# 푸리에변환
# 신호의 주파수해석

# Fourier Representation

## ❑ Fourier Transforms

푸리에 변환은 신호의 주기성, 시간의 연속성 등에 따라 변환식의 형태가 나누어짐

| 시간상 성질 | 주기 | 비주기 |
|---|---|---|
| 연속시간 | 퓨리에 급수 (FS) | 퓨리에 변환 (FT) |
| 이산시간 | 이산시간 퓨리에 급수 (DTFS) | 이산시간 퓨리에 변환 (DTFT) |

# Summary of Fourier Transform

| | 이산시간 신호 | 연속시간 신호 |
|---|---|---|
| 주기신호 | $x[n] = \sum_{n=<N>} X[k]e^{jk\Omega_0 n}$<br>$X[k] = \frac{1}{N}\sum_{n=<N>} x[n]e^{-jk\Omega_0 n}$<br>$x[n] \xleftrightarrow{DTFS;\quad \Omega_0} X[k]$ | $x(t) = \sum_{k=-\infty}^{\infty} X[k]e^{jk\omega_0 t}$<br>$X[k] = \frac{1}{T}\int_0^T x(t)e^{-jk\omega_0 t}dt$<br>$x(t) \xleftrightarrow{FS;\omega_0} X[k]$ |
| 비주기신호 | $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Omega})e^{j\Omega n}d\Omega$<br>$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$<br>$x[n] \xleftrightarrow{DTFT} X(e^{j\Omega})$ | $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$<br>$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$<br>$x(t) \xleftrightarrow{FT} X(j\omega)$ |

# Fourier Representation

## Orthogonality of Sinusoidal Signals : Discrete Time Case

- **Sinuosoidal Function :**

$$\phi_k[n] = e^{jk\Omega_0 n}, \Omega_0 = \frac{2\pi}{N}$$

- **Inner Product :**

$$I_{k,m} = \sum_{n=<N>} \phi_k[n]\phi^*{}_m[n]$$

- **Orthogonality :**

$$I_{k,m} = \sum_{n=0}^{N-1} e^{j(k-m)\Omega_0 n} = \begin{cases} N, & k = m \\ \dfrac{1-e^{jk2\pi}}{1-e^{jk\Omega_0}} = 0 & k \neq m \end{cases}$$

# Fourier Representation

## Orthogonality of Sinusoidal Signals : Continuous Time Case

- **Sinuosoidal Function :** $\phi_k(t) = e^{jk\omega_0 t}, \omega_0 = \dfrac{2\pi}{T}$

- **Inner Product :** $I_{k,m} = \int_{<T>} \phi_k(t)\phi^*_m(t)dt$

- **Orthogonality :**

$$I_{k,m} = \int_0^T e^{j(k-m)\omega_0 t}dt = \begin{cases} T, & k = m \\ \dfrac{1}{j(k-m)\omega_0} e^{j(k-m)\omega_0 t}\Big|_0^T = 0, & k \neq m \end{cases}$$

# Fourier Representation : DTFS(1)

## ❑ Discrete-Time Fourier Series : Discrete Time Periodic Signal

- **Discrete Time Periodic Signal with Period N :** $x[n]$

- **Discrete Time Fourier Series :** $x[n] = \sum_{k=<N>} X[k] e^{jk\Omega_0 n}$

- **DTFS coefficient :** $X[k] = \frac{1}{N} \sum_{n=<N>} x[n] e^{-jk\Omega_0 n}$

$$x[n] \xleftrightarrow{DTFS;\quad \Omega_0} X[k]$$

# Fourier Representation : DTFS(2)

## Idea of Discrete-Time Fourier Series

**A periodic signal can be characterized by one period of that signal. If we consider one period of a signal as a vector, then vector space theory can be applied to describe a periodic signal.**

- **N-Periodic Signal :** $x[n], n = 0,1,\cdots,\infty$
- **N-Dimensional Vector :** $X = \begin{bmatrix} x[0] & x[1] & \cdots & x[N-1] \end{bmatrix}^T$

**N-Dimensional Vector, X, characterized fully the N-periodic signal x.**

# Fourier Representation : DTFS(3)

- **N-Dimensional Basis Vector :**

$$V_k = \frac{1}{\sqrt{N}}\begin{bmatrix} 1 & e^{jk\Omega_0} & \cdots & e^{jk\Omega_0(N-1)} \end{bmatrix}^T, k = 0,1,\cdots,N-1$$

- **Orthonormality** :

$$V_k \cdot V_m = \frac{1}{N}\sum_{n=<N>} V_k[n]V_m^*[n] = \frac{1}{N}\sum_{n=0}^{N-1} e^{j(k-m)\Omega_0 n} = \begin{cases} 1, & k = m \\ \dfrac{1}{N}\dfrac{1-e^{jk2\pi}}{1-e^{jk\Omega_0}} = 0, & k \neq m \end{cases}$$

**Since orthonormal N-dimensional vectors are independent,** $V_k, k = 0,1,\cdots,N-1$ **can be a basis for N-dimensional vector space.**

# Fourier Representation : DTFS(4)

- **Vector representation via linear combination of basis vectors:**

$$X = \sum_{k=0}^{N-1} a_k V_k$$

- **Component value $a_k$ of a vector $X$ along a basis vector $V_k$**

$$a_k = X \cdot V_k$$

$$a_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x[n] e^{-jk\Omega_0 n}$$

# Fourier Representation : DTFS(5)

- **Vector Representation via Linear Combination :**

$$X = \sum_{k=0}^{N-1} a_k V_k = \sum_{k=0}^{N-1} \left( \frac{1}{\sqrt{N}} a_k \times \sqrt{N} V_k \right)$$

$$= \sum_{k=0}^{N-1} \left( X[k] e^{jk\Omega_0 n} \right) \quad \textbf{with} \quad X[k] = \frac{1}{\sqrt{N}} a_k$$

$$x[n] = \sum_{n=<N>} X[k] e^{jk\Omega_0 n}$$

$$X[k] = \frac{1}{N} \sum_{n=<N>} x[n] e^{-jk\Omega_0 n}$$

$$\Longrightarrow x[n] = \sum_{k=0}^{N-1} X[k] e^{jk\Omega_0 n}, n = 0,1,2,\cdots, N-1$$

$$x[n] \xleftrightarrow{DTFS;\quad \Omega_0} X[k]$$

- **Fourier Coefficient** $X[k]$

$$a_k = X \cdot V_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x[n] e^{-jk\Omega_0 n} \Rightarrow X[k] = \frac{1}{\sqrt{N}} a_k$$

# Fourier Representation : DTFS(6)

- **Discrete Time Fourier Series of N-Periodic Signal**

$$x[n] = \sum_{n=<N>} X[k]e^{jk\Omega_0 n}, \Omega_0 = \frac{2\pi}{N}$$

$$X[k] = \frac{1}{N}\sum_{n=<N>} x[n]e^{-jk\Omega_0 n}$$

$$x[n] \xleftrightarrow{DTFS;\quad \Omega_0} X[k]$$

- **Fourier Coefficient** $X[k]$

$$X[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-jk\Omega_0 n}$$