than double
  - AC increases at all levels of output

- □ In the long-run:
  - Firms experience increasing and decreasing returns to scale and therefore long-run average cost is "U" shaped.
  - Source of U-shape is due to returns to scale instead of decreasing returns to scale like the short run curve
  - Long-run marginal cost curve measures the change in long-run total costs as output is increased by 1 unit

 Long-run marginal cost leads long-run average cost:
 If LMC < LAC, LAC will fall</li>
 If LMC > LAC, LAC will rise
 Therefore, LMC = LAC at the minimum of LAC
 In special case where LAC if constant, LAC and LMC are equal

## Long-Run Average and Marginal Cost



- As output increases, firm's AC of producing is likely to decline to a point
  - 1. On a larger scale, workers can better specialize
  - Scale can provide flexibility managers can organize production more effectively
  - Firm may be able to get inputs at lower cost if can get quantity discounts. Lower prices might lead to different input mix

- □ At some point, AC will begin to increase
  - 1. Factory space and machinery may make it more difficult for workers to do their job efficiently
  - 2. Managing a larger firm may become more complex and inefficient as the number of tasks increase
  - 3. Bulk discounts can no longer be utilized. Limited availability of inputs may cause price to rise

- When input proportions change, the firm's expansion path is no longer a straight line
  Concept of return to scale no longer applies
- Economies of scale reflects input proportions that change as the firm change its level of production

#### Economies and Diseconomies of Scale

- Economies of Scale <sup>규모의</sup> 경제
- Increase in output is greater than the increase in inputs.
  Diseconomies of Scale <sup>규모의</sup> 불경제
  - Increase in output is less than the increase in inputs.
- U-shaped LAC shows economies of scale for relatively low output levels and diseconomies of scale for higher levels

## Cost Functions & Measurement of Scale Economies

Scale Economy Index (SCI)

 $\square$  EC = 1, SCI = 0: no economies or diseconomies of scale

- $\square$  EC > 1, SCI is negative: diseconomies of scale
- $\square$  EC < 1, SCI is positive: economies of scale

#### Increasing Returns to Scale

- Output more than doubles when the quantities of all inputs are doubled
- Economies of Scale 규모의 경제(성)
  - Doubling of output requires less than a doubling of cost

#### diseconomies of scale 규모의 불경제(성)

Situation in which a doubling of output requires more than a doubling of cost.

- Economies of scale are measured in terms of cost-output elasticity, E<sub>c</sub> (비용-생산 탄력성)
- EC is the percentage change in the cost of production resulting from a 1-percent increase in output

$$E_{C} = \frac{\Delta C/C}{\Delta Q/Q} = \frac{MC}{AC}$$

- $\square$  EC is equal to 1, MC = AC
  - Costs increase proportionately with output
  - Neither economies nor diseconomies of scale
- $\Box$  EC < 1 when MC < AC
  - Economies of scale
  - Both MC and AC are declining
- $\square$  EC > 1 when MC > AC
  - Diseconomies of scale
  - Both MC and AC are rising

- We will use short and long-run cost to determine the optimal plant size
- We can show the short run average costs for 3 different plant sizes
- This decision is important because once built, the firm may not be able to change plant size for a while

## Long-Run Cost with Constant Returns to Scale

- The optimal plant size will depend on the anticipated output
  - If expect to produce q<sub>0</sub>, then should build smallest plant: AC = 8
  - **I** If produce more, like  $q_1$ , AC rises
  - If expect to produce  $q_2$ , middle plant is least cost
  - $\square$  If expect to produce  $q_3$ , largest plant is best

## Long-Run Cost with Economies and Diseconomies of Scale



# Long-Run Cost with Constant Returns to Scale

- What is the firms' long-run cost curve?
  - Firms can change scale to change output in the longrun.
  - The long-run cost curve is the dark blue portion of the SAC curve which represents the minimum cost for any level of output.
  - Firm will always choose plant that minimizes the average cost of production

# Long-Run Cost with Constant Returns to Scale

- The long-run average cost curve envelopes the short-run average cost curves
- The LAC curve exhibits economies of scale initially but exhibits diseconomies at higher output levels

## Production with Two Outputs – Economies of Scope

- Many firms produce more than one product and those product are closely linked
- Examples:
  - Chicken farm--poultry and eggs
  - Automobile company--cars and trucks
  - University--Teaching and research

Production with Two Outputs – Economies of Scope

- Advantages
- 1. Both use capital and labor.
- 2. The firms share management resources.
- Both use the same labor skills and type of machinery.

# Production with Two Outputs – Economies of Scope 범위(의) 경제(성)

- □ Firms must choose how much of each to produce.
- The alternative quantities can be illustrated using product transformation curves
  - Curves showing the various combinations of two different outputs (products) that can be produced with a given set of inputs
### **Product Transformation Curve**



### Product Transformation Curve

- Product transformation curves are negatively slope
  - To get more of one output, must give up some of the other output
- Constant returns exist in this example
  - Second curve lies twice as far from origin as the first curve
- Curve is concave
  - Joint production has its advantages

# Production with Two Outputs – Economies of Scope

- There is no direct relationship between economies of scope and economies of scale.
  - May experience economies of scope and diseconomies of scale
  - May have economies of scale and not have economies of scope

## Production with Two Outputs – Economies of Scope

The degree of economies of scope (SC) can be measured by percentage of cost saved producing two or more products jointly:

$$SC = \frac{C(q_1) + C(q_2) - C(q_{1,q_2})}{C(q_{1,q_2})}$$

- $\square$  C(q<sub>1</sub>) is the cost of producing q<sub>1</sub>
- **C**( $q_2$ ) is the cost of producing  $q_2$
- $\Box$  C(q<sub>1</sub>,q<sub>2</sub>) is the joint cost of producing both products

# Production with Two Outputs – Economies of Scope

- With economies of scope, the joint cost is less than the sum of the individual costs
- Interpretation:
  - □ If SC > 0 Economies of scope
  - **If** SC < 0 Diseconomies of scope
  - The greater the value of SC, the greater the economies of scope

- Firms may lower their costs not only due to economies of scope, but also due to managers and workers become more experienced at their jobs
- As management and labor gain experience with production, the firm's marginal and average costs may fall

# Dynamic Changes in Costs – The Learning Curve 학습(경험) 곡선

#### Reasons

- 1. Speed of work increases with experience
- 2. Managers learn to schedule production processes more efficiently
- 3. More flexibility is allowed with experience. May include more specialized tools and plant organization
- Suppliers become more efficient passing savings to company

- The learning curve measures the impact of worker's experience on the costs of production.
- It describes the relationship between a firm's cumulative output and amount of inputs needed to produce a unit of output.

### The Learning Curve



## The Learning Curve

- The horizontal axis measures the cumulative number of hours of machine tools the firm has produced
- The vertical axis measures the number of hours of labor needed to produce each lot.

The learning curve in the figure is based on the relationship:

$$L = A + BN^{-\beta}$$

N = cumulative units of output produced

- L = labor input per unit of output
- A, B and  $\beta$  are constants
- A & B are positive and  $\beta$  is between 0 and 1

#### $\Box \text{ If } N = 1$

L equals A + B and this measures labor input to produce the first unit of output

- $\Box \ \mathsf{If} \ \beta = \mathbf{0}$ 
  - Labor input per unit of output remains constant as the cumulative level of output increases, so there is no learning

- □ If  $\beta$  > 0 and N increases,
  - L approaches A, and A represents minimum labor input/unit of output after all learning has taken place.
- $\square$  The larger  $\beta$ ,
  - The more important the learning effect.

### The Learning Curve



#### Observations

- 1. New firms may experience a learning curve, not economies of scale.
  - Should increase production of many lots regardless of individual lot size
- 2. Older firms have relatively small gains from learning.
  - Should produce its machines in very large lots to take advantage of lower costs associated with size

#### **Economies of Scale Versus Learning**



## The Learning Curve in Practice

#### Applying Learning Curves

- 1. To determine if it is profitable to enter an industry.
- 2. To determine when profits will occur based on plant size and cumulative output.

- Estimates of future costs can be obtained from a cost function, which relates the cost of production to the level of output and other variables that the firm can control.
- Suppose we wanted to derive the total cost curve for automobile production.

## Total Cost Curve for the Automobile Industry



□ A linear cost function might be:

$$VC = \beta Q$$

The linear cost function is applicable only if marginal cost is constant.

 $\blacksquare$  Marginal cost is represented by  $\beta$ 

If we wish to allow for a U-shaped average cost curve and a marginal cost that is not constant, we might use a quadratic cost function:



If the marginal cost curve is also not linear, we might use a cubic cost function:

 $VC = \beta Q + \gamma Q^2 + \delta Q^3$ 

#### **Cubic Cost Function**



#### Difficulties in Measuring Cost

- Output data may represent an aggregate of different type of products.
- 2. Cost data may not include opportunity cost.
- Allocating cost to a particular product may be difficult when there is more than one product line.